On ESG Investing: Heterogeneous Preferences, Information, and Asset Prices

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Abstract

We study how environmental, social and governance (ESG) investing reshapes information aggregation and price formation. We develop a rational expectations equilibrium model in which traditional and green investors are informed about monetary and non-monetary risks but have distinct preferences over them. Because of the preference heterogeneity, traditional and green investors trade in opposite directions based on the same information and make the price noisier to each other. We show that an increase in the share of green investors and an improvement in the quality of non-monetary information can reduce overall price informativeness and increase firm's cost of capital. Our analyses provide a rich set of testable implications.

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1 Introduction

Over the recent years, financial markets have witnessed a rapidly increasing appetite for Environmental, Social and Governance (ESG), or sustainable, investing. Across a wide range of geographies and asset classes, a growing share of investors, both institutional and individual, are now integrating ESG principles into their strategies. This trend is likely to sustain its momentum for years to come and was likely to be accelerated by the ongoing COVID-19 pandemic.[1]

In a world where an increasing number of investors start to look beyond firms' conventional monetary performances, important questions naturally arise: how asset prices are formed and how they should be interpreted. The goal of this paper is to shed light on these very issues. We propose a tractable noisy Rational Expectations Equilibrium (REE) model a la Hellwig (1980) where a firm's payoff consists of monetary and non-monetary risky components. The financial market is populated with noise traders and rational risk averse investors of two types. Rational investors receive informative signals about the two payoff components but have distinct preferences about them. Traditional investors value the monetary payoff solely, while green investors value both monetary and non-monetary payoffs.

A key feature of our framework is the strategic interaction between traditional and green investors through learning and trading. Because of their heterogeneous preferences, the two groups of investors trade differently based on the same information and learn differently from the same asset price. In particular, while green investors increase their demand when receiving positive signals about the firm's non-monetary payoff, traditional investors cut back their demand as they infer from the price a worse realization of the firm's monetary payoff. It means that trades by one group of investors make the asset price more aligned with the their preferences but less aligned with preferences of the other group. Therefore, trades by traditional and green investors contaminate price informativeness to each other and hinder the learning from the price.

Heterogeneity in preferences and, associated with it, differential use of information by traditional and green investors have several profound impacts on the stock price. We first show that they give rise to a feedback loop between trading intensities and the price

[1] According to a recent survey of institutions with a total asset under management of $12.9 trillion conducted by J.P. Morgan, the majority of respondents believe that the COVID-19 crisis is going to raise investors' awareness of ESG-related issues. See: https://www.jpmorgan.com/insights/research/covid-19-esg-investing

coefficients, which may lead to multiple equilibria in the trading game. Specifically, on the one hand, when one group of investors trade more intensively, their preferences are reflected more in the price. The price is then more informative to them. On the other hand, when the price is more informative to investors of a particular group, they face less uncertainty and bear less risk when holding the stock. They are then incentivized to trade more intensively. As a result, two equilibria can co-exist. In one equilibrium, the stock is predominately traded by traditional investors, and the equilibrium price primarily loads on the monetary component; while in the other equilibrium, green investors dominate the trade, and the equilibrium price is more aligned with their preferences.[2]

Inspired by the rapidly growing appetite for ESG investment, we then analyze the impact of an increase in the share of green investors in the investor base. In particular, we look at its impacts on two important metrics—the price informativeness and the firm's cost of capital. We show that, as the green investor share increases, the price becomes less informative to traditional investors, i.e. less informative about the monetary component, and more informative to green investors in any stable equilibrium. This intuitive result, however, has important implications for the firm's cost of capital. In particular, the firm's cost of capital is non-monotone in the share of green investors. It is high when the investor base is balanced, i.e. when the masses of traditional and green investors are similar to each other. In this case, the two investor groups trade with similar intensities and add a substantial amount of noise to the price for each other. As a result, the price informativeness is not particularly high for investors of either type. As all rational investors face large residual risk when holding the stock, the firm's cost of capital is high. This result is helpful to reconcile two seemingly contradictory empirical observations. On the one hand, green investors are willing to sacrifice monetary payoff for non-pecuniary benefits (e.g. Martin and Moser, 2016 and Riedl and Smeets, 2017), which implies a lower cost of capital for green firms with a larger fraction of green investors. On the other hand, direct empirical evidence on the cost of capital for green firms is rather mixed.[3] Our result suggests that, although green firms may attract green investors demanding a lower financial return, entry of such investors can amplify the informational risks faced by existing traditional investors.

[2] There exists a third equilibrium where neither group trades the stock actively, and the price is a noisy signal to both green and traditional investors. However, this equilibrium is not stable, and therefore we do not put much focus on it.

[3] For instance, Larcker and Watts (2020) do not find significant differences between prices of green and non-green security issues while Hong and Kacperczyk (2009) document that investors do require a higher premium for sin stocks. Further discussions are provided in Section 4.3.

Our model also speaks to a series of efforts by regulators around the world to improve the quality of ESG-related information. Corresponding to such developments, we investigate how an improvement in the precision of non-monetary information affects the price informativeness and the cost of capital. Better information about the non-monetary payoff benefits traditional investors as it helps them to learn from the price. However, green investors benefit disproportionally more and respond by substantially increasing their trading intensities. When the preference heterogeneity is sufficiently strong, we show that it reduces the price association with the monetary component and, thus, the price informativeness to traditional investors. Furthermore, we show that the decrease in the price informativeness to traditional investors can dominate the increase in that to green investors, leading to an increase in the cost of capital. Overall, our results emphasize the importance of endogenous responses of investors with heterogeneous preferences to improvements in the informational environment.

It is important to notice that our model has a wide range of applications in environments where investors with heterogeneous preferences over different fundamentals interact and affect each other's investment choices. For example, funds pursuing different strategies might care about different components of a stock's payoff to fulfill different investment needs. Investors with different investment horizons assign different weights to short-term payouts and long-term values (Bushee, 2001), which might be driven by distinct shocks. Similarly, investors might have heterogeneous preferences towards dividends and capital gains (e.g. Graham and Kumar, 2006 and Harris, Hartzmark and Solomon, 2015).

Literature Review Our paper contributes to the recent literature that investigates theoretically the impacts of ESG investing on firm behavior and asset prices. Heinkel, Kraus and Zechner (2001) study firms' decision to reform its technology to attract more green investors and, thus, improve risk sharing and lower its cost of capital. Chowdhry, Davies and Waters (2019) study firm financing and contracting in the presence of profit-motivated and socially motivated investors. Oehmke and Opp (2020) emphasize a complementarity between green and traditional capital in the primary market. More on the asset pricing side, Luo and Balvers (2017), Baker, Bergstresser, Serafeim and Wurgler (2018), Pastor, Stambaugh and Taylor (2020) and Pedersen, Fitzgibbons and Pomorski (2020) build a portfolio choice model in the spirit of Fama and French (2007) and demonstrate how green investors' taste for green stocks affect asset prices. Friedman and Heinle (2016) consider a setting featuring investors with heterogeneous valuations who, however, do not learn from price. Different from these papers, we adopt an REE framework and highlight interactions between green and traditional investors in the secondary market via information aggregation and learning.

More generally, our theoretical model also adds to the vast REE literature a la Grossman and Stiglitz (1980), Hellwig (1980) and Diamond and Verrecchia (1981). Two features together distinguish our model from the existing studies. First, there are two groups of investors with distinct preferences over two fundamentals. Second, investors are not restricted to be only informed about the fundamental they value.

There are a few papers that analyze models with multiple fundamentals but under homogeneous preferences. Goldstein and Yang (2015) analyze a model where asset payoff is affected by two fundamentals, like in our case, while investors receives heterogeneous information about fundamentals. Relatedly, Cespa and Foucault (2014) construct a two-asset economy to study cross-asset learning and liquidity spillovers. Ganguli and Yang (2009) and Manzano and Vives (2011) consider settings where investors possess information about asset payoff and aggregate supply shock (see also Amador and Weill, 2010 and Davila and Parlatore, 2020). Introducing preference heterogeneity allows us to capture the idea that investors with different preferences use same information to trade in the opposite directions, thus making the price noisier to each other.[4] This is a key force behind our results about price-related variables and nature of equilibria in the trading game.[5]

Several papers introduce heterogeneous valuations into the REE framework. Vanwalleghem (2017), Rahi and Zigrand (2018) and Rahi (2020) study models where agents have private valuations, but different from us, receive only information about the fundamental they value. Vives (2011) constructs a general environment to study supply schedule competition among agents with correlated private valuations.[6] In our model, heterogeneous asset valuations stem from traders' different preferences towards monetary and non-monetary payoff components. This results in a quite different information struc-

[4]From this perspective, our paper is related to Goldstein, Li and Yang (2014), where investors' objectives might be different due to different investment opportunities.

[5]In our paper, the mechanism giving rise to equilibrium multiplicity in the trading game is quite different from the existing papers: in Ganguli and Yang (2009) and Manzano and Vives (2011), multiplicity emerges because investors with homogeneous preferences might coordinate to trade based on their signals about fundamental or supply shock; in Lundholm (1988), it arises when investors receive correlated private and public signals; in Cespa and Foucault (2014), it is due to liquidity spillovers across different assets; in Glebkin (2019), multiplicity arises because large traders internalize their price impacts.

[6]Relatedly, see Rostek and Weretka (2012), Vives (2014), Babus and Kondor (2018), Bergemann, Heumann and Morris (2019), Glebkin (2019), and Heumann (2020).

ture: Although all investors receive some information about firms' exposures to common risks (for example, by reading firms' reports), their trading activities are affected differently by the same information. This feature is crucial for our results but, to the best of our knowledge, is absent from the existing literature.

Our paper proceeds as follows. Section 2 presents a simplified version of our model to highlight the key mechanisms. Section 3 lays out our benchmark model and characterize equilibria. Sections 4 and 5 study the implications of growing fraction of green investors and improving quality of information about the non-monetary component respectively. Section 6 concludes.

2 A simplified model

To highlight the key mechanisms, we start by presenting a simplified version of our model in which we are able to get closed-form solution. All derivations and proofs are delegated to Appendix.

2.1 Setup

Two assets are traded in the financial market: a risk-free bond and a risky stock of a firm. The bond is in unlimited supply. It pays off one and its price is normalized to one. The stock is a claim on the firm's output which consists of two risky components: a monetary component $\tilde{z}$ and a non-monetary component $\tilde{\delta}$. The monetary part can be interpreted as a cash flow generated by the firm, while the non-monetary part can be interpreted as, for example, the (negative of) environmentally harmful carbon emissions. $\tilde{z}$ and $\tilde{\delta}$ are independent normal random variables, $\tilde{z}, \tilde{\delta} \sim N(0, \tau^{-1})$.[7] The stock is in unit supply, and its price $\tilde{q}$ is determined endogenously by market clearing in equilibrium.

There are two groups of rational investors with a combined mass of $m > 0$. Half of them are traditional investors who only value the firm's monetary output $\tilde{z}$. The other half are green investors who only value the non-monetary output $\tilde{\delta}$.[8] Both traditional and

[7]In reality, firms' non-monetary aspects may be correlated with their monetary performance. For example, low carbon emission in compliance with regulation can also enhance cash flows if it helps avoid regulatory penalties. The key for our paper is that the two payoff components are not perfectly aligned and that investors have heterogeneous preferences over the two components.

[8]In Section 3, we allow for general preferences and unequal masses of investors. All results in the

green investors have constant absolute risk aversion (CARA) utilities with the same risk aversion parameter γ. That is, if an investor of type $j \in \{t, g\}$ has initial wealth W_0 and chooses to hold d units of the stock then her expected utility is

$$\mathbb{E}\left\{-\exp\left(-\gamma\left[W_0 + d\left(\beta_z^j \tilde{z} + \beta_\delta^j \tilde{\delta} - \tilde{q}\right)\right]\right)\right\},$$

where $\beta_z^t = 1$, $\beta_\delta^t = 0$, $\beta_z^g = 0$ and $\beta_\delta^g = 1$. In addition to rational traders, there are noise traders whose stock demand is $\tilde{n} \sim N\left(0, \tau_n^{-1}\right)$.

Rational investors trade based on information contained in the stock price and their private signals. Importantly, traditional and green investors receive signals about both monetary and non-monetary fundamentals, i.e., an investor i of type $j \in \{t, g\}$ observes $\tilde{s}_z^i \sim N\left(\tilde{z}, \tau_s^{-1}\right)$ and $\tilde{s}_\delta^i \sim N\left(\tilde{\delta}, \tau_s^{-1}\right)$. That investors receive signals about both $\tilde{z}$ and $\tilde{\delta}$ reflects the fact that many informational sources, e.g. analyst and investor reports, describe firms' risks comprehensively. For tractability, we also assume that the signal precisions are the same across fundamentals and investors.[9]

2.2 Market clearing and equilibrium

In a CARA-normal setup, the demand for the stock from an investor i of type $j \in \{t, g\}$ is

$$d^j\left(\mathcal{F}^i\right) = \frac{\mathbb{E}\left(\beta_z^j \tilde{z} + \beta_\delta^j \tilde{\delta} | \mathcal{F}^i\right) - \tilde{q}}{\gamma \mathbb{V}\left(\beta_z^j \tilde{z} + \beta_\delta^j \tilde{\delta} | \mathcal{F}^i\right)},$$

where the information set $\mathcal{F}^i = \{\tilde{s}_z^i, \tilde{s}_\delta^i, \tilde{q}\}$ includes agent i's private signals and publicly observable stock price.

Let T_j denote the set of investors of type j. Market clearing implies

$$D^t\left(\tilde{z}, \tilde{\delta}, \tilde{q}\right) + D^g\left(\tilde{z}, \tilde{\delta}, \tilde{q}\right) + \tilde{n} = 1, \tag{1}$$

where $D^j\left(\tilde{z}, \tilde{\delta}, \tilde{q}\right) = \int_{i \in T_j} d^j\left(\mathcal{F}^i\right) di$ is the total demand for the stock from investors of type j.

simplified model hold in the generalized model.

[9] In Appendix D, we consider a general information structure featuring different information precisions for different types of investors and different payoff components. Despite the model becomes much less tractable, we nevertheless show that several key results hold.

We focus on rational expectation equilibria (REE) with linear prices,

$$\tilde{q} = q_0 + q_z\tilde{z} + q_\delta\tilde{\delta} + q_n\tilde{n} = q_0 + q_n\left(\xi_z\tilde{z} + \xi_\delta\tilde{\delta} + \tilde{n}\right),$$

where we define normalized price coefficients, i.e. $\xi_z = \frac{q_z}{q_n}$ and $\xi_\delta = \frac{q_\delta}{q_n}$, to lighten the expressions going forward.

2.3 Equilibrium characterization

2.3.1 Trading intensities and feedback loop

A main ingredient of our model is the heterogeneity in the preferences of traditional and green investors. It has important implications on how investors use their information to trade. Specifically, consider first a traditional investor's demand and define her trading intensities with respect to her private signals $\tilde{s}_z$ and $\tilde{s}_\delta$ as i_z^t and i_δ^t, respectively. We drop the investor-specific superscripts because investors within the same group are identical and their trading intensities are therefore the same. We have

$$\begin{aligned} i_z^t &= \frac{\partial d^t\left(\tilde{s}_z, \tilde{s}_\delta, \tilde{q}\right)}{\partial \tilde{s}_z} = \frac{\tau_s}{\gamma}, \\ i_\delta^t &= \frac{\partial d^t\left(\tilde{s}_z, \tilde{s}_\delta, \tilde{q}\right)}{\partial \tilde{s}_\delta} = -\frac{\tau_s}{\gamma}\frac{\xi_\delta\xi_z}{\xi_\delta^2 + \frac{\tau+\tau_s}{\tau_n}}. \end{aligned} \quad (2)$$

To understand what drives the traditional investor's trading intensities, it is useful to look at how she infers information about $\tilde{z}$, the payoff that she values, from the price and her signals. Specifically, she expects to receive the following payoff from holding one unit of the stock:

$$\mathbb{E}(\tilde{z}|\tilde{s}_z, \tilde{s}_\delta, q) = \underbrace{\tilde{s}_z\frac{\tau_s}{\tau_s+\tau}}_{\text{Signal inference}} + \underbrace{\frac{q_z\frac{1}{\tau+\tau_s}\left[\tilde{q} - \left(q_0 + q_z\tilde{s}_z\frac{\tau_s}{\tau_s+\tau} + q_\delta\tilde{s}_\delta\frac{\tau_s}{\tau_s+\tau}\right)\right]}{q_z^2\frac{1}{\tau+\tau_s} + q_\delta^2\frac{1}{\tau+\tau_s} + q_n^2\frac{1}{\tau_n}}}_{\text{Price inference}}$$

Upon receiving a higher $\tilde{s}_z$, the traditional investor, on the one hand, directly infers from the signal that $\tilde{z}$ is higher. On the other hand, for a given price $\tilde{q}$, a higher $\tilde{s}_z$ implies that the information contained in the price is worse. In other words, other investors are likely to receive lower signals about $\tilde{z}$. The net effect, normalized by the stock payoff variance, is positive and constant, which is a standard result (Hellwig, 1980).

More interestingly, the traditional investor's trading intensity with respect to $\tilde{s}_\delta$ is negative and depends on the equilibrium price coefficients. Since the traditional investor does not value the non-monetary payoff component, she uses her signal on $\tilde{\delta}$ to infer $\tilde{z}$ from the price. A higher $\tilde{s}_\delta$ indicates worse information contained in the price about $\tilde{z}$. Therefore, she reduces her demand in response to a higher $\tilde{s}_\delta$.

The magnitude of i^t_δ is high if, based on her $\tilde{\delta}$-signal, the traditional investor is able to infer a lot about $\tilde{z}$ from the price. This price inference effect is captured by the numerator of the expression (2). At the same time, she reduces her trading activities if she faces a high uncertainty about the payoff component she values. The uncertainty effect is reflected by the denominator of the same expression.

Analogously, the trading intensities of a green investor are

$$
\begin{aligned}
i^g_z &= \frac{\partial d^g\left(\tilde{s}_z, \tilde{s}_\delta, \tilde{q}\right)}{\partial \tilde{s}_z} = -\frac{\tau_s}{\gamma}\frac{\xi_\delta \xi_z}{\xi_z^2 + \frac{\tau+\tau_s}{\tau_n}}, \\
i^g_\delta &= \frac{\partial d^g\left(\tilde{s}_z, \tilde{s}_\delta, \tilde{q}\right)}{\partial \tilde{s}_\delta} = \frac{\tau_s}{\gamma}.
\end{aligned}
$$

Because traditional and green investors value different fundamentals, they trade in opposite directions based the same signals. Both types of investors trade with equal and constant intensities on signals about the payoff components they value: $i^t_z = i^g_\delta = \frac{\tau_s}{\gamma}$. At the same time, their trading intensities based on signals about the fundamentals they do not value (i^t_δ and i^g_z) depend on the equilibrium price coefficients and, in particular, on the riskiness of the stock payoff.[10] Recall that an investor of type j scales up her trading intensity when she faces smaller uncertainty $\mathbb{V}\left(\beta^j_z \tilde{z} + \beta^j_\delta \tilde{\delta} | \mathcal{F}^i\right)$ or, equivalently, when the equilibrium price is more informative to her. Defining the price informativeness to an investor of type j as $PI_j = \mathbb{V}\left(\beta^j_z \tilde{z} + \beta^j_\delta \tilde{\delta} | \mathcal{F}^i\right)^{-1}$, it is easy to see that

$$
\frac{i^t_\delta}{i^g_z} = \frac{\xi_z^2 + \frac{\tau+\tau_s}{\tau_n}}{\xi_\delta^2 + \frac{\tau+\tau_s}{\tau_n}} = \frac{PI_t}{PI_g}. \tag{3}
$$

[10] The price inference effect is the same for the two types of investors. First, it depends on how sensitive the deviation of the price from its expected value is to the signal about the fundamental investors do not value (captured by ξ_δ and ξ_z for traditional and green investors, respectively). Second, it depends on how strongly this deviation comoves with the fundamentals they value (captured by ξ_z for traditional investors and ξ_δ for green investors). The combined effect is therefore the same for the two groups of investors.

Denote the relative price informativeness as $v = \frac{PI_t}{PI_g}$. When $v > 1$, the price is more informative to traditional investors, and their trading against their $\tilde{\delta}$-signals is more intense than trading of green investors against their $\tilde{z}$-signals. The opposite is true when $v < 1$.

The trading intensities of traditional and green investors determine which information is incorporated into the price and, therefore, shape the equilibrium price coefficients. Using the market clearing condition (1), we obtain

$$\begin{aligned} \xi_z &= \frac{m}{2}\left(i_z^t + i_z^g\right), \\ \xi_\delta &= \frac{m}{2}\left(i_\delta^t + i_\delta^g\right). \end{aligned} \tag{4}$$

Expressions (3) and (4) indicate that there exists a feedback loop between the trading intensities i_δ^t, i_z^g and the price coefficients. On the one hand, when traditional investors trade more aggressively against their $\tilde{\delta}$-signals than green investors against their $\tilde{z}$-signals, the price incorporates less information about the non-monetary component, i.e. $\xi_\delta < \xi_z$. On the other hand, if the price reflects less non-monetary information, it is more informative to traditional investors. They face less investment risk which justifies why they trade more aggressively than green investors in the first place.

2.3.2 Equilibrium multiplicity

The feedback loop described above has profound impacts on the equilibrium outcomes. In particular, it might lead to multiple equilibria in the trading stage and, thus, multiple equilibrium pricing functions.

Using the expressions for the trading intensities, the system of equations (4) can be reduced to the following equilibrium condition that pins down ξ_δ:

$$\left(\xi_\delta^2 - \frac{\tau_s}{\gamma}\frac{m}{2}\xi_\delta + \frac{\tau + \tau_s}{\tau_n}\right)\left(\xi_\delta^3 + \frac{\tau + \tau_s}{\tau_n}\xi_\delta - \frac{\tau_s}{\gamma}\frac{m}{2}\frac{\tau + \tau_s}{\tau_n}\right) = 0. \tag{5}$$

Case 1: $\xi_\delta^3 + \frac{\tau+\tau_s}{\tau_n}\xi_\delta - \frac{\tau_s}{\gamma}\frac{m}{2}\frac{\tau+\tau_s}{\tau_n} = 0$.

This equation always has a unique and positive real root. Moreover, this solution corresponds to a symmetric equilibrium, where traditional and green investors use information about the fundamentals they do not value equally actively, $i_\delta^t = i_z^g$. This results in the price being equally informative to the two groups of investors, $v = 1$ and $\xi_\delta = \xi_z$.

Case 2: $\xi_\delta^2 - \frac{\tau_s}{\gamma}\frac{m}{2}\xi_\delta + \frac{\tau+\tau_s}{\tau_n} = 0$.

This equation has two real roots when $\tau_n > \tau_n^* \equiv 4(\tau+\tau_s)\left(\frac{\tau_s}{\gamma}\frac{m}{2}\right)^{-2}$, i.e. when the demand from noise traders is not too volatile.[11] If $\tau > \tau_n^*$, there exist two equilibria in addition to the one in Case 1:

$$\xi_\delta = \frac{1}{2}\left[\frac{\tau_s}{\gamma}\frac{m}{2} \pm \sqrt{\left(\frac{\tau_s}{\gamma}\frac{m}{2}\right)^2 - 4\frac{\tau+\tau_s}{\tau_n}}\right] \quad \text{and} \quad \xi_z = \frac{\tau_s}{\gamma}\frac{m}{2} - \xi_\delta.$$

In the equilibrium where $\xi_\delta > \frac{1}{2}\frac{\tau_s}{\gamma}\frac{m}{2} > \xi_z$, the price is more informative to green investors, $v < 1$. We refer to it as a *G-equilibrium*. The other one is referred to as a *T-equilibrium*: there $\xi_\delta < \frac{1}{2}\frac{\tau_s}{\gamma}\frac{m}{2} < \xi_z$ and the price is more informative to traditional investors, $v > 1$.

The G- and T-equilibria coexist when $\tau_n > \tau_n^*$. It points out that the multiplicity emerges in an economy where the exogenous noise is small—more specifically, the volatility of noise traders' demand τ_n^{-1} is small, signals are precise relative to priors (high τ_s and low τ), and the mass of informed investors m is large. When the exogenous noise is small, the feedback loop described in the previous section is more pronounced. In particular, when τ_n is large, the relative price informativeness (3) is very sensitive to the price coefficients. As a result, multiple equilibria marked by different relative price informativeness arise. If, on the contrary, τ_n is small, the relative price informativeness is always close to one and the feedback loop is weak. In this case, the only possible equilibrium is the one described in Case 1.

To sum up, in this simple model, equilibrium is unique when the exogenous noise is large. Otherwise, there exist three equilibria. In the G- and T-equilibria, trading is dominated by a particular group of investors and the price is more informative to investors of the dominant group. In the third equilibrium, neither of the two groups is dominating, and the price is equally informative to all investors. In what follows, we refer to this equilibrium as an *M-equilibrium*.

[11] If $\tau_n = \tau_n^*$, the root in Case 2 is unique and coincides with that in Case 1, i.e. $\xi_\delta = \xi_z = \frac{1}{2}\frac{\tau_s}{\gamma}\frac{m}{2}$.

2.3.3 Discussions

The key mechanism behind the feedback loop and equilibria multiplicity is that investors trade in the opposite directions when receiving the same signals. This mechanism requires that the two groups of investors have, first, the incentives to trade against each other and, second, the means of doing so.

The incentives arise due to the preference heterogeneity. Because investors value different payoff components, they use the same information differently. By trading against signals about the fundamental they do not value, investors of one group make the price noisier to the other group. Facing riskier stock payoff, investors within the other group choose to trade less actively. The feedback loop between trading intensities and the price informativeness gives rise to multiple equilibria. Without the preference heterogeneity, all investors trade in the same way, and the price is always equally informative to everyone. In that case, our model reduces to a fairly standard REE setting with a unique equilibrium.[12]

The ability of investors to trade in the opposite directions relies on availability of information about payoff components that they value and not. In the context of ESG investing, traditional investors might put less value to firms' ESG performances but still receive related information from news articles or comprehensive disclosure statements. Receiving such information makes it possible for traditional investors to trade against green investors. In the absence of it, investors only trade on signals about the payoff component they value, and the feedback loop disappears. In Appendix D, we show that in the setting with heterogeneous preferences, multiple equilibria are always possible unless investors receive information *only* about the payoffs they value.

[12]The setting where investors with homogeneous preferences trade the stock whose payoff is affected by two independent components has been studied in Goldstein and Yang (2015). They show the uniqueness of the trading-stage equilibrium when each of the two groups of investors is informed only about one fundamental. In Appendix E, we consider a generalized version of their model where investors have the same preference weights over $\tilde{z}$ and $\tilde{\delta}$ but receive signals about both fundamentals. We verify that equilibrium in the trading stage remains unique.

3 A benchmark model

3.1 Setup

This section presents our baseline model, which extends the simplified version in the previous section along two dimensions.

First, we allow for partially aligned preferences of traditional and green investors. In particular, traditional investors still only value the monetary payoff component, i.e. $\beta_z^t = 1, \beta_\delta^t = 0$. However, we now assume that green investors might value both the monetary and non-monetary components. The stock payoff to them is $\beta_z \tilde{z} + \beta_\delta \tilde{\delta}$, where $\beta_z \geq 0$ and $\beta_\delta > 0$ are utility weights. We normalize $\beta_z^2 + \beta_\delta^2 = 1$, so that the ex-ante variance of the stock payoff is the same for traditional and green investors. Note that β_δ captures the degree of preference heterogeneity across traditional and green investors.

Second, we relax the equal-mass assumption and allow for $\alpha \in (0, 1)$ fraction of green investors. The market is thus populated with a mass $m_t = (1 - \alpha)m$ of traditional investors and a mass $m_g = \alpha m$ of green investors.

3.2 Equilibrium characterization

The analyses and intuitions of the simplified model in Section 2.3 can be extended to this benchmark model. In particular, from a green investor's perspective, the stock payoff is now $\tilde{y} = \beta_z \tilde{z} + \beta_\delta \tilde{\delta}$. $\tilde{x} = \beta_\delta \tilde{z} - \beta_z \tilde{\delta}$ is orthogonal to $\tilde{y}$ and thus represents the payoff component that green investors do not value. Using this notation, it is straightforward to derive analogues of Expressions (3) and (4) and establish the existence of the feedback loop between trading intensities and the price coefficients. We do this in Appendix A. We also show there that the market clearing condition yields a fixed point problem for ξ_δ,

$$\xi_\delta = J(\xi_\delta). \tag{6}$$

It can be simplified to the following quintic equation in ξ_δ:

$$\xi_\delta^5 - \frac{\tau_s}{\gamma}\alpha m \beta_\delta \xi_\delta^4 + 2\frac{\tau+\tau_s}{\tau_n}\xi_\delta^3 - 2\frac{\tau_s}{\gamma}\alpha m\beta_\delta \frac{\tau+\tau_s}{\tau_n}\xi_\delta^2 + \left[\left(\frac{\tau+\tau_s}{\tau_n}\right)^2 + \left(\frac{\tau_s}{\gamma}(1-\alpha)m\beta_\delta\right)^2\frac{\tau+\tau_s}{\tau_n}\right]\xi_\delta - \frac{\tau_s}{\gamma}\alpha m\beta_\delta\left(\frac{\tau+\tau_s}{\tau_n}\right)^2 = 0. \quad (7)$$

Note that (7) simplifies to (5) if $\alpha = \frac{1}{2}$ and $\beta_\delta = 1$.

Proposition 1. *There exists a $\tau_n^* = \tau_n^*(\alpha, \beta_\delta) > 0$ such that*[13]

(i) if $\tau_n \in (0, \tau_n^*)$, there is a unique equilibrium;

(ii) if $\tau_n = \tau_n^*$, there are two equilibria if $\alpha \neq \frac{1}{2}$ and one equilibrium if $\alpha = \frac{1}{2}$;

(iii) if $\tau_n > \tau_n^*$, there are three equilibria.

In any equilibrium, $q_0 < 0$, $q_z > 0$, $q_\delta > 0$ and $q_n > 0$.

Proposition 1 confirms that, in our baseline setup, multiple equilibria are possible and arise when the exogenous noise is sufficiently small, i.e. $\tau_n > \tau_n^*$ (panel B in Figure 1). As illustrated earlier, in that case, the price informativeness and trading intensities are sensitive to the equilibrium price coefficients, which strengthens the feedback loop. When τ_n is small (panel A in Figure 1), the feedback loop is weak and the equilibrium is unique.

Proposition 2. *The multiplicity threshold $\tau_n^*(\alpha, \beta_\delta)$ behaves such that (i) $\frac{d\tau_n^*(\alpha,\beta_\delta)}{d(\beta_\delta)} < 0$; (ii) $\frac{d\tau_n^*(\alpha,\beta_\delta)}{d\alpha} < 0$ if $\alpha < \frac{1}{2}$ and $\frac{d\tau_n^*(\alpha,\beta_\delta)}{d\alpha} > 0$ if $\alpha > \frac{1}{2}$.*

Proposition 2 characterizes how the multiplicity threshold τ_n^* varies with the degree of preference heterogeneity β_δ and the green investor share α. The multiplicity is more likely if the effective heterogeneity on the aggregate level is strong, i.e. when the non-monetary utility weight of green investors β_δ is large and the masses of the two groups are similar (α is close to $\frac{1}{2}$). If the investor base tilts towards a particular investor group, or investors' preferences are closely aligned, the effective heterogeneity is small. For example, if there are only a few green investors ($\alpha \to 0$), or green investors mostly value the monetary

[13]We write the threshold τ_n^* as a function of α and β_δ. In general, τ_n^* depends on other parameters, such as the total mass of rational investors m, risk aversion γ, etc. We do not mention them explicitly because they are not our focus here.

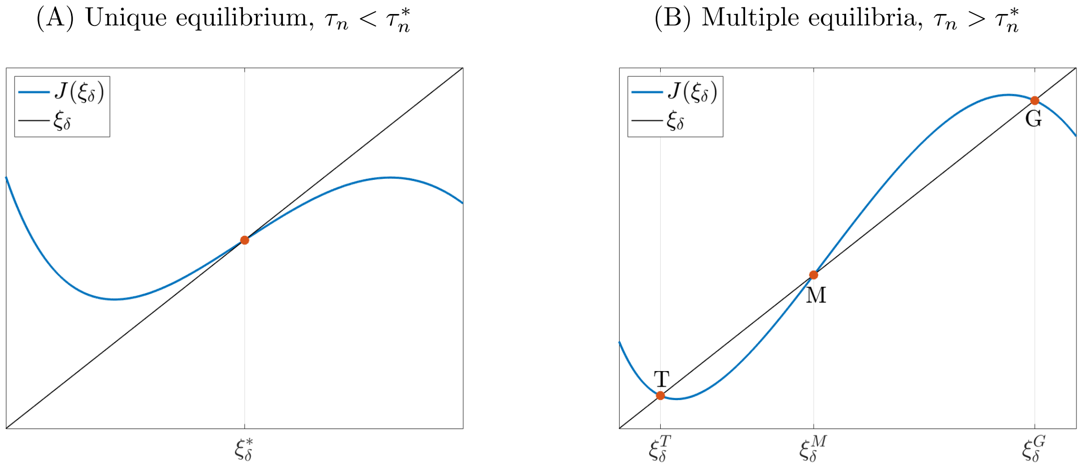

Figure 1: Equilibrium ξ_δ: Solution to equation (6), $\xi_\delta = J(\xi_\delta)$.

payoff ($\beta_\delta \to 0$), the investor base is nearly homogeneous and the model reduces to a standard REE model with a unique pricing function.

When multiple equilibria are possible, they can be ranked according to the relative price informativeness. Formally, price informativeness to traditional and green investors are

$$PI_t = \mathbb{V}\left(\tilde{z}|\mathcal{F}^{ij}\right)^{-1} = (\tau+\tau_s)\frac{\xi_z^2+\xi_\delta^2+\frac{\tau+\tau_s}{\tau_n}}{\xi_\delta^2+\frac{\tau+\tau_s}{\tau_n}},$$
$$PI_g = \mathbb{V}\left(\beta_z\tilde{z}+\beta_\delta\tilde{\delta}|\mathcal{F}^{ij}\right)^{-1} = (\tau+\tau_s)\frac{\xi_z^2+\xi_\delta^2+\frac{\tau+\tau_s}{\tau_n}}{\left(\xi_\delta\beta_z-\xi_z\beta_\delta\right)^2+\frac{\tau+\tau_s}{\tau_n}}.$$

As in the simplified model, we use $v = \frac{PI_t}{PI_g}$ to denote the relative price informativeness to traditional investors. Using the same terminology, whenever there are three equilibria, we call the one with the smallest v the G-equilibrium, the one with the largest v the T-equilibrium, and the one with a medium v the M-equilibrium.[14] Formally, we have

Proposition 3. *When there are three equilibria, they can be ranked according to the relative price informativeness to traditional investors v. In the T-equilibrium $v^T > 1$; in the G-equilibrium $v^G < 1$; in the M-equilibrium $v^M \in \left(v^G, v^T\right)$.*

Possibility of equilibrium multiplicity naturally raises the question about equilibrium selection. A common selection approach suggests that stable equilibria are more likely to be played. Recall that ξ_δ is a fixed point of $J(\xi_\delta)$ as in (6). We call an equilibrium stable if

[14] As we mention in Proposition 1, two equilibria exist when $\tau_n = \tau_n^*$ and $\alpha \neq \frac{1}{2}$. In what follows, we do not analyze this knife-edge case to save space. Results are available upon request.

the dynamics around the equilibrium ξ_δ are locally stable, i.e. $\frac{\partial[J(\xi_\delta)-\xi_\delta]}{\partial\xi_\delta} < 0$.[15] Under this criterion, if the system is pushed to an off-equilibrium point $\xi_\delta^*+\epsilon$, it tends to move back to the equilibrium point ξ_δ^* if $|\epsilon|$ is sufficiently small. In Figure 1, stable/unstable equilibria are those intersections of ξ_δ and $J(\xi_\delta)$ where the derivative of $J(\xi_\delta)$ is below/above one.

Proposition 4. *Define an equilibrium as stable if* $\frac{\partial[J(\xi_\delta)-\xi_\delta]}{\partial\xi_\delta} < 0$, *where* $J(\cdot)$ *is given by* (6). *When equilibrium is unique, it is stable. When there are three equilibria, the T- and G-equilibria are stable and the M-equilibrium is unstable.*

Proposition 4 suggests that investors are unlikely to coordinate on the M-equilibrium when the G- and T-equilibria exist. The M-equilibrium also has counter-intuitive properties. For example, in the M-equilibrium, when the mass of one investor group increases, the price becomes less informative to investors within this group (this is formally established in Proposition 5 below). In other words, investors should coordinate to trade *less* actively when there are more traders with the same preferences. In what follows, we characterize all equilibria but put less focus on the M-equilibrium when the multiplicity is possible.

4 Growth of green investors

In this section, we examine impacts of the recent trend of investors' growing awareness about firms' ESG performances. In particular, we characterize how the price informativeness and the firm's cost of capital respond to an increase in the share of green investor share α in Sections 4.1 and 4.2. We then discuss the empirical implications of our results in Section 4.3.

4.1 Price informativeness

Proposition 5 characterizes how absolute and relative price informativeness, PI_t, PI_g, and v, change with the green investor share α.

[15] Under this criterion, a fixed point of the nonlinear differential equation $\frac{d\xi_{\delta,t}}{dt} = J(\xi_{\delta,t}) - \xi_{\delta,t}$ is locally stable. This criterion implies particular dynamics as the economy converges to the equilibrium point(s), whereas our model is static. Formal evaluation of stability requires a dynamic extension of our current setting, which is beyond the scope of this paper. Nevertheless, our criterion is similar to what is derived in the literature that introduces recursive-least-squares (adaptive) learning in settings a la Grossman and Stiglitz (1980) (e.g. Bray, 1982, Marcet and Sargent, 1989, Heinemann, 2009).

Proposition 5. *If $\tau_n \leq \tau_n^*\left(\frac{1}{2}, \beta_\delta\right)$, there is a unique equilibrium where $\frac{dPI_t}{d\alpha} < 0$, $\frac{dPI_g}{d\alpha} > 0$, and $\frac{dv}{d\alpha} < 0$. If $\tau_n > \tau_n^*\left(\frac{1}{2}, \beta_\delta\right)$, there exists a $\underline{\alpha} \in \left(0, \frac{1}{2}\right)$ and $\bar{\alpha} = 1 - \underline{\alpha}$ such that*[16]

(i) if $\alpha < \underline{\alpha}$, there is a unique T-equilibrium where $v^T > 1$;

(ii) if $\alpha > \bar{\alpha}$, there is a unique G-equilibrium where $v^G < 1$;

(iii) if $\alpha \in (\underline{\alpha}, \bar{\alpha})$, there are three equilibria where $v^T > v^M > v^G$.

Moreover, in the T- and G-equilibria, $\frac{dPI_t}{d\alpha} < 0$, $\frac{dPI_g}{d\alpha} > 0$, and $\frac{dv}{d\alpha} < 0$; in the M-equilibrium, $\frac{dPI_t}{d\alpha} > 0$, $\frac{dPI_g}{d\alpha} < 0$, and $\frac{dv}{d\alpha} > 0$.

If the exogenous noise in the financial market is large, $\tau_n \leq \tau_n^*\left(\frac{1}{2}, \beta_\delta\right)$, equilibrium is unique for any $\alpha \in (0, 1)$.[17] First, for given individual trading intensities, a larger α means that the price becomes more aligned with the preferences of green investors because a larger share of trades in the market are done by them. Furthermore, individual trading intensities adjust. Green investors, facing a lower residual risk, trade more actively, while traditional investors reduce their trading activity. Overall, the equilibrium price coefficients change such that the price becomes more informative to green investors and less informative to traditional investors. Panel (A) in Figure 2 illustrates how the relative price informativeness varies with α.

If the exogenous noise is small, equilibrium multiplicity is possible when the investor base consists of similar masses of traditional and green investors (α is close to $\frac{1}{2}$). Panel (B) in Figure 2 depicts relative price informativeness v in this case. Start from the economy with few green investors ($\alpha < \underline{\alpha}$). Here, traditional investors significantly outweigh green investors. There exists a unique T-equilibrium when the price is predominantly informative about the monetary component, resulting in a large $v > 1$. As α increases and crosses $\underline{\alpha}$, the feedback look becomes sufficiently strong to support the G-equilibrium where the price is more informative to green investors, $v < 1$. Interestingly, this is possible even if the share of green investors is below $\frac{1}{2}$. Eventually, when the share of green investors becomes sufficiently large ($\alpha > \bar{\alpha}$), there is a unique G-equilibrium.

[16] Although there is a unique equilibrium when $\alpha < \underline{\alpha}$ or $\alpha > \bar{\alpha}$, we abuse the notation and refer to it as either a T- or G-equilibrium because the equilibrium outcomes (e.g., the price coefficient) are continuous at $\alpha = \underline{\alpha}$ and $\alpha = \bar{\alpha}$ as shown in panel (B) of Figure 2.

[17] By Proposition 2, multiple equilibria region is largest when the investor base is balanced with equal masses of green and traditional investors, $\alpha = \frac{1}{2}$. When equilibrium is unique for $\alpha = \frac{1}{2}$, i.e. $\tau_n \leq \tau_n^*\left(\frac{1}{2}, \beta_\delta\right)$, equilibrium is unique for all $\alpha \in (0, 1)$.

Similar to the case of large exogenous noise, as α increases, the price becomes more informative to green investors and less informative to traditional investors, irrespective of whether the T- or G-equilibrium is played.[18] Different from the case of large exogenous noise, however, there can be discontinuous jumps in the price informativeness due to switches across equilibria.

(A) Unique equilibrium, $\tau_n \leq \tau_n^*\left(\frac{1}{2}, \beta_\delta\right)$ (B) Multiplicity is possible, $\tau_n > \tau_n^*\left(\frac{1}{2}, \beta_\delta\right)$

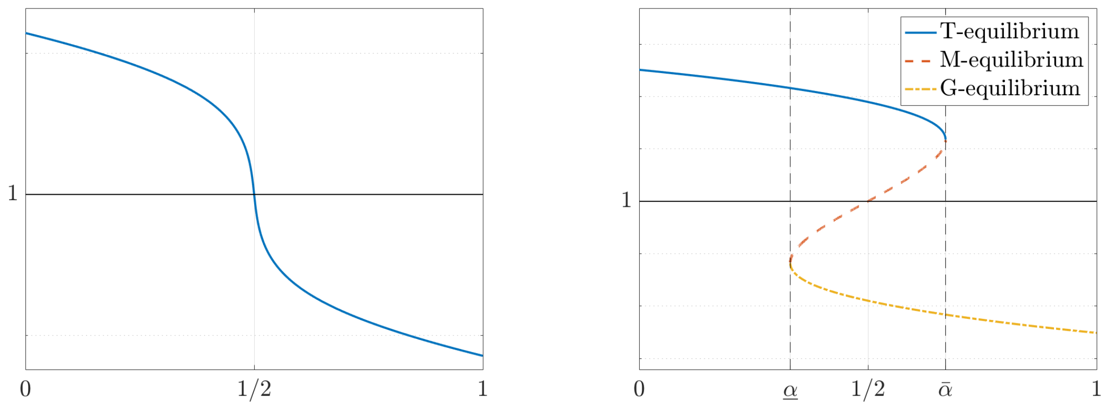

Figure 2: Relative price informativeness to traditional investors v as a function of the green investor share α. Y-axes are in the log scale.

4.2 Cost of capital

We define the cost of capital for the firm as the difference between the stock's expected monetary payoff and its price. As a result, the cost of capital for the firm is given by

$$CoC = \mathbb{E}(\tilde{z} - \tilde{q}) = -q_0 = \frac{\gamma}{m_t PI_t + m_g PI_g}. \tag{8}$$

The cost of capital reflects the compensation required by risk-averse traders for bearing the risk of their investments. In our environment with heterogeneous preferences, it is determined by the weighted average of price informativeness to traditional and green investors. Proposition 6 characterizes how it changes with green investor share α.

Proposition 6. *If $\tau_n \leq \tau_n^*\left(\frac{1}{2}, \beta_\delta\right)$, there is a unique equilibrium where $\frac{dCoC}{d\alpha} \gtreqless 0$ when $\alpha \lesseqgtr \frac{1}{2}$. If $\tau_n > \tau_n^*\left(\frac{1}{2}, \beta_\delta\right)$, in the T-equilibrium, $\frac{dCoC}{d\alpha} > 0$; in the G-equilibrium, $\frac{dCoC}{d\alpha} < 0$; in the M-equilibrium, $\frac{dCoC}{d\alpha} \gtreqless 0$ when $\alpha \lesseqgtr \frac{1}{2}$.*

[18] This is not true in the M-equilibrium. However, as we have discussed at the end of Section 3.2, this equilibrium is unlikely to be played because of its instability.

Consider first the case where the noise is large enough such that $\tau_n \leq \tau_n^*\left(\frac{1}{2}, \beta_\delta\right)$ and equilibrium is always unique. This case is illustrated by panel (A) of Figure 3. For concreteness, focus on the case when the mass of traditional investors is higher than the mass of green investors, $\alpha < \frac{1}{2}$. An increase in α affects the cost of capital in two ways:

$$\frac{dCoC}{d\alpha} = -\frac{\gamma}{\left(m_t PI_t + m_g PI_g\right)^2}\left(\underbrace{PI_g - PI_t}_{\text{Direct effect}} + \underbrace{m_t \frac{dPI_t}{d\alpha} + m_g \frac{dPI_g}{d\alpha}}_{\text{Indirect effect}}\right).$$

The direct effect reflects the change in the cost of capital due to the change in the composition of the investor base holding price informativeness PI_t and PI_g fixed. When $\alpha < \frac{1}{2}$, $PI_t > PI_g$ by Proposition 5, so the direct effect drives the cost of capital up.

The indirect effect captures the change in the cost of capital due to adjustments in the equilibrium price coefficients and price informativeness. By Proposition 5, the price informativeness to traditional investors move in the opposite direction to that of green investors: $\frac{dPI_t}{d\alpha} < 0$ and $\frac{dPI_g}{d\alpha} > 0$. Nevertheless, as we discuss in much details in the proof in Appendix B, the indirect effect also pushes the cost of capital up when $\alpha < \frac{1}{2}$. The key force behind this result is the following. As the share of green investors grow, the price becomes more associated with the non-monetary component. However, this increase in ξ_δ also allows traditional investors to use their $\tilde{\delta}$-signals more actively to trade against green investors along this dimension. Trades by traditional investors thus prevent ξ_δ and PI_g from a sharp increase. This effect is pronounced when the trading intensity of traditional investors is higher than that of green investors, i.e. when $\alpha < \frac{1}{2}$ and $PI_t > PI_g$.

In sum, when the investor base consists mostly of traditional investors, an increase in the share of green investors leads to an increase in the cost of capital. In contrast, when the majority of investors have green preferences ($\alpha > \frac{1}{2}$), the signs of both direct and indirect effects flip, and the cost of capital declines in α. The cost of capital thus reaches its maximum when the masses of the two groups are equal, i.e. when the aggregate level of heterogeneity is high, and trades by green and traditional investors introduce substantial amounts of noise to each other.

The multiple equilibrium case arising when $\tau_n > \tau_n^*\left(\frac{1}{2}, \beta_\delta\right)$ is illustrated by Panel (B) in Figure 3. In the T-equilibrium, traditional investors always dominate and $PI_t > PI_g$. Similar to the unique equilibrium case, an increase in α leads to a higher cost of capital through both direct and indirect channels. The opposite is true in the G-equilibrium, where the stock is primarily traded by green investors.

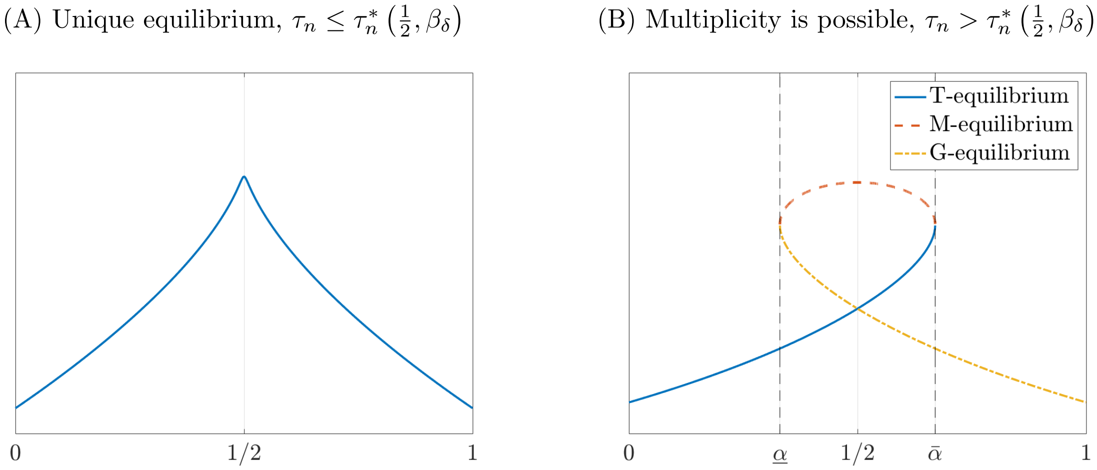

Figure 3: Cost of capital CoC as a function of the green investor share α. Y-axes are in the log scale.

So far, we have analyzed the cost of capital for a firm with zero expected monetary and non-monetary payoffs, i.e. when both $\tilde{z}$ and $\tilde{\delta}$ have zero means. We now characterize how the cost of capital changes with α for a firm with non-zero expected payoffs.

Corollary 1. *Suppose that $\tilde{z} \sim N\left(\mu_z, \tau^{-1}\right)$ and $\tilde{\delta} \sim N\left(\mu_\delta, \tau^{-1}\right)$. The cost of capital is given by*

$$CoC = c_0 + c_z \mu_z + c_\delta \mu_\delta,$$

where $c_0 = \frac{\gamma}{m_t PI_t + m_g PI_g}$, $c_z = \frac{(1-\beta_z)\xi_\delta}{\beta_\delta \xi_z + (1-\beta_z)\xi_\delta} > 0$, and $c_\delta = -\frac{\beta_\delta \xi_\delta}{\beta_\delta \xi_z + (1-\beta_z)\xi_\delta} < 0$. Moreover, $\frac{dc_z}{d\alpha} > 0$ and $\frac{dc_\delta}{d\alpha} < 0$ except for the M-equilibrium. In the M-equilibrium, $\frac{dc_z}{d\alpha} < 0$ and $\frac{dc_\delta}{d\alpha} > 0$.

Corollary 1 delivers two main results. First, the firm's cost of capital increases in its expected monetary output μ_z and decreases in its expected non-monetary output μ_δ. Recall that we focus on the case when the firm's manager values only the monetary output $\tilde{z}$. From the manager's perspective, an increase in μ_z is not fully reflected in the stock price in the presence of green investors who care not only about the monetary performance. In other words, the firm is not fully compensated for an increase in its average monetary output. The opposite is true regarding an increase in μ_δ. As the expected non-monetary output of the firm improves, green investors' demand for the stock increases, the stock price goes up and the cost of capital declines.

Second, as the fraction of green investors α increases, the cost of capital becomes more sensitive to both μ_z and μ_δ (the absolute values of c_z and c_δ increase). With more investors valuing the non-monetary performance, the average preference of investors deviates more from that of the firm manager. As a result, from the manager's perspective, the firm is more under-compensated for an increase in μ_z and more over-compensated for an increase in μ_δ.

We remark that the cost of capital defined in (8) reflects the difference between the firm's monetary payoff and its stock price. Such a metric builds upon the assumption that the firm managers cares only about firm's monetary output. Within our framework, one may consider a general preference for the firm, i.e. $\beta_z^F \tilde{z} + \beta_\delta^F \tilde{\delta}$, under which the results of Corollary 1 will depend on how the preference compare to that of its investors. Our results preserve when the preferences of the firm and traditional investors are sufficiently close.[19]

4.3 Empirical implications

As we establish in the previous sections, the trend of growing ESG awareness is likely to produce profound impacts on the price informativeness and cost of capital. In this section, we discuss the empirical implications of our results and ways to test them.

Price informativeness Proposition 5 states that as the fraction of green investors increases, the price becomes more informative about the non-monetary payoff and less informative about the monetary payoff. These predictions are testable. To do so, one first needs to estimate price informativeness about the monetary and non-monetary fundamentals. In this regard, the work of Davila and Parlatore (2018) can be useful. Using their methodology, one can recover price informativeness about various payoff components via simple regressions of (changes in) individual stock prices on (changes in) earnings and some measures of environmental impact (e.g., changes in firm-level carbon emissions used by Bolton and Kacperczyk, 2020). It then can be tested whether investor composition (e.g., fraction of institutional investors with ESG-related objectives in the overall investor base) is associated with different price informativeness about the monetary and non-monetary fundamentals. Since investor composition is likely to be correlated with characteristics of the production technology (i.e., ESG investors hold on average greener stocks), one can in addition control for average firm-level emissions or ESG ratings.

[19] An interesting question in this respect is how the preferences of firm's manager are connected to those of heterogeneous investors, some of whom have non-pecuniary considerations (Hart and Zingales, 2017). We leave this for future exploration.

Cost of capital Experiments and surveys consistently suggest that many investors do care about non-monetary aspects of firms' operations and are willing to sacrifice financial returns for the societal benefits (e.g. Martin and Moser, 2016, Riedl and Smeets, 2017, and Bauer, Ruof and Smeets, 2020). However, when comparing the cost of capital or the stock returns of green and traditional firms, existing empirical literature documents mixed results. Some papers find a lower cost of capital for green firms (e.g. Hong and Kacperczyk, 2009, El Ghoul, Guedhami, Kwok and Mishra, 2011, Chava, 2014, and Bolton and Kacperczyk, 2020), while a few studies find only a small negative, statistically insignificant or even positive green premium (e.g., Derwall, Guenster, Bauer and Koedijk, 2005 and Henke, 2016, and Larcker and Watts, 2020).

Our model provides a potential resolution to the mixed empirical evidence by highlighting strategic interactions between traditional and green investors, which is not fully revealed by surveying the attitudes of ESG investors. On the one hand, Corollary 1 implies that green firms with a high expected non-monetary output, enjoy a lower cost of capital because green investors are willing to pay a premium for their greenness. On the other hand, green investors tend to divest traditional firms and invest in green firms. As a result, green firms are likely to have a more diverse investor base than traditional firms. According to Proposition 6, this implies a higher cost of capital for green firms due to a lower weighted average price informativeness.

Note that the two channels can be in principle tested separately. For example, to tease out the effect of the investor base diversity, one can compare costs of capital of firms with similar ESG ratings but different investor bases.

Price volatility and trading volume Another robust theoretical prediction of our model is that there might be multiple equilibria in the financial markets, as long as exogenous noise is not prohibitively large. When multiplicity is possible, asset price might experience large fluctuations due to equilibrium switches. Such switches are also likely to be associated with large trading volumes because different equilibria are marked by different trading intensities by traditional and green investors. Our results suggest that multiple equilibria are more likely to arise when the masses of green and traditional investors are similar and exogenous noise is small. One can test if stocks with these properties indeed are more likely to experience price jumps and large flows across ESG and non-ESG investors.

5 Improvements in non-monetary information

In response to the growing trend of ESG investing, policy makers around the world have made a series of efforts to improve the quality of information about firms' ESG performances available to investors. For instance, in May of 2020 the SEC Investor Advisory Committee recommended updating public company reporting requirements to include ESG factors, while the EU regulator has already put in place a disclosure regulation that requires market participants and financial advisers to provide ESG-related information about certain financial products. According to the Carrots & Sticks, there are more than 600 ESG reporting requirements across over 80 countries, including the world's 60 largest economies. Accompanying such regulatory efforts is the fact that 90% of the companies in the S&P 500 Index published sustainability reports in 2019. In addition, there are now hundreds of third-party data providers through which investors are able to obtain ESG-related information of firms.

In this section, we consider an improvement in the precision of non-monetary information. Surprisingly, we show that such an improvement affect trading behaviors of traditional and green investors so that price informativeness about the monetary payoff component and the cost of capital may increase. All proofs for this section can be found in Appendix C.

5.1 Extended setup

For our purpose, we generalize the information structure of our benchmark model in Section 2.1. First, we assume that the prior precisions of the two fundamentals are no longer identical, i.e. $\tilde{z} \sim N\left(0, \tau^{-1}\right)$ and $\tilde{\delta} \sim N\left(0, (\lambda\tau)^{-1}\right)$, where $\lambda > 0$. Second, the precisions of private signals that investors receive also differ by a factor of λ, i.e. $\tilde{s}_z^{ij} \sim N\left(\tilde{z}, \tau_s^{-1}\right)$ and $\tilde{s}_\delta^{ij} \sim N\left(\tilde{\delta}, (\lambda\tau_s)^{-1}\right), \forall i, j$. The extended setup reduces to our benchmark model when $\lambda = 1$. For tractability, we assume a proportional difference of λ for both the prior and signal precisions.[20] Comparative statics with respect to λ reveal the impacts of changes in the overall quality of non-monetary information.

Equilibrium characterization of the extended setup follows that in Section 3.2. We show

[20]The model becomes much more complex when the factor λ for priors is different from that for signals. We have verified the generality of the main results of Section 5.2 with numerical examples in this more general case (unreported to save space).

that, after a proper change of variables, the system of equilibrium conditions takes the same form as that in our benchmark model in Section 3. As a result, main results of that Section hold also in this case. In particular, there are up to three equilibria with two being stable and featuring different relative price informativeness. To save space, we directly proceed to the comparative statics that we are interested in.

Analytically characterizing how the key equilibrium outcomes change with respect to λ for all possible values of this parameter is challenging in this quite general setup. In Section 5.2, we focus on the case where λ is small, i.e. non-monetary information is noisy in comparison with monetary information. While this assumption makes analytical characterization feasible, we believe that it also reflects the current state of things for many companies.[21] In Section 5.3, we show numerically that our main message carries through when λ is not small.

5.2 Price informativeness and cost of capital

Price informativeness The price informativeness to traditional and green investors in the extended setup are given by

$$PI_t = \mathbb{V}\left(\tilde{z}|\mathcal{F}^{ij}\right)^{-1} = (\tau + \tau_s)\frac{\xi_z^2\lambda + \xi_\delta^2 + \lambda\frac{\tau+\tau_s}{\tau_n}}{\xi_\delta^2 + \lambda\frac{\tau+\tau_s}{\tau_n}},$$

$$PI_g = \mathbb{V}\left(\beta_z\tilde{z} + \beta_\delta\tilde{\delta}|\mathcal{F}^{ij}\right)^{-1} = (\tau + \tau_s)\frac{\xi_z^2\lambda + \xi_\delta^2 + \lambda\frac{\tau+\tau_s}{\tau_n}}{\left(\xi_\delta\beta_z - \xi_z\beta_\delta\right)^2 + \left(\beta_z^2\lambda + \beta_\delta^2\right)\frac{\tau+\tau_s}{\tau_n}}.$$

An improvement in the quality of non-monetary information affects price informativeness through two channels. First, a higher λ directly helps investors make better inferences, resulting in an increase in PI_t and PI_g. Specifically, holding price coefficients ξ_z and ξ_δ fixed, it is easy to verify that $\frac{\partial PI_t}{\partial\lambda} > 0$ and $\frac{\partial PI_g}{\partial\lambda} > 0$. Importantly, although traditional investors do not value the non-monetary payoff, more precise information about it allows them to make a better inference about the monetary payoff from the price. Second, there is an indirect effect resulting from changes in investors' trading behavior and thus price coefficients ξ_z and ξ_δ. Proposition 7 describes our comparative statics results regarding

[21] In reality, information about conventional cash flows is likely to be much more precise than about ESG performance as of now. One argument in this regard is the following criticism from SEC Chairman Jay Clayton: "I have not seen circumstances where combining an analysis of E, S and G together, ..., for example with a 'rating' or 'score', ..., would facilitate meaningful investment analysis..."at Meeting of the Asset Management Advisory Committee on May 27, 2020.

the price coefficients and the price informativeness.

Proposition 7. *There exists a $\bar{\lambda} > 0$ such that if $\lambda \in (0, \bar{\lambda})$, equilibrium is unique, and*

(i) $\frac{d\xi_\delta}{d\lambda} > 0$; $\frac{d\xi_z}{d\lambda} \lesseqgtr 0$ *if* $\beta_z \lesseqgtr \frac{\left(\frac{1}{\gamma}\tau_s m_t\right)\left(\frac{1}{\gamma}\tau_s m_g\right)}{\left(\frac{1}{\gamma}m_t\right)^2 + \frac{\tau+\tau_s}{\tau_n}}$;

(ii) $\frac{dPI_g}{d\lambda} > 0$; $\frac{dPI_t}{d\lambda} \lesseqgtr 0$ *if* $\beta_z \lesseqgtr \frac{3}{2}\frac{\left(\frac{1}{\gamma}\tau_s m_t\right)\left(\frac{1}{\gamma}\tau_s m_g\right)}{\left(\frac{1}{\gamma}\tau_s m_t\right)^2 + \frac{\tau+\tau_s}{\tau_n}}$.

Proposition 7 shows that better non-monetary information always leads to an increase in the corresponding price coefficient ξ_δ and makes the price more informative to green investors. Since green investors value the non-monetary payoff, they start to trade actively based on their $\tilde{\delta}$-signals in response to an increase in λ. As a result, more $\tilde{\delta}$-information gets incorporated in the price. Both direct and indirect channels work in favor of green investors, resulting in an increase in the price informativeness to them.

However, the impacts of higher better non-monetary information on ξ_z and the price informativeness to traditional investors is more convoluted. Specifically, if the preference heterogeneity across traditional and green investors is large, green investors not only increase their trading intensity along the $\tilde{\delta}$-dimensions but also trade substantially more aggressively against their $\tilde{z}$-signals in response to an increase in λ. As a result, less monetary information gets incorporated in the price: ξ_z decreases. Therefore, when the preference heterogeneity is sufficiently large, the indirect channel dominates the direct channel, and the price informativeness to traditional investors PI_t declines.

As shown by the cutoffs in Proposition 7, the responses of ξ_z and PI_t depend on other model parameters, in particular, on the mass of green investors m_g. When m_g is high, green investors' aggregate trading against their $\tilde{z}$-information is strong, enforcing the negative indirect channel. Hence, the price informativeness to traditional investors declines even if the preference heterogeneity is not that large.

Cost of capital That price informativeness PI_t and PI_g can respond to changes in λ in opposite directions suggests that the impact of better non-monetary information on the cost of capital may be positive or negative. The expression for the cost of capital in (8) preserves in this extended setup. Differentiating it with respect to λ, we get

$$\frac{dCoC}{d\lambda} = -\gamma \frac{m_t \frac{dPI_t}{d\lambda} + m_g \frac{dPI_g}{d\lambda}}{\left(m_t PI_t + m_g PI_g\right)^2}.$$

The sign of $\frac{dCoC}{d\lambda}$ depends on the weighted average of the changes in the price informativeness across the two investor groups.

Proposition 8. *There exists a* $\bar{\lambda} > 0$ *such that if* $\lambda \in (0, \bar{\lambda})$, $\frac{dCoC}{d\lambda} \gtreqless 0$ *if* $\beta_z \lesseqgtr \frac{3}{2}\frac{\left(\frac{1}{\gamma}\tau_s m_t\right)\left(\frac{1}{\gamma}\tau_s m_g\right)}{\left(\frac{1}{\gamma}\tau_s m_t\right)^2+\frac{\tau+\tau_s}{\tau_n}} - \frac{1}{2}\frac{\left(\frac{1}{\gamma}\tau_s m_t\right)^2+\frac{\tau+\tau_s}{\tau_n}}{\left(\frac{1}{\gamma}\tau_s m_t\right)^2}$.

From Proposition 7 we know that price informativeness PI_g and PI_t move in the opposite directions in response to λ when the preference heterogeneity is large. Proposition 8 establishes a related result for the cost of capital: If β_z is sufficiently small, the reduction in PI_t dominates the improvement in PI_g, and the cost of capital increases in λ.[22]

5.3 Precise non-monetary information

In this section, we demonstrate via numerical example that the results outlined in Propositions 7 and 8 hold for a wide range of λ's. We pick parameters so that $\frac{dPI_t}{d\lambda} < 0$ and $\frac{dCoC}{d\lambda} > 0$ for sufficiently imprecise non-monetary information. We compute PI_t, PI_g and CoC as functions of λ and plot them in Figure 4. We find that multiple equilibria are possible when λ is close to one, that is, when monetary and non-monetary information have similar precisions. When λ is small, the only possible equilibrium is the T-equilibrium, where trading is dominated by traditional investors. Green investors choose not to trade actively because the non-monetary payoff is very uncertain. Naturally, this equilibrium exists as long as λ is not too large. Importantly, we find that the comparative statics results established in Propositions 7 and 8 hold for all values of λ if the T-equilibrium is played. This finding is reassuring because it confirms that our predictions continue to hold even when λ is not small.[23]

5.4 Empirical implications and discussions

Proposition 7 delivers an important message for regulators who push for more ESG-related information provision through, for example, mandatory disclosure. The conventional wisdom that more precise information always helps investors make more informed decisions

[22] In fact, it is possible that CoC always declines in λ because the cutoff for β_z in Proposition 8 can be negative. This is the case, for example, when the mass of green investors is small.

[23] As the goal of this exercise is to illustrate that our main results continue to hold even when non-monetary information is relatively precise, we focus on that and leave further explorations for future work.

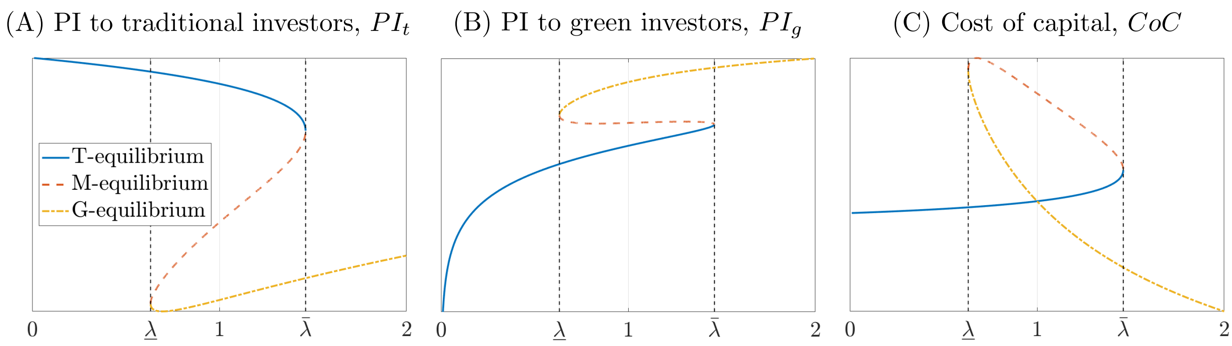

Figure 4: Price informativeness to traditional (panel A) and green (panel B) investors and cost of capital (panel C) as functions of the relative precision of monetary information λ. Y-axes are in the log scale. Parameters used: $m_t = m_g = 1$, $\beta_\delta = \beta_z = \frac{1}{\sqrt{2}}$, $\gamma = 1$, $\tau_s = 5$, $\tau = 1$, $\tau_n = 4$.

does not necessarily hold when investors have heterogeneous preferences. In particular, we show that better ESG-related information may encourage investors to trade against one other. As a result, better ESG-related information can reduce the price informativeness about firms' cash flows, which adversely affects traditional investors' learning from the price and increases their investment risk.[24]

Proposition 8 shows that with sufficient heterogeneity in preferences across investors, improving the precision of ESG-related information can increase the firm's financing cost. This may harm firms' incentive to voluntarily disclose precise ESG-related information. This potentially can explain why—despite regulators' efforts—the quality of ESG-related information is still quite low in the eyes of many market participants.[25] In other words, even though firms are mandated to publish more ESG reports, they can potentially benefit from limiting informativeness of these reports.

Existing empirical evidence does not provide a clear picture of the relationship between voluntary ESG disclosures and cost of capital. For instance, Dhaliwal, Li, Tsang and Yang (2011) document that an issuance of an ESG report is associated with a higher cost of capital after controlling for issuer's ESG performances. Clarkson, Fang, Li and Richardson (2013) use environmental disclosure scores to measure the quality of ESG disclosure and find no relationship between the score and cost of capital. However, since voluntary disclosure is an endogenous choice, predictions of our model might be more

[24]It is important to mention that our paper does not make any welfare statements. Our goal is rather to point out that there might be certain negative consequences of ESG-information-improving policies.

[25]According to the report by the State Street, the correlation between ESG scores provided by MSCI and Sustainalytics, the two biggest ESG data providers, is only about 0.53. Meanwhile, investors frequently blame the unavailability of reliable data for a weak integration of ESG principles into their strategies (Eccles, Kastrapeli and Potter, 2017 and Amel-Zadeh and Serafeim, 2018).

applicable for understanding the impacts of mandatory disclosure. Chen, Hung and Wang (2018) utilize a natural experiment in which Chinese stock exchanges required a subset of firms in 2008 to issue ESG reports and find a consequent reduction in profitability. One can utilize similar policies to study how exogenous changes in the informational environment affect asset prices.

Furthermore, our results in Propositions 7 and 8 that the interactions between information and asset prices vary with the investor base composition provide a rich set of testable implications. One can potentially utilize the cross-sectional variation in ESG funds' holdings and investigate whether improvements in the informational environment lead to different outcomes after controlling for changes in ESG fundamentals.

6 Conclusion

In light of the growing appetite for ESG investing, we analyze the interactions between green and traditional investors in the asset market. Due to preference heterogeneity, trading by one group of investors makes price less informative to the other group, which raises the risk of holding the asset. Such interactions give rise to a number of interesting and important results. First, multiple equilibria with different pricing functions may coexist. Second, an increase in the fraction of green investors may increase firm's cost of capital. It becomes particularly large when the investor base is balanced, i.e. masses of green and traditional investors are similar. Finally, we show that, in contrast to the conventional wisdom, better information about firms' ESG performance can make the price less informative to traditional investors and increase the firm's cost of capital.

We believe that our model has a wide range of applications in asset markets where investors with heterogeneous preferences interact and affect each other's investment choices. In addition to financial markets, our model also might be useful for non-financial assets, such as real estate and arts and collectibles, for which investors are likely to have heterogeneous preferences.[26]

[26] A 2018 survey conducted by the U.S. Trust finds that millennial collectors are more likely to view pieces of arts as investment objects than older generations, who are mostly concerned about their aesthetic values.

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Appendix

For Appendix A: A.1 derives equation (7); A.2 proves results of the simplified model in Section 2; A.3 proves Proposition 1 and Corollary 4; A.4 proves Proposition 2. Appendix B contains proofs for Section 4, as well as of Corollary 3. Appendix C contains proofs for Section 5. Appendix D analyzes the model with a general information structure and discusses conditions required for multiplicity of equilibria in the trading stage. Appendix E shows that trading stage features unique equilibria when investors have homogeneous preferences but heterogeneous information.

Proofs frequently involve some tedious yet straightforward algebraic manipulations, which we perform via Matlab Symbolic Math Toolbox. Therefore, we often omit intermediate steps and present only final results. These omitted derivations are available upon request.

A Baseline proofs

A.1 Preliminary derivations

We start by deriving the quintic equation (7) for ξ_δ. It is straightforward to see that the aggregate demand for stock from investors of group $j \in \{t, g\}$ is given by

$$D^j(\tilde{z}, \tilde{\delta}, \tilde{q}) = m^j \frac{1}{\gamma} \frac{\tilde{z}\beta_z^j \frac{\tau_s}{\tau+\tau_s} + \tilde{\delta}\beta_\delta^j \frac{\tau_s}{\tau+\tau_s} + \left(q_z\beta_z^j \frac{1}{\tau+\tau_s} + q_\delta\beta_\delta^j \frac{1}{\tau+\tau_s}\right) \frac{\tilde{q}-q_0-\tilde{z}q_z\frac{\tau_s}{\tau_s+\tau}-\tilde{\delta}q_\delta\frac{\tau_s}{\tau_s+\tau}}{q_z^2\frac{1}{\tau+\tau_s}+q_\delta^2\frac{1}{\tau+\tau_s}+q_n^2\frac{1}{\tau_n}} - \tilde{q}}{\frac{1}{\tau+\tau_s} - \frac{\left(q_z\beta_z^j\frac{1}{\tau+\tau_s}+q_\delta\beta_\delta^j\frac{1}{\tau+\tau_s}\right)^2}{q_z^2\frac{1}{\tau+\tau_s}+q_\delta^2\frac{1}{\tau+\tau_s}+q_n^2\frac{1}{\tau_n}}}. \tag{9}$$

Plugging (9) in the market clearing condition, $D^t\left(\tilde{z}, \tilde{\delta}, \tilde{q}\right) + D^g\left(\tilde{z}, \tilde{\delta}, \tilde{q}\right) + \tilde{n} = 1$, and then equalizing coefficients in front of $\tilde{z}$, $\tilde{\delta}$ and $\tilde{n}$, we get:

$$\begin{aligned} \xi_z &= \frac{1}{\gamma}\tau_s \left[m_t + m_g \frac{\beta_z\left(\xi_\delta^2 + p\right) - \xi_\delta\xi_z\beta_\delta}{\left(\xi_z\beta_\delta - \xi_\delta\beta_z\right)^2 + p}\right], \\ \xi_\delta &= \frac{1}{\gamma}\tau_s \left[-m_t \frac{\xi_\delta\xi_z}{\xi_\delta^2 + p} + m_g \frac{\beta_\delta\left(\xi_z^2 + p\right) - \xi_\delta\xi_z\beta_z}{\left(\xi_z\beta_\delta - \xi_\delta\beta_z\right)^2 + p}\right], \end{aligned} \tag{10}$$

where we define $p = \frac{\tau+\tau_s}{\tau_n}$ to simplify notations.

Consider a linear combination of the two equations.

$$\xi_z\beta_z + \xi_\delta\beta_\delta = \frac{1}{\gamma}\tau_s\left(m_g + \beta_z m_t - \beta_\delta m_t \frac{\xi_\delta\xi_z}{\xi_\delta^2 + p}\right).$$

ξ_z can be expressed as a function of ξ_δ:

$$\xi_z = \frac{\left(\frac{1}{\gamma}\tau_s m_g + \beta_z \frac{1}{\gamma}\tau_s m_t - \xi_\delta\beta_\delta\right)\left(\xi_\delta^2 + p\right)}{\beta_z\left(\xi_\delta^2 + p\right) + \beta_\delta \frac{1}{\gamma}\tau_s m_t \xi_\delta} \tag{11}$$

Finally, substituting in the expression for ξ_z, we can reduce the system of two equations into equation (7) of one unknown ξ_δ.

Below we work with equation (7). For brevity, we re-scale the masses of investors and define $\hat{m}_g = \frac{1}{\gamma}\tau_s\beta_\delta m_g$ and $\hat{m}_t = \frac{1}{\gamma}\tau_s\beta_\delta m_t$. Then equation (7) can be written as

$$\xi_\delta^5 - \hat{m}_g\xi_\delta^4 + 2p\xi_\delta^3 - 2\hat{m}_g p\xi_\delta^2 + \left(p^2 + \hat{m}_t^2 p\right)\xi_\delta - \hat{m}_g p^2 = 0. \tag{12}$$

A.2 The simplified model

Section 2 considers a special case where (i) $\beta_z = 0$ and $\beta_\delta = 1$ and (ii) $m_t = m_g = \frac{m}{2}$, or equivalently, $\hat{m}_t = \hat{m}_g = \frac{\hat{m}}{2}$ with $\hat{m} = \hat{m}_t + \hat{m}_g$. Equation (12) reduces to

$$\left(\xi_\delta^2 - \frac{\hat{m}}{2}\xi_\delta + p\right)\left(\xi_\delta^3 + p\xi_\delta - \frac{\hat{m}}{2}p\right) = 0.$$

Case 1: $\xi_\delta^3 + p\xi_\delta - \frac{\hat{m}}{2}p = 0 \Leftrightarrow \xi_\delta^2 + p = \frac{1}{\xi_\delta}\frac{\hat{m}}{2}p$. Then, using (11), we get

$$\xi_z = \frac{\left(\frac{\hat{m}}{2} - \xi_\delta\right)\left(\xi_\delta^2 + p\right)}{\frac{\hat{m}}{2}\xi_\delta} = \frac{\left(\frac{\hat{m}}{2} - \xi_\delta\right)p}{\xi_\delta^2} = \frac{\left(\frac{\hat{m}}{2} - \xi_\delta\right)}{\frac{\hat{m}}{2} - \xi_\delta}\xi_\delta = \xi_\delta.$$

Case 2: $\xi_\delta^2 - \frac{\hat{m}}{2}\xi_\delta + p = 0 \Leftrightarrow \xi_\delta^2 + p = \frac{\hat{m}}{2}\xi_\delta$. Again, using (11), we get

$$\xi_z = \frac{\left(\frac{\hat{m}}{2} - \xi_\delta\right)\left(\xi_\delta^2 + p\right)}{\frac{\hat{m}}{2}\xi_\delta} = \frac{\hat{m}}{2} - \xi_\delta.$$

Solving the quadratic equation for ξ_δ, we find

$$\xi_\delta = \frac{1}{2}\left[\frac{\hat{m}}{2} \pm \sqrt{\left(\frac{\hat{m}}{2}\right)^2 - 4p}\right].$$

In a solution with a higher ξ_δ, $v < 1$ because $\xi_\delta > \frac{\hat{m}}{4}$ and the price is more informative to the green investors. Conversely, in the solution with a lower ξ_δ, $v > 1$ because $\xi_\delta < \frac{\hat{m}}{4}$ and the price is more informative to traditional investors.

A.3 General case and equilibria stability

A.3.1 Number of equilibria and noise precision

Here we prove Proposition 1 in the general case with $m_t > 0$ and $m_g > 0$.

Proof.

The proof consists of three parts. In Part 1, we establish that equation (12) can have at least one and at most three real roots. In Part 2, we prove the existence of the threshold τ_n^*. Finally, in Part 3, we show that non-normalized price coefficients have conventional signs, i.e. $q_0 < 0$, $q_z, q_\delta, q_n > 0$.

Part 1: Number and signs of roots of (12).

Statement: Equation (12) has at least one and at most three real roots. All real roots are positive and are below $\hat{m}_g$.

Proof.

All real roots of equation (12) are positive because coefficients of odd powers are positive and coefficients of even powers are negative. It is also easy to see that all roots are below $\hat{m}_g$ because the left-hand side of equation (12) is clearly positive for all $\xi_\delta > \hat{m}_g$.

In principle, equation (12) can have from one to five real roots. Below we show that it can have at most three real roots. Denote the left-hand side of (12) by $f(\xi_\delta) = \xi_\delta^5 - \hat{m}_g\xi_\delta^4 + 2p\xi_\delta^3 - 2p\hat{m}_g\xi_\delta^2 + (p^2 + \hat{m}_t^2 p)\,\xi_\delta - \hat{m}_g p^2$. Taking first and second derivatives of $f(\xi_\delta)$, we get

$$\frac{\partial f}{\partial \xi_\delta} = 5\xi_\delta^4 - 4\hat{m}_g\xi_\delta^3 + 6p\xi_\delta^2 - 4\hat{m}_g p\xi_\delta + p^2 + \hat{m}_t^2 p,$$
$$\frac{\partial^2 f}{\partial \xi_\delta^2} = 20\xi_\delta^3 - 12\hat{m}_g\xi_\delta^2 + 12p\xi_\delta - 4\hat{m}_g p.$$

The equation $\frac{\partial^2 f}{\partial \xi_\delta^2} = 0$ has a unique real root because its discriminant is negative: $\Delta \propto -p\left(\left(m_g^2 - p\right)^2 + 4p^2\right) < 0$, where $\propto$ denotes proportionality up to a positive constant. Moreover, this root is positive because coefficients of odd powers are positive and coefficients of even powers are negative. Hence, $f(\xi_\delta)$ has a unique inflection point $\xi_\delta^{infl} > 0$ such that $f(\xi_\delta)$ is concave if $\xi_\delta < \xi_\delta^{infl}$ and convex if $\xi_\delta > \xi_\delta^{infl}$. Given also that $f(\xi_\delta)$ is a continuous function, it follows that it can have at most three intersections with the zero line. □

Part 2: Number of roots and precision of noise trading τ_n.

Statement: For any $\alpha = \frac{m_g}{m_t+m_g} \in (0,1)$, $\hat{m} = \hat{m}_t + \hat{m}_g > 0$, $\beta_\delta \in (0,1]$, $\exists \tau_n^*(\alpha, \hat{m}) > 0$ such that $\forall \tau_n \in (0, \tau_n^*)$ equation (12) has a unique solution; for $\tau_n = \tau_n^*$ it has two solutions when $\alpha \neq \frac{1}{2}$ and a unique solution when $\alpha = \frac{1}{2}$; $\forall \tau_n > \tau_n^*$ it has three solutions.

Proof.

The proof of this statement for a special case $\alpha = \frac{1}{2}$ is done in Section 2; in particular, $\tau_n^*\left(\frac{1}{2}, \hat{m}\right) = \frac{16(\tau+\tau_s)}{\hat{m}^2}$, where $\hat{m} = \hat{m}_t + \hat{m}_g = \frac{1}{\gamma}\tau_s\beta_\delta m$. Below we focus on the case when $\alpha \neq \frac{1}{2}$.

It is convenient to rewrite (12) as

$$\frac{1}{p}\left(\xi_\delta^3 + p\xi_\delta - \alpha\hat{m}p\right)\left(\xi_\delta^2 - \alpha\hat{m}\xi_\delta + p\right) = -\left(1-2\alpha\right)\hat{m}^2\xi_\delta. \quad (13)$$

Denote the left-hand side of the expression above by

$$g(\xi_\delta) = \frac{1}{p}\left(\xi_\delta^3 + p\xi_\delta - \alpha\hat{m}p\right)\left(\xi_\delta^2 - \alpha\hat{m}\xi_\delta + p\right).$$

The proof of Part 2 proceeds in several steps. In Lemmas 1 and 2, we show that there exist $\underline{p}$ and $\bar{p}$ such that equation (13) has one solution when $p > \bar{p}$ and three solutions when $p < \underline{p}$. Then, in Lemma 3, we show that if for a given p equation (13) has one or three solutions, then it has one or three solutions for any $\hat{p}$ above or below the given p, respectively. At the end, we show that there exists p^* such that equation (13) has two solutions, and any increase or decrease in p leaves with one or three solutions, respectively. Since $p = \frac{\tau+\tau_s}{\tau_n}$, there is a one-to-one mapping between p and τ_n for given τ and τ_s. The conditions on p then can be translated into conditions on τ_n.

Lemma 1. $\forall p \geq \bar{p} = \frac{1}{4}\alpha^2\hat{m}^2$ *equation* (13) *has a unique solution.*

Proof.

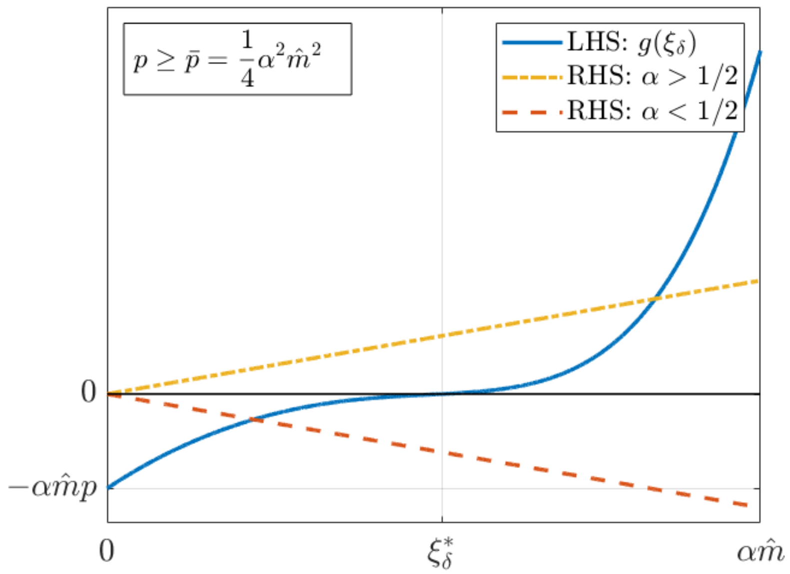

Figure 5: Unique solution to equation (13).

Take the first derivative of $g(\xi_\delta)$:

$$p\frac{\partial g}{\partial \xi_\delta} = 5\xi_\delta^4 - 4\alpha\hat{m}\xi_\delta^3 + 6p\xi_\delta^2 - 4\alpha\hat{m}p\xi_\delta + p^2 + \alpha\hat{m}^2 p = \tag{14}$$
$$\frac{1}{16}(2\xi_\delta - \alpha\hat{m})^2\left(20\xi_\delta^2 + 4\alpha\hat{m}\xi_\delta + 5\alpha^2\hat{m}^2\right) + \left(p - \frac{1}{4}\alpha^2\hat{m}^2\right)\left(6\xi_\delta^2 - 4\alpha\hat{m}\xi_\delta + p + \frac{5}{4}\alpha^2\hat{m}^2\right).$$

First of all, if $p \geq \frac{1}{4}\alpha^2\hat{m}^2$ then $g(\xi_\delta)$ is an increasing function. If, in addition, $\alpha < \frac{1}{2}$, equation (13) has a unique solution because the left-hand side is increasing in ξ_δ, $g(0) = -\alpha\hat{m}p < 0$ and $g(\alpha\hat{m}) > 0$, while the right-hand side is decreasing in ξ_δ and its value at

$\xi_\delta = 0$ is zero. This case is illustrated by the intersecting solid blue line and dashed red line in Figure 5.

Assume now that $\alpha > \frac{1}{2}$ and $p \geq \frac{1}{4}\alpha^2\hat{m}^2$. In this case, both the left-hand side and the right-hand side of equation (13) are increasing in ξ_δ. They still have only one intersection because, as we show below, the left-hand side—$g(\xi_\delta)$—is an increasing convex function $\forall \xi_\delta \geq \xi_\delta^* > 0$ where ξ_δ^* is the (unique) real root of equation $g(\xi_\delta) = 0$. This case is illustrated by the intersecting solid blue line and dot-dashed yellow line in Figure 5.

Take the second derivative of $g(\xi_\delta)$:

$$\frac{1}{4}p\frac{\partial^2 g}{\partial \xi_\delta} = 5\xi_\delta^3 - 3\alpha\hat{m}\xi_\delta^2 + 3p\xi_\delta - \alpha\hat{m}p.$$

Recall that ξ_δ^* solves $g(\xi_\delta) = 0$, which for $p \geq \frac{1}{4}\hat{m}_g^2$ implies that $(\xi_\delta^*)^3 + p\xi_\delta^* - \alpha\hat{m}p = 0$. Then for $\xi_\delta \geq \xi_\delta^*$

$$\frac{1}{4}p\frac{\partial^2 g}{\partial \xi_\delta} = 5\xi_\delta^3 - 3\alpha\hat{m}\xi_\delta^2 + 3p\xi_\delta - (\xi_\delta^*)^3 - p\xi_\delta^* \overset{\xi_\delta \geq \xi_\delta^*}{\geq} \left(4\xi_\delta^2 - 3\alpha\hat{m}\xi_\delta + 2p\right)\xi_\delta.$$

The term in parentheses has real roots if $9\alpha^2\hat{m}^2 - 32p \geq 0$. The largest real root (if exists) is given by

$$\frac{3\alpha\hat{m} + \sqrt{9\alpha^2\hat{m}^2 - 32p}}{8} \overset{p \geq \frac{1}{4}\hat{\alpha}^2 m^2}{\leq} \frac{1}{2}\alpha\hat{m}.$$

However, $\xi_\delta^* \geq \frac{1}{2}\alpha\hat{m}$ when $p \geq \frac{1}{4}\alpha^2\hat{m}^2$: if $p = \frac{1}{4}\alpha^2\hat{m}^2$, $\xi_\delta^* = \frac{1}{2}\alpha\hat{m}$, and $\frac{\partial \xi_\delta^*}{\partial p} \geq 0$ for all viable values of ξ_δ^*, i.e. $\xi_\delta^* \in (0, \alpha\hat{m})$. Therefore, $\frac{\partial^2 g}{\partial \xi_\delta} \geq 0$ whenever $\xi_\delta \geq \xi_\delta^*$. □

Lemma 2. $\exists \underline{p} \in (0, \bar{p})$ *such that* $\forall p \in (0, \underline{p})$ *equation* (13) *has 3 solutions.*

Proof.

Write (13) in its original form as in (12):

$$\begin{aligned} f(\xi_\delta) &= \xi_\delta^5 - \alpha\hat{m}\xi_\delta^4 + 2p\xi_\delta^3 - 2\alpha\hat{m}p\xi_\delta^2 + \left(p^2 + (1-\alpha)^2\hat{m}^2 p\right)\xi_\delta - \alpha\hat{m}p^2 \\ &= (\xi_\delta - \alpha\hat{m})\left(\xi_\delta^4 + 2p\xi_\delta^2 + p^2\right) + (1-\alpha)^2\hat{m}^2 p\xi_\delta = 0. \end{aligned}$$

Notice that $f(\alpha\hat{m}) = (1-\alpha)^2\hat{m}^2 p\xi_\delta > 0$. At the same time, $\exists p_1 > 0$ such that $\forall p \in (0, p_1]$, $f\left(\alpha\hat{m} - \sqrt{p}\right) = -\sqrt{p}\left(\left(\alpha\hat{m} - \sqrt{p}\right)^4 + 2p\left(\alpha\hat{m} - \sqrt{p}\right)^2 + p^2\right) + (1-\alpha)^2\hat{m}^2 p\left(\alpha\hat{m} - \sqrt{p}\right) <$

0. This is because we can always pick a sufficiently small p_1, so that $(1-\alpha)^2\hat{m}^2 p\left(\alpha\hat{m}-\sqrt{p}\right)$ is smaller than $\sqrt{p}\left(\left(\alpha\hat{m}-\sqrt{p}\right)^4+2p\left(\alpha\hat{m}-\sqrt{p}\right)^2+p^2\right)$ $\forall p\in(0,p_1)$.

Notice also that $f(0)=-\alpha\hat{m}p^2<0$. Evaluate $f(\cdot)$ at $\frac{\alpha\hat{m}}{(1-\alpha)^2\hat{m}^2}p+ap$, where a is an arbitrary positive constant:

$$f\left(\frac{\alpha\hat{m}}{(1-\alpha)^2\hat{m}^2}p+ap\right)=p^2\Bigg[p^3\left(\frac{\alpha\hat{m}}{(1-\alpha)^2\hat{m}^2}+a\right)^5-\alpha\hat{m}p^2\left(\frac{\alpha\hat{m}}{(1-\alpha)^2\hat{m}^2}+a\right)^4+$$
$$2p^2\left(\frac{\alpha\hat{m}}{(1-\alpha)^2\hat{m}^2}+a\right)^3-2\alpha\hat{m}p\left(\frac{\alpha\hat{m}}{(1-\alpha)^2\hat{m}^2}+a\right)^2+p\left(\frac{\alpha\hat{m}}{(1-\alpha)^2\hat{m}^2}+a\right)+(1-\alpha)^2\hat{m}^2a\Bigg].$$

$\exists p_2>0$ such that $\forall p\in(0,p_2)$, $f\left(\frac{\alpha\hat{m}}{(1-\alpha)^2\hat{m}^2}p+ap\right)>0$ because the last term in the expression in brackets, $(1-\alpha)^2\hat{m}^2a$, does not depend on p, while the other terms are proportional to p^b, $b=1,2,3$.

Finally, define p_3 such that $\frac{\alpha\hat{m}}{(1-\alpha)^2\hat{m}^2}p+ap=\alpha\hat{m}-\sqrt{p}$. Therefore, $\forall p\in(0,p_3)$, $\frac{\alpha\hat{m}}{(1-\alpha)^2\hat{m}^2}p+ap<\alpha\hat{m}-\sqrt{p}$. Pick $\underline{p}=\min\left[p_1,p_2,p_3\right]$; then $\forall p\in(0,\underline{p})$, a continuous function $f(\xi_\delta)$ changes its sign from negative to positive (at least) twice, hence equation (13) has (at least) three solutions. Since it cannot have more than three solutions, it must be that it has exactly three solutions. □

Lemma 3. *For any $p>0$, if equation (13) has 3 solutions at p, then it has 3 solutions $\forall\hat{p}\in(0,p)$; if equation (13) has 1 solution at p, then it has 1 solution $\forall\hat{p}>p$.*

Proof.

Since the result trivially holds when $p\in(0,\underline{p}]$ and $p\geq\bar{p}$, we focus on the case when $p\in(\underline{p},\bar{p})$. In particular, $p<\bar{p}=\frac{1}{4}\alpha^2\hat{m}^2$. For such p, equation $g(\xi_\delta)=0$ has three solutions (this is a special case $m_g=m_t$ studied in Section 2).

Differentiate $g(\xi_\delta)$ with respect to p:

$$\frac{\partial g}{\partial p}=-\frac{1}{p^2}\xi_\delta^5+\frac{1}{p^2}\alpha\hat{m}\xi_\delta^4+\xi_\delta-\alpha\hat{m}=-\frac{1}{p^2}\left(\xi_\delta^2+p\right)\left(\xi_\delta+\sqrt{p}\right)\left(\xi_\delta-\sqrt{p}\right)\left(\xi_\delta-\alpha\hat{m}\right)$$

Then $\frac{\partial g}{\partial p}<0$ when $\xi_\delta\in(0,\sqrt{p})$ and $\frac{\partial g}{\partial p}>0$ when $\xi_\delta\in(\sqrt{p},\alpha\hat{m})$. In particular, notice that $\frac{\partial g}{\partial p}(\xi_\delta^*)>0$, where $\xi_\delta^*>\sqrt{p}$ solves $\xi_\delta^3+p\xi_\delta-\alpha\hat{m}p=0$. The latter is true because $\xi_\delta^3+p\xi_\delta-\alpha\hat{m}p$ is an increasing function which is negative at $\xi_\delta=\sqrt{p}$: $p\sqrt{p}+p\sqrt{p}-\alpha\hat{m}p=$

$p\left(2\sqrt{p}-\alpha\hat{m}\right) \overset{p<\frac{1}{4}\alpha^2\hat{m}^2}{<} 0.$

Case 1: $\alpha < \frac{1}{2}$

Suppose $\exists p \in (\underline{p}, \bar{p})$ such that equation (13) has three solutions. This case is illustrated in Figure 6. The first root ξ_δ^i is smaller than the smallest root of $g(\xi_\delta) = 0$, i.e. $\frac{\alpha\hat{m}-\sqrt{\alpha^2\hat{m}^2-4p}}{2}$. Since $\frac{\alpha\hat{m}-\sqrt{\alpha^2\hat{m}^2-4p}}{2} \overset{p<\frac{1}{4}\alpha^2\hat{m}^2}{<} \sqrt{p}$, we have $\xi_\delta^i < \sqrt{p}$. The other two roots are above ξ_δ^* and $\sqrt{p}$: $\xi_\delta^{iii} > \xi_\delta^{ii} > \xi_\delta^* > \sqrt{p}$. A marginal decrease in p is going to shift $g(\xi_\delta)$ (blue solid line) downwards $\forall \xi_\delta \in (\sqrt{p}, \alpha\hat{m})$. At the same time, the right-hand side of equation (13) (yellow dot-dashed line) does not depend on p and, thus, is not going to move. Therefore, for a marginally smaller p equation (13) still has three solutions.

An analogous argument holds when for a given p there is a unique solution to (13): then a marginal increase in p is going to shift the relevant part of $g(\theta_\delta)$ up, while the right-hand side line is not going to move. Then equation (13) still has one solution.

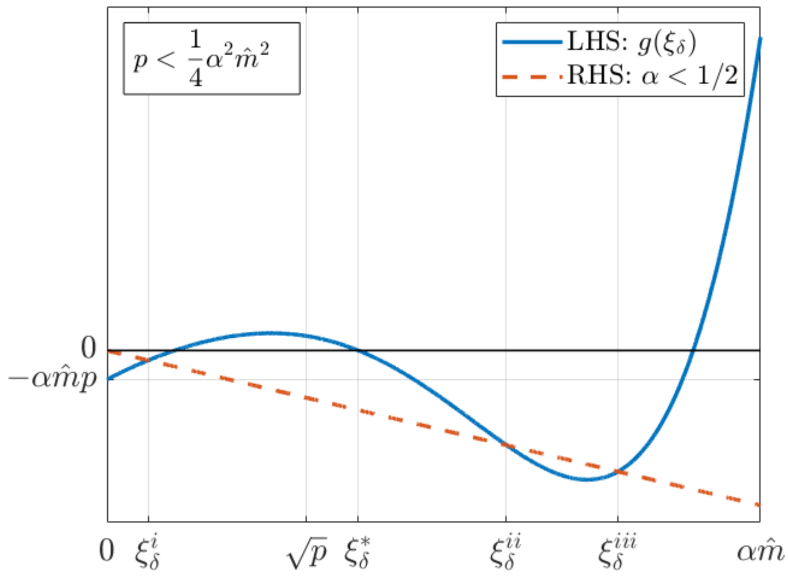

Figure 6: Three solutions to equation (13): $\alpha < \frac{1}{2}$.

Case 2: $\alpha > \frac{1}{2}$

Suppose $\exists p \in (\underline{p}, \bar{p})$ such that equation (13) has three solutions. This case is illustrated in Figure 7. In this graph, two thin black lines (marked with crosses and circles) are tangent to the concave and the convex part of $g(\xi_\delta)$: As was established in Part 1 of this proof, $g(\xi_\delta)$ has a unique inflection point ξ_δ^{infl}, and it is concave on $\xi_\delta \in (0, \xi_\delta^{infl})$ and convex on $(\xi_\delta^{infl}, \alpha\hat{m})$. Two tangent points, $\xi_\delta^{tang,1} < \xi_\delta^{infl} < \xi_\delta^{tang,2}$, solve

$$h(\xi_\delta) = \frac{\partial g(\xi_\delta)}{\partial \xi_\delta}\xi_\delta - g(\xi_\delta) = \frac{1}{p}\left(4\xi_\delta^5 - 3\alpha\hat{m}\xi_\delta^4 + 4p\xi_\delta^3 - 2\alpha\hat{m}p\xi_\delta^2 + \hat{\alpha}mp^2\right) = 0.$$

Notice that $\frac{\partial h}{\partial \xi_\delta} = \xi_\delta \frac{\partial^2 g}{\partial \xi_\delta^2}$. Therefore, $h(\xi_\delta)$ is decreasing on $\xi_\delta \in (0, \xi_\delta^{infl})$ and is increasing on $\xi_\delta \in (\xi_\delta^{infl}, \alpha\hat{m})$.

Evaluate $h(\xi_\delta)$ at $\sqrt{p}$:

$$h\left(\sqrt{p}\right) = 4p\left(2\sqrt{p} - \alpha\hat{m}\right) \overset{p<\frac{1}{4}\alpha^2\hat{m}^2}{<} 0.$$

Because $h(0) > 0$, $h(\alpha\hat{m}) > 0$ and $h\left(\sqrt{p}\right) < 0$, $h(\xi_\delta) = 0$ has two solutions $\xi_\delta^{tang,1} < \sqrt{p} < \xi_\delta^{tang,2}$.[27] The two tangent lines shown in Figure 7 lines going through zero are described by equations $f^{tang,k}(\xi_\delta) = \frac{g\left(\xi_\delta^{tang,k}\right)}{\xi_\delta^{tang,k}}\xi_\delta$, $k = 1, 2$.

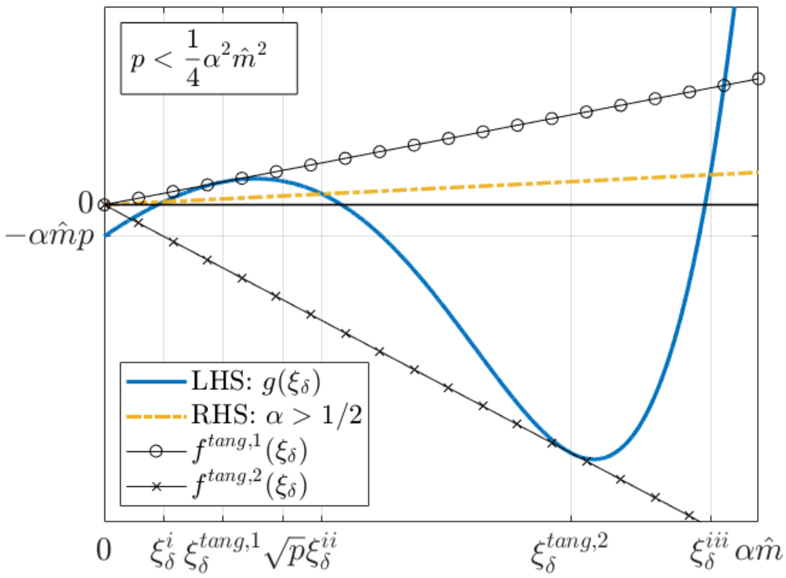

Figure 7: Three solutions to equation (13): $\alpha > \frac{1}{2}$.

Importantly, the right-hand side of equation (13), $-(1-2\alpha)\,\hat{m}^2\xi_\delta$, intersects $g(\xi_\delta)$ three times when its slope is smaller than the slope of the tangent line $f^{tang,1}(\xi_\delta)$, so that $\xi_\delta^i < \xi_\delta^{tang,1} < \sqrt{p}$. A marginal decrease in p is going to shift $g(\xi_\delta)$ (blue solid line)

[27]Notice also that $\xi_\delta^{tang,1} < \frac{\alpha\hat{m}}{2} < \xi_\delta^{tang,2}$ because $h\left(\frac{\alpha\hat{m}}{2}\right) = \alpha\hat{m}\frac{1}{p}\left(-\frac{\alpha^2\hat{m}^2}{4} + p\right)\left(\frac{\alpha^2\hat{m}^2}{4} + p\right) \overset{\frac{\alpha^2\hat{m}^2}{4} > p}{<} 0$. This will be used in the proof of Proposition 2.

upwards $\forall \xi_\delta \in (0, \sqrt{p})$. At the same time, the right-hand side of equation (13) (red dashed line) does not depend on p and, thus, is not going to move. Therefore, for a marginally smaller p equation (13) still has three solutions.

An analogous argument holds when for a given p there is a unique solution to (13). Then a marginal increase in p is going to shift the relevant part of $g(\xi_\delta)$ down, while the right-hand side line is not going to move. Then equation (13) still has one solution. □

Having proved Lemmas 1-3, we are now ready to complete the proof of Proposition 1. From Lemmas 1 and 2, it follows that $\exists \bar{p}^* \geq \underline{p}^* > 0$ such that equation (13) has 3 solutions when $p \in (0, \underline{p}^*)$ and 1 solution when $p \in (\bar{p}^*, \infty)$. Moreover, by Lemma 3, it must be the case that for $p \in [\underline{p}^*, \bar{p}^*]$, equation (13) has two solutions. It must be the case, however, that $\underline{p}^* = \bar{p}^*$. To see this, focus on the case $\alpha > \frac{1}{2}$ without loss of generality. The equation (13) then has two solutions if and only if the left-hand side of (13) coincides with the tangent line $f^{tang,1}(\xi_\delta)$ (see Figure 7). However, any marginal increase or decrease in p leaves equation (13) with one or three solutions, respectively.

Finally, since by definition $p = \frac{\tau+\tau_s}{\tau_n}$, given τ and τ_s, define $\tau_n^* = \frac{\tau+\tau_s}{p^*}$. Then equation (13) has two solutions if $\tau_n = \tau_n^*$, one solution if $\tau_n \in (0, \tau_n^*)$ and three solutions if $\tau_n > \tau_n^*$. □

Part 3: Signs of price coefficients.

Statement: $q_0 < 0$, $q_z > 0$, $q_\delta > 0$, $q_n > 0$.

Proof.

From Part 1, all solutions to (12) are positive and below $\hat{m}_g = \alpha \hat{m}$. By equation (11), we have

$$\xi_z = \frac{(\hat{m}_g + \beta_z \hat{m}_t - \xi_\delta \beta_\delta^2)(\xi_\delta^2 + p)}{\beta_\delta \beta_z (\xi_\delta^2 + p) + \beta_\delta \hat{m}_t \xi_\delta} > \frac{(\hat{m}_g + \beta_z \hat{m}_t - \hat{m}_g \beta_\delta^2)(\xi_\delta^2 + p)}{\beta_\delta \beta_z (\xi_\delta^2 + p) + \beta_\delta \hat{m}_t \xi_\delta} > 0.$$

Recall that $\xi_\delta = \frac{q_\delta}{q_n}$ and $\xi_z = \frac{q_z}{q_n}$. Therefore, q_z, q_δ and q_n have the same sign.

When we match price coefficients in the market clearing condition, we have:

$$\frac{\gamma}{\tau + \tau_s} = q_n \left[m_g \frac{(1 + q_n^2 p) - (q_z \beta_z + q_\delta \beta_\delta)}{(q_z \beta_\delta - q_\delta \beta_z)^2 + q_n^2 p} + m_t \frac{(1 + q_n^2 p) - q_z}{q_\delta^2 + q_n^2 p} \right].$$

One can easily see that if q_z, q_δ and q_n are all negative, the right-hand side is negative. Thus, by contradiction, we conclude that q_z, q_δ and q_n are all positive.

We are left to show that $q_0 < 0$. Again, by matching coefficients in the market clearing condition, we have:

$$-\frac{\gamma}{\tau + \tau_s} = q_0 \left[\frac{m_g}{(\beta_z q_\delta - \beta_\delta q_z)^2 + q_n^2 p} + \frac{m_t}{q_\delta^2 + q_n^2 p} \right] (1 + q_n^2 p).$$

Therefore, $q_0 < 0$. □

Proposition 1 follows from Parts 1-3. □

A.3.2 Stability of equilibria

Plugging expression (11) for $\xi_z(\xi_\delta)$ in the right-hand side of the second equation of (10), we can write $\xi_\delta = J(\xi_\delta)$. Moreover, $J(\xi_\delta) - \xi_\delta = -k(\xi_\delta) \times f(\xi_\delta)$, where $k(\xi_\delta) > 0\ \forall \xi_\delta$ and $f(\xi_\delta)$ is the left-hand side of (7). Then at any solution ξ_δ^{root} we have $\left.\frac{\partial[J(\xi_\delta) - \xi_\delta]}{\partial \xi_\delta}\right|_{\xi_\delta = \xi_\delta^{root}} = -k(\xi_\delta^{root}) f'(\xi_\delta^{root})$. From our analysis in Appendix A.3.1, it follows that when solution is unique, $f'(\xi_\delta^{root}) > 0$ (see Figure 5). When there are three roots, $f'(\xi_\delta^{root}) > 0$ for $\xi_\delta^{root} = \xi_\delta^{i}, \xi_\delta^{iii}$ and $f'(\xi_\delta^{root}) < 0$ for $\xi_\delta^{root} = \xi_\delta^{ii}$ (see Figures 6 and 7). Since ξ_δ^{i}, ξ_δ^{ii} and ξ_δ^{iii} correspond to T-, M- and G-equilibria, respectively, Corollary 4 follows.

A.4 Comparative statics of τ_n^*

We proceed by establishing comparative statics properties of τ_n^* stated in Proposition 2.

Proof.

Comparative statics with respect to α

Fix $\alpha_1 < \frac{1}{2}$. Adding $\alpha_1^2 \hat{m}^2 \xi_\delta$ on both sides, equation (13) can be written as

$$\frac{1}{p}\left(\xi_\delta^5 - \alpha \hat{m} \xi_\delta^4 + 2p\xi_\delta^3 - 2\alpha \hat{m} p \xi_\delta^2 + \left(p^2 + \alpha_1^2 \hat{m}^2 p\right)\xi_\delta - \alpha \hat{m} p^2\right) = -\left((1-\alpha)^2 - \alpha_1^2\right)\hat{m}^2 \xi_\delta, \tag{15}$$

By definition, at $\alpha = \alpha_1$ and $\tau_n = \tau_n^*(\alpha_1, \hat{m})$ equation (15) has two solutions (this is illustrated by $f^{tang,2}(\xi_\delta)$ and $g(\xi_\delta)$ in Figure 7). Denote the left-hand side of (15) by $l(\xi_\delta, \alpha, \alpha_1, p)$. Note that $l(\xi_\delta, \alpha, \alpha, p) = g(\xi_\delta, \alpha, p)$, where $g(\xi_\delta, \alpha, p)$ is defined by (14).

Consider a marginal increase in α from α_1 to $\alpha_1+d\alpha$. That shifts the left-hand side of (15) down and the right-hand side of (15) up. Then equation $l\left(\xi_\delta, \alpha_1 + d\alpha, \alpha_1, \frac{\tau+\tau_s}{\tau_n^*(\alpha_1,\hat{m})}\right) = -\left((1-\alpha_1-d\alpha)^2 - \alpha_1^2\right)\hat{m}^2\xi_\delta$ has three solutions. From Lemma 3, it then follows that $\tau_n^*(\alpha_1+d\alpha,\hat{m}) < \tau_n^*(\alpha_1,\hat{m})$. Hence, $\frac{\partial\tau_n^*}{\partial\alpha} < 0$ when $\alpha < \frac{1}{2}$.

Analogous arguments can be made to show that $\frac{\partial\tau_n^*}{\partial\alpha} > 0$ when $\alpha > \frac{1}{2}$.

Comparative statics with respect to $\hat{m}$

Divide (13) by $\hat{m}^2$ to get

$$\frac{1}{\hat{m}^2} g(\xi_\delta, \hat{m}, p) = \frac{1}{p\hat{m}^2}(\xi_\delta - \alpha\hat{m})\left(\xi_\delta^2 + p\right)^2 + \alpha^2\xi_\delta = -(1-2\alpha)\xi_\delta. \tag{16}$$

Then

$$\frac{\partial\left[\frac{p}{\hat{m}^2}g\right]}{\partial\hat{m}} = \frac{\alpha\hat{m} - 2\xi_\delta}{\hat{m}^3}\left(\xi_\delta^2 + p\right)^2,$$

so that $\frac{p}{\hat{m}^2}g$ increases in $\hat{m}$ when $\xi_\delta \in \left(0, \frac{\alpha\hat{m}}{2}\right)$ and decreases in $\hat{m}$ when $\xi_\delta \in \left(\frac{\alpha\hat{m}}{2}, \alpha\hat{m}\right)$.

Suppose that $\alpha < \frac{1}{2}$. Fix $\hat{m}_1 > 0$. By definition, at $\tau_n = \tau_n^*(\alpha, \hat{m}_1)$ equation (16) has two solutions. This is illustrated by the solid blue and the red dashed lines in Figure 8, which intersect twice (in particular, the largest intersection $\xi_\delta^{tang,2}$ is a tangent point). Recall that $\xi_\delta^{tang,2} > \frac{\alpha\hat{m}_1}{2}$ (see footnote 27). Therefore, holding all other parameters fixed, a marginal increase in $\hat{m}$ from $\hat{m}_1$ to $\hat{m}_2$ shifts the curve $\frac{1}{\hat{m}^2}g(\xi_\delta, \hat{m}_1, p^*(\alpha, \hat{m}_1))$ down $\forall\xi_\delta \in \left(\frac{\alpha\hat{m}_1}{2}, \alpha\hat{m}_1\right)$, as shown in Figure 8 (crossed blue solid line). Therefore, there exist three solutions to (13) when $\hat{m} = \hat{m}_2$. From Lemma 3, it then follows that $\frac{\partial\tau_n^*}{\partial\hat{m}} < 0$ when $\alpha < \frac{1}{2}$.

Analogous arguments can be made to show that $\frac{\partial\tau_n^*}{\partial\hat{m}} < 0$ when $\alpha > \frac{1}{2}$. □

B Growth of green investors

B.1 Price informativeness

In this section, we analyze how price informativeness changes as the fraction of green investors α increases and prove Proposition 5. Notice that Corollary 3 directly follows from Proposition 5.

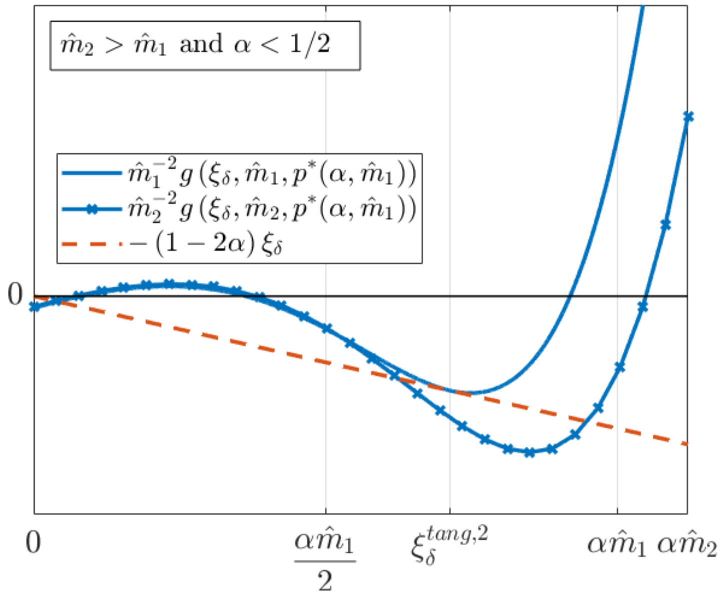

Figure 8: Comparative statics of $p^*(\alpha, \hat{m})$ with respect to $\hat{m}$.

Proof.

Denote $x = \beta_\delta \xi_z - \beta_z \xi_\delta$. The system of equations (10) can then be written as

$$\begin{aligned} x^2 + p &= \alpha \frac{1}{\gamma} \tau_s \beta_\delta m \frac{p}{\xi_\delta}, \\ \xi_\delta^2 + p &= (1-\alpha) \frac{1}{\gamma} \tau_s \beta_\delta m \frac{p}{x}, \end{aligned} \tag{17}$$

Both x and ξ_δ are positive. Take the first order derivative with respect to α for the system (17) to obtain

$$\begin{cases} 2x\xi_\delta x' + (x^2 + p)\xi_\delta' - p\frac{1}{\gamma}\tau_s \beta_\delta m = 0, \\ (\beta_\delta^2 + p)x' + 2x\beta_\delta \beta_\delta' + p\frac{1}{\gamma}\tau_s \beta_\delta m = 0. \end{cases}$$

Here we use x' and ξ_δ' to denote derivatives with respect to α. Summing the two equations, we have

$$\left(\beta_\delta^2 + 2x\xi_\delta + p\right) x' + (x^2 + 2x\xi_\delta + p)\xi_\delta' = 0. \tag{18}$$

The absolute and relative price informativeness can be expressed as

$$PI_t = \frac{\tau + \tau_s}{\beta_\delta^2} \frac{2\beta_z x \xi_\delta + \xi_\delta^2 + x^2 + \beta_\delta^2 p}{\xi_\delta^2 + p},$$
$$PI_g = \frac{\tau + \tau_s}{\beta_\delta^2} \frac{2\beta_z x \xi_\delta + \xi_\delta^2 + x^2 + \beta_\delta^2 p}{x^2 + p},$$
$$v = \frac{PI_t}{PI_g} = \frac{x^2 + p}{\xi_\delta^2 + p}.$$

Below we analyze the comparative statics of PI_t, PI_g and v with respect to α.

Comparative statics of PI_t with respect to α

$$\frac{dPI_t}{d\alpha} = \frac{\tau + \tau_s}{\beta_\delta^2} \frac{(2\beta_z x' \xi_\delta + 2\beta_z x \xi_\delta' + 2\xi_\delta \xi_\delta' + 2xx')(\xi_\delta^2 + p) - (2\beta_z x \xi_\delta + \xi_\delta^2 + x^2 + \beta_\delta^2 p) 2\xi_\delta \xi_\delta'}{(\xi_\delta^2 + p)^2}.$$

Using (18) to express $x' = x'(\xi_\delta', \xi_\delta, x) = -\frac{(x^2 + 2x\xi_\delta + p)\xi_\delta'}{\beta_\delta^2 + 2x\xi_\delta + p}$, we can write $\frac{dPI_t}{d\alpha}$ as

$$\frac{dPI_t}{d\alpha} = -\xi_\delta' \times A_1(\xi_\delta, x),$$

where $A_1(\xi_\delta, x)$ is a function that takes positive values for $\xi_\delta > 0$ and $x > 0$. Hence, the sign of $\frac{dPI_t}{d\alpha}$ is the same as the sign of $-\xi_\delta'$.

Comparative statics of PI_g with respect to α

$$\frac{dPI_g}{d\alpha} = \frac{\tau + \tau_s}{\beta_\delta^2} \frac{(2\beta_z x' \xi_\delta + 2\beta_z x \xi_\delta' + 2\xi_\delta \xi_\delta' + 2xx')(x^2 + p) - (2\beta_z x \xi_\delta + \xi_\delta^2 + x^2 + \beta_\delta^2 p) 2xx'}{(x^2 + p)^2}.$$

Using (18) to express $x' = x'(\xi_\delta', \xi_\delta, x)$, we can write $\frac{dPI_g}{d\alpha}$ as

$$\frac{dPI_g}{d\alpha} = \xi_\delta' \times A_2(\xi_\delta, x),$$

where $A_2(\xi_\delta, x)$ is a function that takes positive values for $\xi_\delta > 0$ and $x > 0$. Hence, the sign of $\frac{dPI_g}{d\alpha}$ is the same as the sign of ξ_δ'.

Comparative statics of v with respect to α

$$\frac{dv}{d\alpha} = \frac{\frac{dPI_t}{d\alpha}PI_g - \frac{dPI_g}{d\alpha}PI_t}{PI_g^2} = \frac{-\xi_\delta'\left(A_1 PI_g + A_2 PI_t\right)}{PI_g^2}.$$

Hence, the sign of $\frac{dv}{d\alpha}$ is the same as the sign of $-\xi_\delta'$.

Comparative statics of ξ_δ with respect to α

ξ_δ is implicitly defined by equation (7), which we also show below.

$$f(\xi_\delta, \alpha) = \xi_\delta^5 - \alpha\hat{m}\xi_\delta^4 + 2p\xi_\delta^3 - 2\alpha\hat{m}p\xi_\delta^2 + \left[p^2 + (1-\alpha)^2\hat{m}^2 p\right]\xi_\delta - \alpha\hat{m}p^2 = 0,$$

where we again denote $\hat{m} = \frac{1}{\gamma}\tau_s\beta_\delta m$. Using the implicit function theorem, we get

$$\xi_\delta' = \frac{\partial\xi_\delta}{\partial\alpha} = \frac{\hat{m}\xi_\delta^4 + 2\hat{m}p\xi_\delta^2 + 2(1-\alpha)\hat{m}^2 p\xi_\delta + \hat{m}p^2}{\frac{\partial f}{\partial\xi_\delta}}.$$

Therefore, the sign of ξ_δ' is the same as the sign of $\frac{\partial f}{\partial\xi_\delta}$. From our analysis in Appendix A.3.1, it follows that when solution is unique, $f'(\xi_\delta^{root}) > 0$ (see Figure 5). When there are three roots, $f'(\xi_\delta^{root}) > 0$ for $\xi_\delta^{root} = \xi_\delta^{i}, \xi_\delta^{iii}$ and $f'(\xi_\delta^{root}) < 0$ for $\xi_\delta^{root} = \xi_\delta^{ii}$ (see Figures 6 and 7). This proves the comparative statics part of Proposition 5.

We are left to prove that when $\tau_n > \tau_n^*\left(\frac{1}{2}, m\beta_\delta\right)$ and multiplicity is possible, equilibria can be ranked by their relative price informativeness v. First of all, notice that existence of $\bar{\alpha}$ and $\underline{\alpha}$ follows from Proposition 2. Second, from (17) it is clear that ξ_δ and x are symmetric in the sense that $\xi_\delta(\alpha) = x(1-\alpha)$. As a result, $\underline{\alpha}$ and $\bar{\alpha}$ are symmetric around $\frac{1}{2}$: $\bar{\alpha} = 1 - \underline{\alpha}$. Symmetry of $\xi_\delta(\alpha)$ and $x(\alpha)$ is illustrated in Figure 9 (monotonicity properties of ξ_δ with respect to α have been established above). Fix $\alpha = \alpha_1 < \underline{\alpha}$ (other α's can be considered analogously). Then $x(\alpha_1) = \xi_\delta(1-\alpha_1) > \xi_\delta(\alpha_1)$. From this Figure, it is clear that $\xi_\delta(\alpha) < x(\alpha)$ when $\alpha < \bar{\alpha}$ and $\xi_\delta(\alpha) > x(\alpha)$ when $\alpha > \underline{\alpha}$. Therefore, in the T-equilibrium $v^T > 1$, and in the G-equilibrium $v^G < 1$. Finally, $v^T > v^M > v^G$. $\square$

B.2 Cost of capital

In this section, we prove Proposition 6 and Corollary 1. First, we express the cost of capital in its general form when the firm's expected output is non-zero. Recall that the

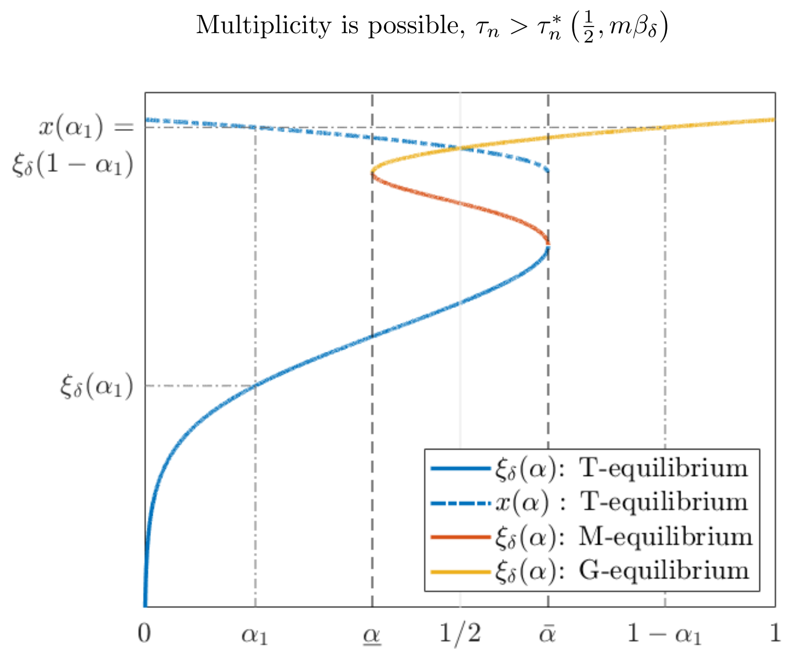

Figure 9: ξ_δ and $x = \beta_\delta \xi_z - \beta_z \xi_\delta$ as functions of the fraction of green investors α. Y-axes are in the log scale.

cost of capital is defined as $CoC = \mathbb{E}[\tilde{z} - \tilde{q}] = \mu_z - q_0 - q_z\mu_z - q_\delta\mu_\delta$. Note that in this general case, the coefficients q_z, q_δ and q_n of the pricing function remain the same as the zero-mean case. However, the expression for q_0 is slightly different. Specifically,

$$q_0 = \frac{\gamma\left(\frac{\tau}{\tau_s}\left(\mu_z\xi_z + \mu_\delta\xi_\delta\right) - 1\right)}{m_t PI_t + m_g PI_g}.$$

Note that if $\mu_z = \mu_\delta = 0$, we get back to the zero-mean case analyzed in the main text.

Substituting in the expressions for the pricing coefficients, the cost of capital can be expressed as

$$CoC = c_z\mu_z + c_\delta\mu_\delta + \frac{\gamma}{m_t PI_t + m_g PI_g},$$

where $c_z = \frac{(1-\beta_z)\xi_\delta}{\beta_\delta\xi_z + (1-\beta_z)\xi_\delta}$ and $c_\delta = -\frac{\beta_\delta\xi_\delta}{\beta_\delta\xi_z + (1-\beta_z)\xi_\delta}$.

Proof of Proposition 6. When $\mu_z = \mu_\delta = 0$,

$$CoC = \frac{\gamma}{m_t PI_t + m_g PI_g}.$$

Using (18) to differentiate CoC with respect to α, we get

$$\frac{dCoC}{d\alpha} = -\left(\xi_\delta - x\right)\xi_\delta' \times A_3(\xi_\delta, x),$$

where $A_3(\xi_\delta, x)$ is a function that takes positive values for $\xi_\delta > 0$ and $x > 0$. Since x and ξ_δ are symmetric, we conclude: When equilibrium is unique, $\xi_\delta' > 0$ and $\xi_\delta \gtreqqless x$ when $\alpha \gtreqqless \frac{1}{2}$. When multiplicity is possible, $\xi_\delta' > 0$ and $\xi_\delta < x$ in the T-equilibrium, $\xi_\delta' > 0$ and $\xi_\delta > x$ in the G-equilibrium, and $\xi_\delta' < 0$ and $\xi_\delta \gtreqqless x$ when $\alpha \lesseqqgtr \frac{1}{2}$ in the M-equilibrium. □

Proof of Corollary 1. As we show at the beginning of this section, cost of capital is linear in μ_z and μ_δ. Below we analyze the comparative statics of c_z and c_δ with respect to α. First, note that $c_z = -\frac{1-\beta_z}{\beta_\delta} c_\delta$. Therefore, below we focus on $\frac{dc_\delta}{d\alpha}$, and $\frac{dc_z}{d\alpha}$ always has the opposite sign.

Recall that $x = \beta_\delta \xi_z - \beta_z \xi_\delta$. Hence,

$$c_\delta = -\frac{\beta_\delta \xi_\delta}{x + \xi_\delta} \Rightarrow c_\delta' = \frac{dc_\delta}{d\alpha} = -\beta_\delta \frac{\xi_\delta' x - \xi_\delta x'}{(x + \xi_\delta)^2}$$

Substitute $x' = x'(\xi_\delta', \xi_\delta, x)$ from (18) to obtain $c_\delta' = -A_4(\xi_\delta, x)\xi_\delta'$, where $A_4(\xi_\delta, x)$ is a function that takes positive values for $\xi_\delta > 0$ and $x > 0$. □

C Improvements in non-monetary information

In this Appendix, we consider the setting discussed in Section 5. When $\lambda > 0$, demand for the stock from investors of type j is

$$D^j(\tilde{z}, \tilde{\delta}, \tilde{q}) = m^j \frac{1}{\gamma} \frac{\tilde{z}\beta_z^j \frac{\tau_s}{\tau+\tau_s} + \tilde{\delta}\beta_\delta^j \frac{\lambda\tau_s}{\lambda\tau+\lambda\tau_s} + \left(q_z \beta_z^j \frac{1}{\tau+\tau_s} + q_\delta \beta_\delta^j \frac{1}{\lambda\tau+\lambda\tau_s}\right) \frac{\tilde{q} - q_0 - \tilde{z} q_z \frac{\tau_s}{\tau_s+\tau} - \tilde{\delta} q_\delta \frac{\lambda\tau_s}{\lambda\tau_s+\lambda\tau}}{q_z^2 \frac{1}{\tau+\tau_s} + q_\delta^2 \frac{1}{\lambda\tau+\lambda\tau_s} + q_n^2 \frac{1}{\tau_n}} - \tilde{q}}{\left(\beta_z^j\right)^2 \frac{1}{\tau+\tau_s} + \left(\beta_\delta^j\right)^2 \frac{1}{\lambda\tau+\lambda\tau_s} - \frac{\left(q_z \beta_z^j \frac{1}{\tau+\tau_s} + q_\delta \beta_\delta^j \frac{1}{\lambda\tau+\lambda\tau_s}\right)^2}{q_z^2 \frac{1}{\tau+\tau_s} + q_\delta^2 \frac{1}{\lambda\tau+\lambda\tau_s} + q_n^2 \frac{1}{\tau_n}}}.$$

After imposing the market clearing condition (1), we obtain the system:

$$\begin{aligned} \xi_z &= \frac{1}{\gamma}\tau_s \left[m_t + m_g \frac{\beta_z \left(\xi_\delta^2 + \lambda p\right) - \xi_\delta \xi_z \beta_\delta}{\left(\xi_z \beta_\delta - \xi_\delta \beta_z\right)^2 + \left(\beta_z^2 \lambda + \beta_\delta^2\right) p} \right], \\ \xi_\delta &= \frac{1}{\gamma}\lambda\tau_s \left[-m_t \frac{\xi_\delta \xi_z}{\xi_\delta^2 + \lambda p} + m_g \frac{\beta_\delta \left(\xi_z^2 + p\right) - \xi_\delta \xi_z \beta_z}{\left(\xi_z \beta_\delta - \xi_\delta \beta_z\right)^2 + \left(\beta_z^2 \lambda + \beta_\delta^2\right) p} \right]. \end{aligned} \tag{19}$$

Denote $\hat{p} = \lambda p, \hat{\beta}_\delta = \frac{\beta_\delta}{\sqrt{\lambda\beta_z^2+\beta_\delta^2}}$, $\hat{\beta}_z = \sqrt{1-\hat{\beta}_\delta^2} = \frac{\sqrt{\lambda}\beta_z}{\sqrt{\lambda\beta_z^2+\beta_\delta^2}}$, $\hat{m}_g = m_g\lambda\frac{1}{\sqrt{\lambda\beta_z^2+\beta_\delta^2}}$, $\hat{m}_t = m_t\sqrt{\lambda}$ and $\hat{\xi}_z = \xi_z\sqrt{\lambda}$. Then the system becomes

$$\hat{\xi}_z = \frac{1}{\gamma}\tau_s\left[\hat{m}_t + \hat{m}_g\frac{\hat{\beta}_z\left(\xi_\delta^2+\hat{p}\right)-\xi_\delta\hat{\xi}_z\hat{\beta}_\delta}{\left(\hat{\xi}_z\hat{\beta}_\delta-\xi_\delta\hat{\beta}_z\right)^2+\hat{p}}\right],$$

$$\xi_\delta = \frac{1}{\gamma}\tau_s\left[-\hat{m}_t\frac{\xi_\delta\hat{\xi}_z}{\xi_\delta^2+\hat{p}} + \hat{m}_g\frac{\hat{\beta}_\delta\left(\hat{\xi}_z^2+\hat{p}\right)-\xi_\delta\hat{\xi}_z\hat{\beta}_z}{\left(\hat{\xi}_z\hat{\beta}_\delta-\xi_\delta\hat{\beta}_z\right)^2+\hat{p}}\right].$$

Note that it has the same structure as (10). Therefore, adjusted versions of Propositions 1 and 2 hold, where m, α and β_δ are substituted by, respectively, $\hat{m} = \hat{m}_t + \hat{m}_g$, $\hat{\alpha} = \frac{\hat{m}_g}{\hat{m}}$ and $\hat{\beta}_\delta$.

Fully characterizing comparative statics of the endogenous objects such as price coefficients, price informativeness and cost of capital is nontrivial. In what follows, we investigate the model under assumption that λ is small. We are going to linearize price coefficients ξ_z and ξ_δ around $\lambda = 0$ and investigate the comparative statics of the linearized solution. To do so, we proceed in three steps. First, we separately solve the model for the case $\lambda = 0$. Second, we use system (19), derived under assumption $\lambda > 0$, to get equation (20) for ξ_δ. We are going to show that, if λ is sufficiently small, there exists a unique solution to this equation that is smooth in λ around 0. Moreover, the solution to this equation coincides with the solution derived in Step 1 when $\lambda = 0$. In the third step, we linearize the solution of equation (20) around $\lambda = 0$ and prove Propositions 7 and 8.

Step 1: Solving the model when $\lambda = 0$.

When $\lambda = 0$, prior and signals about the non-monetary component $\tilde{\delta}$ are infinitely imprecise. Therefore, the price cannot be informative about $\tilde{\delta}$ in any equilibrium so that $q_\delta = 0$ and $\tilde{q} = q_0 + q_z\tilde{z} + q_n\tilde{n} = q_0 + q_n\left(\xi_z\tilde{z} + \tilde{n}\right)$. Green investors do not trade the stock because its payoff is infinitely risky to them. Then the equilibrium price coefficients are shaped by trading activities of traditional and noise investors only. In particular, demand for the stock from traditional investors is

$$D^t(\tilde{z},\tilde{q}) = m_t \frac{1}{\gamma} \frac{\tilde{z}\frac{\tau_s}{\tau+\tau_s} + \left(q_z \frac{1}{\tau+\tau_s}\right) \frac{\tilde{q}-q_0-\tilde{z}q_z\frac{\tau_s}{\tau_s+\tau}}{q_z^2\frac{1}{\tau+\tau_s}+q_n^2\frac{1}{\tau_n}} - \tilde{q}}{\frac{1}{\tau+\tau_s} - \frac{\left(q_z\frac{1}{\tau+\tau_s}\right)^2}{q_z^2\frac{1}{\tau+\tau_s}+q_n^2\frac{1}{\tau_n}}}.$$

The market clearing condition is $D^t(\tilde{z},\tilde{q}) + \tilde{n} = 1$. Matching the price coefficients, it is straightforward to derive that $q_n > 0$, $q_z > 0$, $q_0 < 0$. Moreover, $\xi_z = \frac{1}{\gamma}\tau_s m_t$ and $\xi_\delta = 0$.

Step 2: Equation for ξ_δ when $\lambda > 0$.

When $\lambda > 0$, we can use system (19) to get a quintic equation of ξ_δ, analogous to equation (7) in the main text:

$$f(\xi_\delta) = \xi_\delta^5 - \left(\frac{1}{\gamma}\tau_s m_g\right)\lambda\frac{\beta_\delta}{\lambda\beta_z^2+\beta_\delta^2}\xi_\delta^4 + 2\lambda p\xi_\delta^3 - 2\left(\frac{1}{\gamma}\tau_s m_g\right)\lambda^2 p\frac{\beta_\delta}{\lambda\beta_z^2+\beta_\delta^2}\xi_\delta^2 + \left[\lambda^2p^2 + \left(\frac{1}{\gamma}\tau_s m_t\right)^2\lambda^2 p\frac{\beta_\delta^2}{\lambda\beta_z^2+\beta_\delta^2}\right]\xi_\delta - \left(\frac{1}{\gamma}\tau_s m_g\right)\lambda^3p^2\frac{\beta_\delta}{\lambda\beta_z^2+\beta_\delta^2} = 0, \tag{20}$$

which can be rewritten as

$$f(\xi_\delta) = \left(\xi_\delta - \left(\frac{1}{\gamma}\tau_s m_g\right)\lambda\frac{\beta_\delta}{\lambda\beta_z^2+\beta_\delta^2}\right)\left(\xi_\delta^2+\lambda p\right)^2 + \left(\frac{1}{\gamma}\tau_s m_t\right)^2\lambda^2 p\frac{\beta_\delta^2}{\lambda\beta_z^2+\beta_\delta^2}\xi_\delta = 0.$$

When $\lambda = 0$, this equation has a unique solution $\xi_\delta = 0$, which coincides with the one derived in Step 1. When $\lambda > 0$, there always exists a positive solution because $f(0) < 0$ and $f(\infty) > 0$. Moreover, all solutions are below $\frac{1}{\gamma}\tau_s m_g\lambda\frac{\beta_\delta}{\lambda\beta_z^2+\beta_\delta^2}$. When λ is sufficiently small, the solution is unique. Indeed, differentiate (20) and observe that for $\xi_\delta \in \left(0, \frac{1}{\gamma}\tau_s m_g\lambda\frac{\beta_\delta}{\lambda\beta_z^2+\beta_\delta^2}\right)$,

$$\frac{\partial f}{\partial \xi_\delta} > -4\left(\frac{1}{\gamma}\tau_s m_g\lambda\frac{\beta_\delta}{\lambda\beta_z^2+\beta_\delta^2}\right)^4 - 4\left(\frac{1}{\gamma}\tau_s m_g\lambda\frac{\beta_\delta}{\lambda\beta_z^2+\beta_\delta^2}\right)^2\lambda p + \lambda^2 p\left(p + \left(\frac{1}{\gamma}\tau_s m_t\right)^2\frac{\beta_\delta^2}{\lambda\beta_z^2+\beta_\delta^2}\right).$$

When λ is sufficiently small, the last positive term is larger in absolute terms than the first two negative terms. Therefore, $f(\xi_\delta)$ is strictly increasing on the relevant interval, which guarantees that there exists a unique solution $\xi_\delta(\lambda)$. Moreover, the function $\xi_\delta(\lambda)$

is smooth in the neighborhood of zero because $f(\xi_\delta, \lambda)$ is smooth in the neighborhood of $(0,0)$.[28]

Step 3: Linearization.

Since $\xi_\delta(\lambda)$ is smooth around $\lambda = 0$, we can use its Taylor series to approximate it around this point. Write $\xi_\delta = \xi_{\delta,1}\lambda + o(\lambda)$, where $\xi_{\delta,1}$ does not depend on λ, and plug it in (20). Omitting higher order terms, we obtain

$$\xi_{\delta,1} = \frac{\left(\frac{1}{\gamma}\tau_s m_g\right) p}{\beta_\delta \left(\left(\frac{1}{\gamma}\tau_s m_t\right)^2 + p\right)} > 0.$$

Similarly, we have $\xi_z = \xi_{z,0} + \xi_{z,1}\lambda + o(\lambda)$. Using the first equation of system (19), we get

$$\xi_{z,0} = \frac{1}{\gamma}\tau_s m_t \quad \text{and} \quad \xi_{z,1} = \xi_{\delta,1}\frac{1}{\beta_\delta}\left[\beta_z - \frac{\left(\frac{1}{\gamma}\tau_s m_t\right)\left(\frac{1}{\gamma}\tau_s m_g\right)}{\left(\frac{1}{\gamma}\tau_s m_t\right)^2 + p}\right].$$

The linear term of ξ_z is negative when $\beta_z < \frac{\left(\frac{1}{\gamma}\tau_s m_t\right)\left(\frac{1}{\gamma}\tau_s m_g\right)}{\left(\frac{1}{\gamma}\tau_s m_t\right)^2 + p}$, which proves part (i) of Proposition 7.

Next, we linearize price informativeness. Recall that price informativeness are given by

$$PI_t = (\tau + \tau_s)\frac{\xi_z^2\lambda + \xi_\delta^2 + \lambda p}{\xi_\delta^2 + \lambda p} \quad \text{and} \quad PI_g = (\tau + \tau_s)\frac{\xi_z^2\lambda + \xi_\delta^2 + \lambda p}{(\xi_\delta\beta_z - \xi_z\beta_\delta)^2 + (\beta_z^2\lambda + \beta_\delta^2)\, p}.$$

For green investors, we have

$$PI_{g,0} + PI_{g,1}\lambda = (\tau + \tau_s)\frac{\xi_{z,0}^2\lambda + \lambda p}{\xi_{z,0}^2\beta_\delta^2 + \beta_\delta^2 p} \Rightarrow PI_{g,0} = 0, \quad PI_{g,1} = (\tau + \tau_s)\frac{1}{\beta_\delta^2} > 0.$$

[28] Although we only considered economically meaningful case $\lambda > 0$, we can similarly consider the case $\lambda < 0$.

For traditional investors, we have

$$PI_{t,0}+PI_{t,1}\lambda=(\tau+\tau_s)\frac{\xi_{z,0}^2\lambda+2\xi_{z,1}\xi_{z,0}\lambda^2+\xi_{\delta,1}^2\lambda^2+\lambda p}{\xi_{\delta,1}^2\lambda^2+\lambda p}\Rightarrow$$

$$PI_{t,0}=(\tau+\tau_s)\frac{\left(\frac{1}{\gamma}\tau_s m_t\right)^2+p}{p},\ PI_{t,1}=(\tau+\tau_s)\frac{2\left(\frac{1}{\gamma}\tau_s m_t\right)\xi_{\delta,1}}{p\beta_\delta}\left(\beta_z-\frac{3}{2}\frac{\left(\frac{1}{\gamma}\tau_s m_g\right)\left(\frac{1}{\gamma}\tau_s m_t\right)}{\left(\frac{1}{\gamma}\tau_s m_t\right)^2+p}\right).$$

Clearly, $PI_{t,1}<0$ when $\beta_z-\frac{3}{2}\frac{\left(\frac{1}{\gamma}\tau_s m_g\right)\left(\frac{1}{\gamma}\tau_s m_t\right)}{\left(\frac{1}{\gamma}\tau_s m_t\right)^2+p}<0$, which proves part (ii) of Proposition 7.

Finally, recall that $CoC=\frac{\gamma}{m_t PI_t+m_g PI_g}$ and so $\frac{dCoC}{d\lambda}\propto-\left(\left(\frac{1}{\gamma}\tau_s m_t\right)\frac{dPI_t}{d\lambda}+\left(\frac{1}{\gamma}\tau_s m_g\right)\frac{dPI_g}{d\lambda}\right)$. Using $PI_{g,1}$ and $PI_{t,1}$ derived above, we have

$$m_t\frac{dPI_t}{d\lambda}+m_g\frac{dPI_g}{d\lambda}=m_t PI_{t,1}+m_g PI_{g,1}+o(1)=$$

$$(\tau+\tau_s)\frac{2\left(\frac{1}{\gamma}\tau_s m_g\right)\left(\frac{1}{\gamma}\tau_s m_t\right)^2}{\beta_\delta^2\left(\left(\frac{1}{\gamma}\tau_s m_t\right)^2+p\right)}\left[\beta_z-\frac{3}{2}\frac{\left(\frac{1}{\gamma}\tau_s m_t\right)\left(\frac{1}{\gamma}\tau_s m_g\right)}{\left(\frac{1}{\gamma}\tau_s m_t\right)^2+p}+\frac{1}{2}\frac{\left(\frac{1}{\gamma}\tau_s m_t\right)^2+p}{\left(\frac{1}{\gamma}\tau_s m_t\right)^2}\right]+o(1).$$

When the expression in the brackets is negative, CoC declines in λ when λ is sufficiently close to zero. This proves Proposition 8.

D General information structure

In our main analyses, we consider an analytically tractable case when traditional and green investors have access to information of the same quality. In this section, we explore the role of information structure for our main results. In particular, we establish that our results about the existence of multiple equilibria in the trading game and the nature of these equilibria are robust to rather general assumptions about information available to investors.

First, we allow $\tilde{z}$ and $\tilde{\delta}$ to have different ex ante variances, τ_z^{-1} and τ_δ^{-1}, respectively. Second, traditional and green investors receive informative signals about $\tilde{z}$ and $\tilde{\delta}$ of potentially different precisions. In particular, investor i of type $j\in\{t,g\}$ receives two private signals, $\tilde{s}_z^{ij}\sim N\left(\tilde{z},\left(\tau_{s_z}^j\right)^{-1}\right)$ and $\tilde{s}_\delta^{ij}\sim N\left(\tilde{\delta},\left(\tau_{s_\delta}^j\right)^{-1}\right)$. Given their preferences, we assume that traditional/green investors receive some useful information about $\tilde{z}/\tilde{\delta}$, namely, $\tau^t_{s_z} > 0$ and $\tau^g_{s_\delta} > 0$. Other signals can be in principle uninformative, $\tau^t_{s_\delta} \geq 0$ and $\tau^g_{s_z} \geq 0$. Finally, we maintain our baseline assumptions: masses of traditional and green investors are positive, $m_t > 0$ and $m_g > 0$; investors' risk aversion parameter is $\gamma > 0$; traditional investors care only about $\tilde{z}$ and green investors care about $\beta_z \tilde{z} + \beta_\delta \tilde{\delta}$, where $\beta_z \geq 0$ and $\beta_\delta > 0$; noise traders trade $\tilde{n} \sim N\left(0, \tau_n^{-1}\right)$.

Under general information structure, the system of equations (10) for ξ_z and ξ_δ becomes

$$\begin{aligned}
\xi_z &= m_t \tau^t_{s_z} + m_g \tau^g_{s_z} \frac{\beta_z \left(\xi_\delta^2 + \frac{\tau_\delta + \tau^g_{s_\delta}}{\tau_n}\right) - \xi_\delta \xi_z \beta_\delta}{\left(\xi_z \beta_\delta - \xi_\delta \beta_z\right)^2 + \left(\beta_z^2 \left(\tau_\delta + \tau^g_{s_\delta}\right) + \beta_\delta^2 \left(\tau_z + \tau^g_{s_z}\right)\right) \frac{1}{\tau_n}}, \\
\xi_\delta &= -m_t \tau^t_{s_\delta} \frac{\xi_\delta \xi_z}{\xi_\delta^2 + \frac{\tau_\delta + \tau^t_{s_\delta}}{\tau_n}} + m_g \tau^g_{s_\delta} \frac{\beta_\delta \left(\xi_z^2 + \frac{\tau_z + \tau^g_{s_z}}{\tau_n}\right) - \xi_\delta \xi_z \beta_z}{\left(\xi_z \beta_\delta - \xi_\delta \beta_z\right)^2 + \left(\beta_z^2 \left(\tau_\delta + \tau^g_{s_\delta}\right) + \beta_\delta^2 \left(\tau_z + \tau^g_{s_z}\right)\right) \frac{1}{\tau_n}}.
\end{aligned} \tag{21}$$

In (21), we set $\gamma = 1$. This is without loss of generality because it is equivalent to redefining the masses of traditional and green investors.

Proposition 9. *Fix $m_t > 0$, $m_g > 0$, $\gamma > 0$, $\beta_z \geq 0$, $\beta_\delta > 0$, $\tau^t_{s_z} > 0$, $\tau^t_{s_\delta} \geq 0$, $\tau^g_{s_z} \geq 0$, $\tau^g_{s_\delta} > 0$. For any $\tau_n > 0$, an equilibrium with a linear price $\tilde{q} = q_0 + q_z \tilde{z} + q_\delta \tilde{\delta} + q_n \tilde{n}$ exists. Moreover, for a sufficiently large τ_n multiple equilibria exist if one of the following conditions is satisfied:*

(i) $\tau^t_{s_\delta} > 0$ *and* $\tau^g_{s_z} > 0$;

(ii) $\tau^t_{s_\delta} > 0$, $\tau^g_{s_z} = 0$, *and either* $\dfrac{4\beta_\delta^2 m_t^2 \tau^t_{s_z} \tau^t_{s_\delta}}{m_g^2 \left(\tau^g_{s_\delta}\right)^2} < 1$ *or* $\beta_z > 0$;

(iii) $\tau^t_{s_\delta} = 0$, $\tau^g_{s_z} > 0$, *and* $\dfrac{4 m_g \tau^g_{s_\delta} \left(\tau^g_{s_z} m_g + \beta_z m_t \tau^t_{s_z}\right)}{\beta_\delta^2 m_t^2 \left(\tau^t_{s_z}\right)^2} < 1$;

(iv) $\tau^t_{s_\delta} = 0$, $\tau^g_{s_z} = 0$, $\beta_z > 0$ *and* $\dfrac{4\beta_z m_g \tau^g_{s_\delta}}{\beta_\delta^2 m_t \tau^t_{s_z}} < 1$.

Below, we first discuss Proposition 9 and then formally prove it at the end of this section.

Discussion

Proposition 9 emphasizes the importance of the information structure for the existence of multiple equilibria in the trading stage. In particular, they arise when investors have access to information about fundamentals that they value differently.[29] To see it clearly, it is instructive to consider a special case when green investors care only about the $\tilde{\delta}$-component, i.e. $\beta_z = 0$. For a sufficiently small exogenous noise τ_n, multiple equilibria always arise as long as traditional and green investors receive *some* informative signals about $\tilde{\delta}$ and $\tilde{z}$, respectively. In an equilibrium that resembles the T-equilibrium, the price is closely associated with $\tilde{z}$ and is thus very informative to traditional investors. This incentivizes them to trade the stock intensively. In particular, they actively trade against their $\tilde{\delta}$-signals, virtually offsetting the impact of green investors who trade in the opposite direction. The price is, therefore, weakly associated with $\tilde{\delta}$. Analogously, there is an equilibrium that resembles the G-equilibrium, where the price is closely associated with $\tilde{z}$.

Notice that the multiplicity is possible even if only one group of investors receive signals about the factor they do not particularly value, e.g. $\tau^t_{s_\delta} > 0$ and $\tau^g_{s_z} = 0$ (the case where $\tau^t_{s_\delta} = 0$ and $\tau^g_{s_z} > 0$ is analogous). In the absence of relevant signals about $\tilde{z}$, green investors are not able to offset traditional investors' trading along the $\tilde{z}$-dimension. The price is always informative to traditional investors because the price coefficient ξ_z is shaped solely by their trading activities. The multiplicity is still possible due to trading in the opposite directions along the $\tilde{\delta}$-dimension. It requires, however, that the mass of traditional investors is small and their private signals are not precise relative to those of green investors, i.e. $\frac{4\beta_\delta^2 m_t^2 \tau^t_{s_z} \tau^t_{s_\delta}}{m_g^2 \left(\tau^g_{s_\delta}\right)^2} < 1$. If this is not the case, traditional investors dominate in trading along the $\tilde{\delta}$-dimension and the price is uniquely defined.[30]

Finally, the equilibrium is always unique when investors are informed only about the factors they care about, i.e. $\tau^t_{s_\delta} = \tau^g_{s_z} = 0$. In this case, there is no trading in the opposite directions because the investors' information sets are orthogonal.[31,32]

Overall, Proposition 9 shows that, under fairly general assumptions on the information

[29] For instance, traditional investors are likely to possess some information about the firm's ESG performance from reading analyst reports or news articles, even though they do not necessarily value the ESG performance directly.

[30] Note that when green investors care about the $\tilde{z}$-component, multiple equilibria are always possible for a sufficiently small noise. When $\beta_z > 0$, preferences of green and traditional investors are partially aligned. Green investors benefit to some extent from traditional investors' trading as they can learn about $\tilde{z}$ from the price. The price is less noisy to them, and they trade more aggressively based on their $\tilde{\delta}$-signals.

[31] This case is studied in Rahi and Zigrand (2018) and Rahi (2020).

[32] Again, multiple equilibria might arise when $\beta_z > 0$. In this case, signals received by green investors are not perfectly aligned with what they care about and, therefore, they benefit from the information about $\tilde{z}$ contained in the price.

structure, the price might not be uniquely pinned down if the stock is traded by investors with heterogeneous valuations. Potential equilibria differ in which group of investors most actively trade the stock and which factors the price is mostly informative about. There are two key requirements for the multiplicity to emerge. First, investors of one group need to possess *some* information about the fundamental that investors of the other group care about. That allows investors with heterogeneous preferences to trade against each other based on the same information. Second, the amount of exogenous noise should be small; otherwise, the price is always an imprecise signal to all investors.

Proof

As in other proofs, this one involves many tedious yet straightforward algebraic manipulations, which we frequently perform via Matlab Symbolic Math Toolbox and do not show.

The first part of the proof involves the reduction of (21) to a polynomial equation either for ξ_δ or ξ_z. Depending on the values of signal precisions, this equation is either cubic or have a higher odd order. For cubic equations, we investigate the number of roots using the sign of the discriminant. For higher order equations, the analysis is conceptually similar to the proof of Lemma 2. In particular, we prove that, for a sufficiently large τ_n, there are at least three distinct real roots by showing that the polynomial changes its sign at least three times.

Getting a polynomial equation for either ξ_z or ξ_δ from the system (21) involves different steps when $\beta_z = 0$ and $\beta_z > 0$, so we analyze these two scenarios separately. Each scenario is further split into four cases that jointly cover all possible values of signal precisions. In some of those cases, we introduce new notation. Since cases are independent from one another, the additional notation is case-specific, i.e. we might use the same notation in other cases to denote different objects.

Proof.

Case 1: $\beta_z = 0$.

The system (21) simplifies to

$$\begin{aligned}\xi_z &= m_t \tau^t_{s_z} - \hat{m}_g \tau^g_{s_z} \frac{\xi_\delta \xi_z}{\xi_z^2 + \frac{\tau_z + \tau^g_{s_z}}{\tau_n}},\\ \xi_\delta &= \hat{m}_g \tau^g_{s_\delta} - m_t \tau^t_{s_\delta} \frac{\xi_\delta \xi_z}{\xi_\delta^2 + \frac{\tau_\delta + \tau^t_{s_\delta}}{\tau_n}},\end{aligned} \tag{22}$$

where we denote $\hat{m}_g = \frac{1}{\beta_\delta} m_g$.

Case 1.1: $\tau^t_{s_\delta} = \tau^g_{s_z} = 0$.

If investors receive signals only about fundamentals they care about, the equilibrium in the trading stage is trivially unique: $\xi_z = m_t \tau^t_{s_z}$ and $\xi_\delta = \hat{m}_g \tau^g_{s_\delta}$.

Case 1.2: $\tau^t_{s_\delta} = 0$ **and** $\tau^g_{s_z} > 0$**.**

If only green investors receive informative signals about $\tilde{\delta}$, their trading activity solely defines the corresponding price coefficient, $\xi_\delta = \hat{m}_g \tau^g_{s_\delta}$. ξ_z solves the following equation:

$$\xi_z^3 - \xi_z^2 \left[m_t \tau^t_{s_z}\right] + \xi_z \left[\frac{\tau_z + \tau^g_{s_z}}{\tau_n} + \hat{m}_g^2 \tau^g_{s_z} \tau^g_{s_\delta}\right] - m^t \tau^t_{s_z} \frac{\tau_z + \tau^g_{s_z}}{\tau_n} = 0. \tag{23}$$

This equation has a real root because it is cubic. It has only positive real roots since the coefficients of odd/even powers are positive/negative. It has three distinct real roots when its discriminant is positive. The discriminant can be written as a polynomial of $\frac{1}{\tau_n}$:

$$D = \sum_{i=0}^{3} d_i \left(\frac{1}{\tau_n}\right)^i,$$

where $d_0 = \hat{m}_g^4 m_t^2 \left(\tau^g_{s_z} \tau^g_{s_\delta} \tau^t_{s_z}\right)^2 - 4\hat{m}_g^6 \left(\tau^g_{s_z} \tau^g_{s_\delta}\right)^3$. For a sufficiently large τ_n, $D > 0$ if $d_0 > 0$. Therefore, for a sufficiently large τ_n, (23) has three distinct real roots when

$$\frac{4\hat{m}_g^2 \tau^g_{s_\delta} \tau^g_{s_z}}{m_t^2 \left(\tau^t_{s_z}\right)^2} = \frac{4 m_g^2 \tau^g_{s_\delta} \tau^g_{s_z}}{\beta_\delta^2 m_t^2 \left(\tau^t_{s_z}\right)^2} < 1.$$

Case 1.3: $\tau^t_{s_\delta} > 0$ **and** $\tau^g_{s_z} = 0$**.**

This case is analogous to **Case 1.2**. There are three solutions to (22) if τ_n is sufficiently large and

$$\frac{4\beta_\delta^2 m_t^2 \tau_{s_\delta}^t \tau_{s_z}^t}{m_g^2 \left(\tau_{s_z}^g\right)^2} < 1.$$

Case 1.4: $\tau_{s_\delta}^t, \tau_{s_z}^g > 0$.

Since the first equation of (22) is linear in ξ_δ, we can straightforwardly write $\xi_\delta = \xi_\delta(\xi_z)$. Plugging it in the second equation of the system, we obtain the following equation for ξ_z:

$$f(\xi_z) = \sum_{i=0}^{9} a_i \xi_z^i = 0. \tag{24}$$

Moreover, $a_9 = 1$ and $a_0 = a_{0,3}\left(\frac{1}{\tau_n}\right)^3$, where $a_{0,3} < 0$ does not depend on τ_n. Then there exists at least one positive real root. Let's now show that there exists at least three positive real roots for a sufficiently large τ_n. Our approach is analogous to the proof of Lemma 2, so we keep the proof here brief.

We can write

$$\begin{aligned}
a_0 &= a_{0,3}\left(\frac{1}{\tau_n}\right)^3, \\
a_1 &= a_{1,2}\left(\frac{1}{\tau_n}\right)^2 + a_{1,3}\left(\frac{1}{\tau_n}\right)^3, \\
a_2 &= a_{2,2}\left(\frac{1}{\tau_n}\right)^2 + a_{2,3}\left(\frac{1}{\tau_n}\right)^3, \\
a_3 &= a_{3,1}\frac{1}{\tau_n} + a_{3,2}\left(\frac{1}{\tau_n}\right)^2 + a_{3,3}\left(\frac{1}{\tau_n}\right)^3,
\end{aligned}$$

where $a_{i,j}$ are coefficients that do not depend on τ_n. Moreover, $a_{0,3} < 0$ and $a_{1,2} > 0$. Then, evaluating $f(\cdot)$ at $-\frac{a_{0,3}}{a_{1,2}}\frac{1}{\tau_n} + c_1\frac{1}{\tau_n} > 0$ for some $c_1 > 0$, we obtain

$$f\left(-\frac{a_{0,3}}{a_{1,2}}\frac{1}{\tau_n} + c_1\frac{1}{\tau_n}\right) = a_{1,2}c_1\left(\frac{1}{\tau_n}\right)^3 + o\left(\left(\frac{1}{\tau_n}\right)^3\right).$$

For a sufficiently large τ_n, $f\left(-\frac{a_{0,3}}{a_{1,2}}\frac{1}{\tau_n} + c_1\frac{1}{\tau_n}\right) > 0$.

Next, we can write (24) also as a polynomial of $\frac{1}{\tau_n}$:

$$f\left(\xi_z\right)=\sum_{i=0}^{3} b_i(\xi_z)\left(\frac{1}{\tau_n}\right)^i,$$

where

$$b_0(\xi_z)=\xi_z^4\left(\xi_z-m_t\tau_{s_z}^t\right)^2\left(\xi_z^2\left(\xi_z-m_t\tau_{s_z}^t\right)+\hat{m}_g^2\tau_{s_z}^g\tau_{s_\delta}^g\xi_z\right)+\hat{m}_g^2m_t\left(\tau_{s_z}^g\right)^2\tau_{s_\delta}^t\xi_z^5\left(\xi_z-m_t\tau_{s_z}^t\right).$$

Then, evaluating $f\left(\cdot\right)$ at $m_t\tau_{s_z}^t-\left(\frac{1}{\tau_n}\right)^{1/2}$, we obtain

$$f\left(m_t\tau_{s_z}^t-\left(\frac{1}{\tau_n}\right)^{1/2},\frac{1}{\tau_n}\right)=-\hat{m}_g^2m_t\left(\tau_{s_z}^g\right)^2\tau_{s_\delta}^t\left(m_t\tau_{s_z}^t\right)^5\left(\frac{1}{\tau_n}\right)^{1/2}+o\left(\left(\frac{1}{\tau_n}\right)^{1/2}\right).$$

Therefore, for a sufficiently large τ_n, $f\left(m_t\tau_{s_z}^t-\left(\frac{1}{\tau_n}\right)^{1/2}\right)<0$ and $m_t\tau_{s_z}^t-\left(\frac{1}{\tau_n}\right)^{1/2}>-\frac{a_{0,3}}{a_{1,2}}\frac{1}{\tau_n}+c_1\frac{1}{\tau_n}>0$. Furthermore, because $a_9>0$, for any $\tau_n>0$ $f\left(\xi_z\right)>0$ if ξ_z is sufficiently large. Hence, we have shown that (24) has at least three (positive real) solutions for ξ_z if τ_n is sufficiently large.

Case 2: $\beta_z>0$.

We now work with the system (21).

Case 2.1: $\tau_{s_\delta}^t=\tau_{s_z}^g=0$.

The price coefficient ξ_z is $m_t\tau_{s_z}^t$. ξ_δ solves

$$\xi_\delta^3[\beta_z^2]-\xi_z^2\left[2\beta_z\beta_\delta m_t\tau_{s_z}^t\right]+\xi_z\left[\left(\beta_z^2\left(\tau_\delta+\tau_{s_\delta}^g\right)+\beta_\delta^2\tau_z\right)\frac{1}{\tau_n}+m_gm_t\tau_{s_\delta}^g\tau_{s_z}^t\beta_z+\left(\beta_\delta m_t\tau_{s_z}^t\right)^2\right]-m_g\tau_{s_\delta}^g\beta_\delta\left(\left(m_t\tau_{s_z}^t\right)^2+\frac{\tau_z}{\tau_n}\right)=0. \tag{25}$$

This equation has a real root because it is cubic. It has only positive real roots since the coefficients of odd/even powers are positive/negative. It has three distinct real roots when its discriminant is positive. The discriminant can be written as a polynomial of $\frac{1}{\tau_n}$:

$$D = \sum_{i=0}^{3} d_i \left(\frac{1}{\tau_n}\right)^i,$$

where $d_0 = \beta_z^4 m_g^2 m_t^3 \left(\tau_{s_z}^t\right)^3 \left(\tau_{s_\delta}^g\right)^2 \left(m_t \tau_{s_z}^t \beta_\delta^2 - 4\beta_z m_g \tau_{s_\delta}^g\right)$. For a sufficiently large τ_n, $D > 0$ if $d_0 > 0$. Therefore, for a sufficiently large τ_n, (25) has three distinct real roots when

$$\frac{4\beta_z m_g \tau_{s_\delta}^g}{\beta_\delta^2 m_t \tau_{s_z}^t} < 1.$$

Case 2.2: $\tau_{s_\delta}^t = 0$ **and** $\tau_{s_z}^g > 0$.

Notice that $\xi_\delta \beta_\delta \tau_{s_z}^g + \xi_z \beta_z \tau_{s_\delta}^g$ is constant, so $\xi_z(\xi_\delta)$ is a linear function. Plugging it back to the second equation of the system (21), we obtain the following equation for ξ_δ:

$$f(\xi_\delta) = \sum_{i=0}^{3} a_i \xi_\delta^i = 0, \qquad (26)$$

where $a_1, a_3 > 0$ and $a_0, a_2 < 0$. This equation has a real root because it is cubic. It has only positive real roots since the coefficients of odd/even powers are positive/negative. It has three distinct real roots when its discriminant is positive. The discriminant can be written as a polynomial of $\frac{1}{\tau_n}$:

$$D = \sum_{i=0}^{3} d_i \left(\frac{1}{\tau_n}\right)^i,$$

where

$$d_0 = \frac{m_g^2}{\beta_z^2}\left(m_g \tau_{s_z}^g + \beta_z m_t \tau_{s_z}^t\right)^2 \left(\tau_{s_\delta}^g \beta_z^2 + \tau_{s_z}^g \beta_\delta^2\right)^2 \left(\beta_\delta^2 m_t^2 \left(\tau_{s_z}^t\right)^2 - 4\tau_{s_z}^g \tau_{s_\delta}^g m_g^2 - 4\beta_z m_g m_t \tau_{s_\delta}^g \tau_{s_z}^t\right).$$

For a sufficiently large τ_n, $D > 0$ if $d_0 > 0$. Therefore, for a sufficiently large τ_n, (26) has three distinct real roots when

$$\frac{4\tau_{s_z}^g \tau_{s_\delta}^g m_g^2 + 4\beta_z m_g m_t \tau_{s_\delta}^g \tau_{s_z}^t}{\beta_\delta^2 m_t^2 \left(\tau_{s_z}^t\right)^2} = \frac{4 m_g \tau_{s_\delta}^g \left(\tau_{s_z}^g m_g + \beta_z m_t \tau_{s_z}^t\right)}{\beta_\delta^2 m_t^2 \left(\tau_{s_z}^t\right)^2} < 1.$$

Case 2.3: $\tau_{s_\delta}^t > 0$ **and** $\tau_{s_z}^g = 0$.

The price coefficient ξ_z is $m_t \tau_{s_z}^t$. ξ_δ solves

$$f(\xi_\delta) = \sum_{i=1}^{5} a_i \xi_\delta^i = 0. \tag{27}$$

Moreover, $a_5 = \beta_z^2$ and $a_0 = a_{0,1}\frac{1}{\tau_n} + a_{0,2}\left(\frac{1}{\tau_n}\right)^2$, where $a_{0,1}, a_{0,2} < 0$ do not depend on τ_n. Then there exists at least one positive real root. Let's now show that there exists at least three positive real roots for a sufficiently large τ_n. Our approach is analogous to the proof of Lemma 2, so we keep the proof here brief.

We can write

$$a_0 = a_{0,1}\frac{1}{\tau_n} + a_{0,2}\left(\frac{1}{\tau_n}\right)^2,$$
$$a_1 = a_{1,0} + a_{1,1}\frac{1}{\tau_n} + a_{1,2}\left(\frac{1}{\tau_n}\right)^2,$$

where $a_{i,j}$ are coefficients that do not depend on τ_n. Moreover, $a_{0,1} < 0$ and $a_{1,0} > 0$. Then, evaluating $f(\cdot)$ at $-\frac{a_{0,1}}{a_{1,0}}\frac{1}{\tau_n} + c_1\frac{1}{\tau_n} > 0$ for some $c_1 > 0$, we obtain

$$f\left(-\frac{a_{0,1}}{a_{1,0}}\frac{1}{\tau_n} + c_1\frac{1}{\tau_n}\right) = a_{1,0}c_1\frac{1}{\tau_n} + o\left(\frac{1}{\tau_n}\right).$$

For a sufficiently large τ_n, $f\left(-\frac{a_{0,1}}{a_{1,0}}\frac{1}{\tau_n} + c_1\frac{1}{\tau_n}\right) > 0$.

Next, we can write (27) also as a polynomial of $\frac{1}{\tau_n}$:

$$f\left(\xi_\delta\right) = \sum_{i=0}^{2} b_i(\xi_\delta)\left(\frac{1}{\tau_n}\right)^i,$$

where

$$b_0(\xi_\delta) = \xi_z\left(\xi_z^2 + m_t^2\tau_{s_z}^t\tau_{s_\delta}^t\right)\left(\beta_z\xi_\delta - \beta_\delta m_t\tau_{s_z}^t\right)^2 + m_t m_g \tau_{s_\delta}^g \tau_{s_z}^t \xi_\delta^2\left(\beta_z\xi_\delta - \beta_\delta m_t\tau_{s_z}^t\right).$$

Then, evaluating $f(\cdot)$ at $\frac{1}{\beta_z}\left(\beta_\delta m_t \tau^t_{s_z} - \left(\frac{1}{\tau_n}\right)^{1/2}\right)$, we obtain

$$f\left(\frac{1}{\beta_z}\left(\beta_\delta m_t \tau^t_{s_z} - \left(\frac{1}{\tau_n}\right)^{1/2}\right)\right) = -m_t m_g \tau^t_{s_\delta}\tau^t_{s_z}\left(\frac{1}{\beta_z}\beta_\delta m_t \tau^t_{s_z}\right)^2\left(\frac{1}{\tau_n}\right)^{1/2} + o\left(\left(\frac{1}{\tau_n}\right)^{1/2}\right).$$

Therefore, for a sufficiently large τ_n, $f\left(\frac{1}{\beta_z}\left(\beta_\delta m_t \tau^t_{s_z} - \left(\frac{1}{\tau_n}\right)^{1/2}\right)\right) < 0$ and, at the same time, $\frac{1}{\beta_z}\left(\beta_\delta m_t \tau^t_{s_z} - \left(\frac{1}{\tau_n}\right)^{1/2}\right) > -\frac{a_{0,1}}{a_{1,0}}\frac{1}{\tau_n} + c_1\frac{1}{\tau_n} > 0$. Furthermore, because $a_5 > 0$, for any $\tau_n > 0$ $f(\xi_\delta) > 0$ if ξ_δ is sufficiently large. Hence, we have shown that (27) has at least three (positive real) solutions for ξ_δ if τ_n is sufficiently large.

Case 2.4: $\tau^t_{s_\delta}, \tau^g_{s_z} > 0$**.**

Notice that $\xi_z\beta_z\tau^g_{s_\delta} + \xi_\delta\beta_\delta\tau^g_{s_z}$ is linear in ξ_z, so we can straightforwardly write $\xi_z = \xi_z(\xi_\delta)$. Plugging it back to the second equation of the system (21), we obtain the following equation for ξ_δ:

$$f(\xi_\delta) = \sum_{i=1}^{9} a_i \xi_\delta^i = 0. \tag{28}$$

Moreover, $a_{11} > 0$ and $a_0 < 0$. Then there exists at least one positive real root. Let's now show that there exists at least three real roots for a sufficiently large τ_n. Our approach is analogous to the proof of Lemma 2, so we keep the proof here brief.

We can write

$$\begin{aligned}
a_0 &= a_{0,4}\left(\frac{1}{\tau_n}\right)^3 + a_{0,5}\left(\frac{1}{\tau_n}\right)^4,\\
a_1 &= a_{1,3}\left(\frac{1}{\tau_n}\right)^2 + a_{1,4}\left(\frac{1}{\tau_n}\right)^3 + a_{1,5}\left(\frac{1}{\tau_n}\right)^4,\\
a_2 &= a_{2,3}\left(\frac{1}{\tau_n}\right)^2 + a_{2,4}\left(\frac{1}{\tau_n}\right)^3,\\
a_3 &= a_{3,2}\left(\frac{1}{\tau_n}\right) + a_{3,3}\left(\frac{1}{\tau_n}\right)^2 + a_{3,4}\left(\frac{1}{\tau_n}\right)^3,\\
a_4 &= a_{4,2}\left(\frac{1}{\tau_n}\right) + a_{4,3}\left(\frac{1}{\tau_n}\right)^2,
\end{aligned}$$

where $a_{i,j}$ are coefficients that do not depend on τ_n. Moreover, $a_{0,4} < 0$ and $a_{1,3} > 0$. Then, evaluating $f(\cdot)$ at $-\frac{a_{0,4}}{a_{1,3}}\frac{1}{\tau_n} + c_1\frac{1}{\tau_n} > 0$ for some $c_1 > 0$, we obtain

$$f\left(-\frac{a_{0,4}}{a_{1,3}}\frac{1}{\tau_n} + c_1\frac{1}{\tau_n}\right) = a_{1,3}c_1\left(\frac{1}{\tau_n}\right)^4 + o\left(\left(\frac{1}{\tau_n}\right)^4\right).$$

For a sufficiently large τ_n, $f\left(-\frac{a_{0,4}}{a_{1,3}}\frac{1}{\tau_n} + c_1\frac{1}{\tau_n}\right) > 0$.

Next, we can write (28) also as a polynomial of $\frac{1}{\tau_n}$:

$$f(\xi_\delta) = \sum_{i=0}^{5} b_i(\xi_\delta)\left(\frac{1}{\tau_n}\right)^i,$$

where $b_0(\xi_\delta)$ has a root at $\xi_\delta = \check{\xi}_\delta = \frac{\beta_\delta\left[m_g\tau^g_{s_z}\tau^g_{s_\delta} + \beta_z m_t\left(\tau^t_{s_z}\tau^g_{s_\delta} - \tau^g_{s_z}\tau^t_{s_\delta}\right)\right]}{\left(\beta_z^2\tau^g_{s_\delta} + \beta_\delta^2\tau^g_{s_z}\right)}$. Note that under our benchmark assumptions, $\tau^t_{s_z}\tau^g_{s_\delta} - \tau^g_{s_z}\tau^t_{s_\delta} = 0$ and $\check{\xi}_\delta > 0$. Moreover, $\check{\xi}_\delta > 0$ as long as traditional/green investors are relatively more informed about $\tilde{z}/\tilde{\delta}$-payoff component. Therefore, we consider $\check{\xi}_\delta > 0$ as a more empirically relevant case. However, for the sake of completeness, we also study the case $\check{\xi}_\delta \leq 0$ separately.

Case 2.4.1: $\check{\xi}_\delta > 0$. Evaluate $b_0(\cdot)$ at $\check{\xi}_\delta - \left(\frac{1}{\tau_n}\right)^{1/2}$ to obtain

$$b_0\left(\check{\xi}_\delta - \left(\frac{1}{\tau_n}\right)^{1/2}\right) = -c_2\left(\frac{1}{\tau_n}\right)^{1/2} + o\left(\left(\frac{1}{\tau_n}\right)^{1/2}\right),$$

where c_2 is a positive coefficient which does not depend on τ_n. Then, evaluating $f(\cdot)$ at the same point, we obtain

$$f\left(\check{\xi}_\delta - \left(\frac{1}{\tau_n}\right)^{1/2}\right) = -c_2\left(\frac{1}{\tau_n}\right)^{1/2} + o\left(\left(\frac{1}{\tau_n}\right)^{1/2}\right).$$

For a sufficiently large τ_n, the above expression is negative and, at the same time, $\check{\xi}_\delta - \left(\frac{1}{\tau_n}\right)^{1/2} > -\frac{a_{0,4}}{a_{1,3}}\frac{1}{\tau_n} + c_1\frac{1}{\tau_n} > 0$. Furthermore, because $a_9 > 0$, for any $\tau_n > 0$ $f(\xi_\delta) > 0$ if ξ_δ is sufficiently large. Hence, we have shown that (28) has at least three (positive real) solutions for ξ_δ if τ_n is sufficiently large.

Case 2.4.2: $\check{\xi}_\delta < 0$. Evaluate $b_0(\cdot)$ at $\check{\xi}_\delta + \left(\frac{1}{\tau_n}\right)^{1/2}$ to obtain

$$b_0\left(\check{\xi}_\delta + \left(\frac{1}{\tau_n}\right)^{1/2}\right) = c_2\left(\frac{1}{\tau_n}\right)^{1/2} + o\left(\left(\frac{1}{\tau_n}\right)^{1/2}\right),$$

where c_2 is the same positive coefficient as in Case 2.4.1. Then, evaluating $f(\cdot)$ at the same point, we obtain

$$f\left(\check{\xi}_\delta + \left(\frac{1}{\tau_n}\right)^{1/2}\right) = c_2\left(\frac{1}{\tau_n}\right)^{1/2} + o\left(\left(\frac{1}{\tau_n}\right)^{1/2}\right).$$

For a sufficiently large τ_n, the above expression is positive and, at the same time, $\check{\xi}_\delta + \left(\frac{1}{\tau_n}\right)^{1/2} < 0 < -\frac{a_{0,4}}{a_{1,3}}\frac{1}{\tau_n} + c_1\frac{1}{\tau_n}$.[33] Furthermore, because $a_{11} > 0$, for any $\tau_n > 0$ $f(\xi_\delta) < 0$ if ξ_δ is sufficiently large in absolute terms and negative. Hence, we have shown that (28) has at least three real solutions for ξ_δ if τ_n is sufficiently large (recall that $f(0) = a_0 < 0$).

Case 2.4.3: $\check{\xi}_\delta = 0$.

In this case, $b_0(\cdot)$ can be written as

$$b_0(\xi_\delta) \overset{\check{\xi}_\delta=0}{=} A\xi_\delta^6 \sum_{i=0}^{3} b_{0,i}\xi_\delta^i,$$

where $A > 0, b_{0,3} > 0, b_{0,0} > 0$. Then there exists $\hat{\xi}_\delta < 0$ that solves $b_0(\xi_\delta) = 0$ such that

$$b_0\left(\hat{\xi}_\delta + \left(\frac{1}{\tau_n}\right)^{1/2}\right) = c_3\left(\frac{1}{\tau_n}\right)^{1/2} + o\left(\left(\frac{1}{\tau_n}\right)^{1/2}\right),$$

where c_3 is a positive constant. Moreover, at this point

$$f\left(\hat{\xi}_\delta + \left(\frac{1}{\tau_n}\right)^{1/2}\right) = c_3\left(\frac{1}{\tau_n}\right)^{1/2} + o\left(\left(\frac{1}{\tau_n}\right)^{1/2}\right).$$

For a sufficiently large τ_n, the above expression is positive and, at the same time, $\hat{\xi}_\delta + \left(\frac{1}{\tau_n}\right)^{1/2} < 0 < -\frac{a_{0,4}}{a_{1,3}}\frac{1}{\tau_n} + c_1\frac{1}{\tau_n}$. Furthermore, because $a_{11} > 0$, for any $\tau_n > 0$ $f(\xi_\delta) < 0$ if ξ_δ is sufficiently large in absolute terms and negative. Hence, we have shown that (28)

[33] Note that if a negative ξ_δ solves (28), it does not necessarily imply that the price is negatively associated with $\tilde{\delta}$, i.e. that $q_\delta < 0$, because by definition $\xi_\delta = \frac{q_\delta}{q_n}$.

has at least three real solutions for ξ_δ if τ_n is sufficiently large (recall that $f(0) = a_0 < 0$).

$\square$

E Investors with homogeneous preferences

The key assumption we make throughout the paper is that there are two groups of investors with heterogeneous stock valuations. Because of the preference heterogeneity, they use information about the same fundamentals differently and trade in the opposite directions, which might give rise to multiple equilibria that differ in the relative price informativeness about the two fundamentals. We show the robustness of this result to general assumptions on the information structure in Appendix D.

The goal of this Appendix is to show that the preference heterogeneity is an essential ingredient for the equilibrium multiplicity. In particular, we explore a model that features two groups of investors that have homogeneous preferences but might have different information about the two fundamentals. The key difference between our setting and Goldstein and Yang (2015) is that we allow investors of both groups to receive informative signals about both fundamentals. As we discuss in Appendix D, this is crucial to support multiple equilibria in the trading stage when investors' preferences are heterogeneous. Our key result here is that equilibrium in the trading stage is unique when preferences are homogeneous.

We consider the same framework as described in Section 3 with several differences. First, we assume that both groups of investors have the same stock valuation, $\beta_z \tilde{z} + \beta_\delta \tilde{\delta}$. For consistency, we keep denoting the two groups using t and g subscripts. The masses of the two groups are m_t and m_g. Without loss of generality, we set the utility weights $\beta_z = \beta_\delta = 1$ and the risk aversion parameter $\gamma = 1$. Further, we assume that t/g-investors specialize in particular types of information and, thus, receive signals about $\tilde{z}$ and $\tilde{\delta}$ with precisions of $\tau_s/\rho\tau_s$ and $\rho\tau_s/\tau_s$, respectively. Without loss of generality, we assume $\rho \in [0, 1]$. The priors for $\tilde{z}$ and $\tilde{\delta}$ are assumed to be the same, $\tau_z = \tau_\delta = \tau$.[34]

Following the same steps as in Section 3.2, we arrive at the following system of equations

[34]These assumptions on the information structure can be further relaxed (at the expense of tractability but without changing the final result) by allowing for different prior precisions and more general signal precisions. The analyses are available upon request.

for ξ_z and ξ_δ:

$$\xi_z = \tau_s \left[m_t \frac{\xi_\delta^2 + \frac{\tau+\rho\tau_s}{\tau_n} - \xi_\delta \xi_z}{(\xi_z - \xi_\delta)^2 + (2\tau + \tau_s(1+\rho))\frac{1}{\tau_n}} + m_g \rho \frac{\xi_\delta^2 + \frac{\tau+\tau_s}{\tau_n} - \xi_\delta \xi_z}{(\xi_z - \xi_\delta)^2 + (2\tau + \tau_s(1+\rho))\frac{1}{\tau_n}} \right],$$

$$\xi_\delta = \tau_s \left[m_t \rho \frac{\xi_z^2 + \frac{\tau+\tau_s}{\tau_n} - \xi_\delta \xi_z}{(\xi_z - \xi_\delta)^2 + (2\tau + \tau_s(1+\rho))\frac{1}{\tau_n}} + m_g \frac{\xi_z^2 + \frac{\tau+\rho\tau_s}{\tau_n} - \xi_\delta \xi_z}{(\xi_z - \xi_\delta)^2 + (2\tau + \tau_s(1+\rho))\frac{1}{\tau_n}} \right].$$

Denote $x \equiv \xi_\delta - \xi_z$. It is easy to see that, given x, ξ_z and ξ_δ are uniquely pinned down. Furthermore, the system can be simplified to the following quintic equation for x:

$$f(x)g(x) = (1-\rho)^2 (1+\rho)\, \tau_n \tau_s^3 m_g m_t x, \tag{29}$$

where

$$f(x) = x\left(x^2\tau_n + 2\tau + \tau_s(1+\rho)\right) + x\tau_n\tau_s^2\left(\rho\left(m_g^2 + m_t^2\right) + m_g m_t\left(1+\rho^2\right)\right) - \tau\tau_s(m_g - m_t)(1-\rho),$$

$$g(x) = x^2\tau_n + 2\tau + \tau_s(1+\rho).$$

Clearly, (29) has a unique solution $x = 0$ when $\rho = 1$. Suppose now that $\rho < 1$ and $m_g > m_t$ (case of $m_g \leq m_t$ can be considered analogously). Our goal is to show that (29) has unique solution.

Note that there exists a unique $\underline{x} > 0$ such that $f(\underline{x}) = 0$. Moreover, $\forall x \geq \underline{x}$ $f(x)g(x)$ is an increasing convex function. Then there exists exactly one solution to (29) when $x \geq 0$.

Let's now show that there is no solutions when $x < 0$. First, $f(0)g(0) < 0$. Second, $f(x)g(x)$ is increasing and concave when $x < 0$. Finally, the derivative of $f(x)g(x)$ at 0 is $f'(0)g(0) + f(0)g'(0) > (1+\rho^2)(1+\rho)\tau_n\tau_s^3 m_g m_t > (1-\rho)^2(1+\rho)\tau_n\tau_s^3 m_g m_t$. So the right-hand side of (29) is always above the left-hand side when $x < 0$. We, therefore, have established the following proposition.

Proposition 10. *If investors have homogeneous preferences, there exists a unique equilibrium with a linear stock price.*

We conclude that the equilibrium multiplicity in the trading game requires investors to have heterogeneous stock valuations. Otherwise, trading behaviors of investors are closely aligned and the price can never be informative to one group but not to the other.