Exploration Activity, Long Run Decisions, and the Risk Premium in Energy Futures*

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Abstract

We present evidence that the aggregate capital stock of firms in oil and gas exploration and development (E&D) as well as firms' inventories help in explaining the dynamics of the slope of the futures curve for crude oil. Standard structural approaches for modeling the futures curve either highlight the role of inventory (storage models) or the rate of extraction (production models), but both decisions are seldom modeled simultaneously. Here we build a new equilibrium model that has both features, and in addition, models the process of E&D capital accumulation, which can affect the cost of extraction as the oil industry drills in increasingly expensive fields. We show how the three decisions interact in a world of exhaustible resources. In a nutshell, a steeper futures slope not only increases the attractiveness of carrying inventory, but also provides greater value to accumulating E&D capital. Our model sheds light on the role of exhaustibility of resources on the increasing trend of real oil prices and capital accumulation, and the peaking of consumption. Its also helps understand why inventories and E&D capital each negatively predict returns on oil futures, and is thus able to shed light on the negative relation between the slope and risk premium on oil futures.

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Introduction

Recent years have seen the development of increasingly sophisticated technologies for the extraction of natural resources from costlier fields.[1] These new technologies brought to fruition by investments by the resource extraction industry have changed the current and expected future prices of resources and have important consequences for energy self sufficiently and stability of growth for North America. In this paper we ask if the investment in exploration and development (E&D) of resources has an impact or is affected by the keenly watched market statistics of current and future prices of the resource.

One of the most widely watched statistics in the futures market is the slope of the futures curve. We measure it as the weak relative basis, which is the proportional difference between the discounted value of the futures price and the current spot price of the resource (see also Litzenberger and Rabinowitz (1995)). When this quantity is positive (negative) we say the futures market is in weak contango (backwardation). Of interest to practitioners and researchers is the economic information that determines the relative basis. The theory of storage (Kaldor (1939) and Working (1948)) implies that the futures relative basis is strongly positively related to inventories. We call this the "short-run" information about resource prices in the futures relative basis. However, as we will see below, inventory data, though very useful, is unable to explain the basis in certain periods. In addition in this paper, we argue that the futures relative basis also contains "long-run" information about resource prices, which has important implications for decisions such as the exploration and development of the resource extraction process. In particular, we will develop four stylized properties of oil futures prices that arise from the long-run risks faced by energy producers.

Stylized Fact 1: The aggregate capital stock of firms in E&D as well as firms' inventories help in explaining the dynamics of the slope of the futures curve for crude oil.

The top left panel of Figure 1 shows the seasonally adjusted futures basis of crude oil. As can been seen the futures curve has mostly been in contango for the period from 2008 to 2013,

[1]In Alberta, Canada, new techniques have been developed to extract crude oil from bitumen using less water and energy and damage to the environment than previously envisaged. In the United States, new hydraulic fracturing technology has made the oil and gas trapped in previously inaccessible shale rock now economically feasible to extract. A recent survey by Maugeri (2012) provides several interesting highlights of the recent revolution in energy production.

Table 1: What Explains the Futures Weak Relative Basis For Crude Oil (1986:7 - 2014:9)?

No.	α	β_1	β_2	R^2
1	-0.933	0.850		0.284
	[-7.402]	[6.837]		
2	-0.149		0.866	0.246
	[0.0215]		[4.796]	
3	-0.821	0.678	0.611	0.401
	[6.704]	[5.459]	[3.159]	

We report the coefficients of the fitted monthly regression:

$$\text{Weak Relative Basis}(t) = \alpha + \beta_1 \text{ Inventory}(t-1) + \beta_2 \text{ Capital Stock/GDP}(t-1) + \epsilon(t).$$

The weak relative basis on 1-year contracts in quarter t is $[e^{-r(t)}F(t) - S(t)]/S(t)$, where $F(t)$ is the 1-year futures prices at the beginning of each quarter and $S(t)$ is the spot price of WTI oil in Cushing, Oklahoma. Seasonal adjustment is done using the X-12 procedure (used by the US Department of Commerce). The explanatory variable "Inventory" stands for the seasonally adjusted total US stock of crude oil and petroleum products (in billions of barrels) excluding special purpose reserves at the end of each month. The "Capital Stock" is the sum of the "Property Plant and Equipment" variable in Compustat of firms in oil and gas field exploration services (SIC code 1382). T-statistics are in parenthesis and are adjusted for heteroskedasticity and autocorrelation.

while backwardation was more frequently prevalent earlier. The bottom left and right panels show seasonally adjusted inventory and the capital stock to GDP ratios, respectively. While inventory has been higher in recent years as well relative to the earlier part of the sample, the growth of the capital stock of firms in E&D firms has been far more spectacular. The ratio of capital stock to GDP was quite stable in our sample from 1986 to around 2001, but has grown very rapidly since then. This has been the period of rapid development of the shale oil plays in the US.

Table 1 reports simple linear regressions at a monthly frequency of the futures relative basis on inventory and the capital stock of E&D firms as a share of U.S. GDP (see the data appendix for sources of data).[2] As can be seen, while one quarter lagged inventory explains

[2]Throughout this paper we look at the statistics of the one-year futures relative basis. While it would be of interest to study longer maturity futures, we are constrained by the lack of long historical times series on these longer term contracts. Two-year contracts started trading actively in mid 1990 and four-year contracts only in 1997. The correlations of the relative basis of the two-year and four-year contracts with the one-year contract are 99.7 and 98.7 over the subsamples, respectively.

about 28 percent of the variation in the basis, the lagged capital to GDP ratio explains about 25% of the variation in the relative basis (lines 1 and 2). Both variables have positive beta coefficients. When both variables are considered, we explain about 40% of the variation in the relative basis, and each variable remains significant. This suggests that both short and long run decision by firms are important determinants of the futures relative basis. The periods when the discounted futures price is higher than the spot price, inventory accumulates. In addition, firms raise more capital for E&D expenses in response perhaps to higher futures prices.

Table 2: Risk Premium on Crude Oil Futures at Alternative Horizons (1986:7 - 2014:9)

h	α	β_1	β_2	R^2
1	0.101	-0.073	-1.113	0.024
	[0.900]	[-0.509]	[-1.758]	
3	0.231	-0.155	-0.322	0.037
	[0.821]	[-0.454]	[-1.472]	
12	-0.501	0.795	-1.688	0.065
	[0.579]	[0.941	[-2.336]	

We report the coefficients of the fitted monthly regression:

$$\sum_{i=0}^{h} \text{return}(t-1+i) = \alpha + \beta_1 \, \text{Inventory}(t-2) + \beta_2 \, \text{Capital Stock/GDP}(t-2) + \epsilon(t),$$

for $h = 1, 3,$ and 12 months. return(t) is the return on purchasing a 2-month maturity futures contract at the beginning of the month $t-1$ and closing it at the spot price, which is the 1-month maturity futures contract at t. The explanatory variable "Inventory" stands for the total US stock of crude oil and petroleum products (in billions of barrels) excluding special purpose reserves at the end of the month The "Capital Stock" is the sum of the "Property Plant and Equipment" variable in Compustat of firms in oil and gas field exploration services (SIC code 1382). T-statistics are in parenthesis and are adjusted for heteroskedasticity and autocorrelation.

Stylized Fact 2: The risk premium on crude oil futures is negatively related to both inventory and capital of E&D firms.

As seen in Table 2, the two variables explain 2.4 percent of the excess return variation at a monthly frequency, with each variable having negative coefficients. At longer horizons of 3 months and 12 month rolling returns (holding a sequence of 2-month contracts for 1 month) are predicted with R^2 of 3.7 percent and 6.5 percent, respectively.

Stylized Fact 3: As seen in the top panel of Figure 2, the aggregate consumption of petroleum products trended upwards from 1986 to 2007, but has fallen off since then. The bottom panel shows that the real spot price of oil has trended up around some fluctuations in the entire sample from 1986 to 2014. Therefore, despite increasing capital expenditures and inventories, consumption appears to have at least a local peak amid rising prices.

Stylized Fact 4: It is natural to think of long-run risk as pertaining to events that happen less frequently. The top left panel of Figure 3 shows the decomposition of the time series of the real price of oil into its trend and cycle components using the Hodrick-Prescott procedure. As seen, the trend in real oil prices has been strongly positive since around 2000. The panel also shows the cycle component, which declined during the past two recessions, and shows the large fluctuations around the positive trend. The bottom left panel of Figure 3 shows the variance frequency decomposition of the real spot price of oil. As seen in the figure, there is a large amount of variance of the real spot price that is of low frequency movements (every 56 years or less). There is also a large proportion of the variation at the business cycle frequency of around 2.5 to 3 years, which reflects the variation in the cycle components above. The cyclical variation at the business cycle frequency is consistent with the evidence in Hamilton, who points out the role of oil prices in most past recessions. It is also consistent with the "mean-reversion" view of real oil prices modeled in finance. The trend component that we show here suggests that the commodity pricing cycle may have additional components to the business cycle, and we will examine this more in detail in this paper.

In this paper, we build a model of the long run decision making of resource producing firms that in equilibrium will lead to firms' decisions and spot and futures prices having these stylized properties. We model demand shocks that drive the business cycle fluctuations in oil prices, but also build in the implications of the exhaustibility of the total resource base, and firms' decisions on capital accumulation that manage their extraction costs as they extract from increasingly costly fields. In fact the new feature that we model in this paper relative to the existing literature, is to have both exogenous and endogenous components of the extraction costs of the resource. To tractably capture the social learning aspect of technological innovations in energy production (see e.g. Covert (2015)), we model capital expenditures by firms that lower the extraction costs of multiple wells owned by the firm. In addition, firms' choose inventories to smooth fluctuations in demand and extraction.

We start with a 2-period version of the model for which we can provide closed form expressions for the value of extraction options and can characterize the optimal investment and inventory policy quite tractably. The closed-form expressions for the futures basis and risk premium show explicitly the role of the real decisions variables in determining these variables. With comparative static exercises we show how the relationship between the decision and financial variables varies with the level of the capital stock. In particular, we highlight the aggressiveness of extraction decisions and inventory choices and their role in affecting the financial variables. The comparative static results are at least qualitatively similar to the first two stylized facts.

To study the dynamics of these variables and their relationship to the data, we next build an infinite horizon model. In dynamic models with inventory restricted to be positive, it is not possible to obtain closed-forms as has been pointed out by several authors. So, we solve the model using projection methods as has been popularized by the work of Judd (1999). Simulations of the model show that both inventory and capital accumulation impact the basis and the risk premium with the same signs as in the data. The model also shows that even though consumption peaks, and real prices of the resource trend upwards, the cycles of inventory and capital accumulation are quite stable, as increasingly costlier resources are extracted until a point when new extraction stops. While the resource is not completely exhausted even after more than a hundred years, consumption drops significantly after about a third of the resource base is exhausted, while capital fluctuates with the demand cycle for long periods before firms stop accumulating it. Therefore, as noted by Pindyck (1980), the resource in the long term in not renewed, rather than exhausted. The model also implies that after capital accumulation slows, the futures basis declines and the risk premium increases.

Our model contributes to the literature on resource extraction and storage. Most existing models have either one of these features. Models of storage assume exogenous extraction decisions (e.g. Deaton and Laroque (1992) and Routledge, Seppi, and Spatt (2000)). Starting with the seminal work of Hotelling (1931), models of exhaustible resource extraction on the other hand, allow no storage (e.g. Pindyck (1980), Litzenberger and Rabinowitz (1995), Carlson, Khokher, and Titman (2007) and Kogan, Livdan, and Yaron (2009)). The same is true of models that allow production of commodities (see e.g. Casassus, Collin-Dufresne, and Routledge (2008)). In the context of agricultural commodities, there is an older literature that has production and storage in equilibrium, but the analysis in such models does not apply to exhaustible resources, where equilibrium profits are compatible with competitive equilibrium due to limitations in supply (e.g Scheinkman and Schechtman (1983)). With exhaustible resources, as pointed out in Litzenberger and Rabinowitz (1995), there are profits at the time of extraction that are optimized by the extraction timing decision of resource firms, a feature that we model explicitly. The model sheds light on the negative relation between the futures basis and the risk premium as has been pointed out by several authors (e.g. Fama and French (1987), Gorton and Rouwenhourst (2006), Erb and Harvey (2006), and Baker and Routledge (2012)).

The paper structure is as follows. In Section 1, we formulate the 2-period model of optimal resource extraction and the relative basis, and study its comparative statics. In Section 3, we extend our analysis to the infinite horizon model and study the dynamic properties of the model using simulations. Section 4 concludes. Three appendices contain the data description and sources, some analytically results, and the numerical procedure for solving the infinite horizon model, respectively.

1 A Simple Two Period Model of Resource Extraction and Exploration Activity

We build on the two period version of the model of LR, with several generalizations. The most significant addition is of an E&D (investment) decision that reduces costs of future extraction of the resources. To tractably analyze the investment decision with technology spillover, we introduce multi-plant firms. Assume a continuum of price taking identical resource production multi-plant firms, each of which owns an equal share of reserves. We will focus our analysis on the representative firm.

We start with a description of the demand side of the model. The demand function for the resource at time t is given by simple function $q_t = f(S_t, \epsilon_t)$, where ϵ_t is a demand shock realization for the resource at date t. Without loss of generality, we set $\epsilon_0 = 0$, and $\epsilon_1 = \epsilon$. Conditional on a realization of ϵ, the inverse demand function is $s = f^{-1}(q_t; \epsilon_t)$.

Supply of the resource is optimally determined by the firm. The resource can be extracted from wells of varying quality, which is parsimoniously captured by a variable x. Wells x are uniformly distributed $x \in [0, \bar{x}]$ in period 0. Well x is operated by plant x owned by the firm and has access to technology with extraction cost in period 0 of $x\, g(K_0)$, where K_0 is the amount of capital in the industry (to be discussed below). Let R_0 be the reserves available at date 0. At date 0, the plant level decisions determine the cutoff reserve quality (extensive margin), x_0^e.

As discussed in the introduction, there are interesting relationships between the slope of the futures curve, real decisions, and expected returns on futures strategies. To address these, our model must build in the price of risk in energy commodities. Following a long literature in asset pricing, we specify an exogenous pricing kernel with a constant price of risk of the form:

$$M_{t+1} \;=\; M_t \cdot \exp(-r - \sigma_M\, \epsilon). \tag{1}$$

To keep things simple, we have specified that the kernel depends on the shock to energy consumption, so that marginal utility is high in periods of low energy demand. While oil and total consumption are not perfectly correlated in real data, we could generalize this assumption with an increase in computational complexity by having a second not perfectly correlated shock to the kernel. Using the kernel, we can compute all expectations under the risk-neutral measure, which we will denote as $E^Q[\cdot]$.

We assume that all investments in technology are made at the firm level, and affects extraction costs for resources of all grades. In particular, extraction cost for well x at time t is given by $x\, g(K_t)$, where K_t is the amount of capital in E&D. The timing of capital installation is as follows: At date 0, the firm inherits capital of K_0 from past decisions. The firm can augment this capital stock by incurring E&D expenses, which we call investing. The new capital will follow the standard process

$$K_{t+1} = e^{-\delta}\, K_t + I_t. \tag{2}$$

The investment choice is made *before* before any extraction decisions are made. Conditional on the investment choice at the firm level, each plant chooses its extraction decision to maximize the profits of the plant. Conditional on the firm level investment, the plant level maximization can be written as:

$$\pi_0^x = \max_{0 \le Q_0^x \le \frac{R_0}{\bar{x}}} S_0\, Q_0^x - Q_0^x\, x\, g(K_0) \;+\; e^{-r} E^Q[(S_1 - x\, g(K_1))^+]\, (\frac{R_0}{\bar{x}} - Q_0^x). \tag{3}$$

In particular, for a firm with positive and interior production

$$S_0 - x\, g(K_0) = C(x\, g(K_1)), \tag{4}$$

where $C(x\, g(K_1))$ is the value of a 1-period call option with exercise price of $x\, g(K_1)$. The left-hand side is the net gain to current extraction, while the right-hand side is the value of delaying extraction. It is useful to note at this point that the call option valuation in (4) is quite similar to a regular American option, with the only difference being that the price at each date of the resource is determined by the aggregate optimal extraction decision of all producers using the inverse demand function. The Kuhn-Tucker optimality condition at the boundaries for the extraction choice of plant x satisfy

$$[S_0 - x\, g(K_0) - \; C(x\, g(K_1))]\; Q_0^x = 0 \quad \text{or} \quad [S_0 - x\, g(K_0) - \; C(x\, g(K_1))]\; (\frac{R_0}{\bar{x}} - \; Q_0^x) = 0. \tag{5}$$

We complete the analysis of the model by determining the investment choice at date 0 in the context of the model without and with storage in the following subsections.

1.1 Model Without Storage

We now show how the cutoff resource quality (the extensive margin is determined) x_0^e. At date 1 since there are no further options and no inventory, all plants with available resource and extraction costs smaller than the price ($x\, g(K_1) < S_1$) will extract. Given installed capital of K_1, therefore, aggregate production at date 1 will be

$$Q_1(x^e, \epsilon) = \left(\int_{x_0^e}^{S_1/g(K_1)} \frac{1}{\bar{x}} \, dx \right) R_0 = \frac{S_1/g(K_1) \, - x_0^e}{\bar{x}} \, R_0. \tag{6}$$

Hence, the date 1 price is $\tilde{S}_1 = s(Q_1(x^e, \epsilon); \epsilon)$. Let $C(x|x_0^e, K_0, I_0)$ be the value of the extraction call option for the firm when the extensive margin is x_0^e, K_0 is capital at date 0, and I_0 is investment of at date 0. Then x_0^e satisfies the fixed-point condition:

$$S_0(x_0^e) - x_0^e g(K_0) = C(x_0^e \, g(K_1)|x_0^e, K_0, I_0), \tag{7}$$

when it lies in the interior of the interval $[0, \bar{x}]$, and with the boundary conditions:

$$x_0^e(K_0, I_0) \;=\; 0 \;\text{ if }\; s(0)\, g(K_0) < C(0|0, K_0, I_0), \tag{8}$$

$$\;=\; \bar{x} \;\text{ if }\; s(\bar{x})\, g(K_0) - \bar{x} > C(\bar{x}|\bar{x}, K_0, I_0). \tag{9}$$

The firm maximizes total profit at date 0

$$\begin{aligned} \pi_0 &= \max_{I_0 > 0} E^Q \left[\int_0^{\bar{x}} \pi_0^x dx - P_0 \, I_0 \right] \qquad (10) \\ &= \max_{I_0 > 0} \left[S_0 \, \frac{x_0^e}{\bar{x}} - \frac{(x_0^e)^2}{\bar{x}} \, g(K_0) \right] \, R_0 \; + \; \left(\int_{x_0^e}^{\bar{x}} C(x \, g(K_1)|x_0^e, K_0, I_0) \, dx \right) \, \frac{R_0}{\bar{x}} \; - \; P_0 \, I_0 \end{aligned}$$

where P_0 is the price of capital at date 0 in consumption goods at that date. To compute expected profit we calculate the maximal investment choice numerically by choosing over a grid of values.

We now make specific assumptions on the demand function and the distribution of shocks that enable us to solve for the firm value in closed form. Specifically, we assume a linear demand function in each period of the form: $q_0 = a - b\, S_0$, and $q_1 = a \cdot e^{\mu + \sigma \, \epsilon} - b\, S_1$, where the demand shock ϵ is distributed $N(0, 1)$. The assumption implies resource expected demand growth at the rate μ. Now using the demand function at date 1, equilibrium entails that:

$$\frac{1}{\bar{x}} \left(S_1/g(K_1) - x_0^e \right) \, R_0 = a \, e^{\mu + \sigma \, \epsilon} - b\, S_1.$$

Solving for S_1 we have

$$S_1 = \frac{a\, e^{\mu+\sigma\epsilon} + \frac{x_0^e}{\bar{x}}\, R_0}{b + \frac{R_0}{\bar{x}\, g(K_1)}}. \tag{11}$$

In addition, we have

$$S_0 = 1/b \left(a - \frac{x_0^e}{\bar{x}} R_0 \right). \tag{12}$$

Since the resource prices at each date are dependent on the extraction choices of firms, which in turn depends on capital inherited at date 0, and their investment choice, we first formulate the value of the extraction option conditional on both these variables.

Proposition 1 *The value of the extraction call option at date 0, given installed capital K_0, investment I_0, and cut-off resource quality $x_0^e \in [0, \bar{x}]$ for plant x is*

$$\begin{aligned}
C(x\, g(K_1)|x_0^e, K_0, I_0) &= \frac{a\, e^{-r}}{D} \left[e^{(\mu-\sigma_M\, \sigma+0.5\, \sigma^2)} N(-d_1) - k N(-d_2) \right], \\
d_1 &= \frac{\log(k) - \mu + \sigma_M\, \sigma - \sigma^2}{\sigma}; \qquad d_2 = \frac{\log(k) - \mu + \sigma_M\, \sigma}{\sigma}; \\
k &= \frac{1}{a} \left(D\, x\, g(K_1) - \frac{x_0^e}{\bar{x}}\, R_0 \right); \\
D &= b + \frac{R_0}{\bar{x}\, g(K_1)}.
\end{aligned}$$

The value of a put option is

$$P(x|x^e, K_1) = \frac{a\, e^{-r}}{D} \left[k\, N(d_2) - e^{(\mu-\sigma_M\, \sigma+0.5\, \sigma^2)} N(d_1) \right].$$

The proof is in the appendix.

Using the stock price in (11) implies that the forward price for the linear demand case satisfies:

$$F_0 = E^Q[s(Q_1; \epsilon)] = \frac{a\, e^{\mu-\sigma_M\, \sigma+0.5\sigma^2} + \frac{x_0^e}{\bar{x}}\, R_0}{b + \frac{R_0}{\bar{x}\, g(K_1)}}. \tag{13}$$

What does this simple two-period model imply about the relationship between investment and futures basis? It is hard to sign this relationship in general, we can for given extraction x_0^e decisions. In this case, as seen above, the futures price is increasing in extraction costs, while the spot price, conditional on x_0^e does not depend on it. Therefore, the futures basis is increasing in $g(K_1)$. An increase in the extraction costs implies a lower expected supply in the future, so that prices will be higher in the future, Under the assumption that $g'(K_1) < 0$, we will have a negative relationship between capital and the futures basis, which is counterfactual. However, if x_0^e is higher in periods of high capital, which is reasonable, since the firm is likely to be more aggressive with its extraction policy in periods of low future extraction costs, the relationship can well turn positive. We will look at this relationship further in Section 1.3 below.

1.2 Model With Storage

As mentioned in the introduction, existing models of resource extraction do not allow for storage, while models with inventory do not have optimal resource extraction. In addition, none of these models have exploration activity. Here we provide the analysis of a model with production, storage and exploration. The model will help us address the stylized facts noted in the introduction on the positive comovement of exploration activity, extraction, and inventory accumulation.

We continue to formulate the decisions of the multi-plant firm in the subsection 1.1 assuming once again that E&D investment decisions are made before extraction and inventory decisions. We assume that investment and inventory decisions are made at the firm level, while extraction decisions are made at the plant level. Essentially, in the model with storage, the firm has two substitutable ways of providing the resource to consumers at date 1: it can either defer date 0 extraction and extract in date 1, or it can extract in date 0, and carry inventory to date 1. Which strategy is more profitable? Each has its own advantages, and the tradeoff is to a large part determined by storage costs and the expected change in extraction costs. If the latter are expected to increase rapidly, for example, it might be worthwhile for the firm to extract in date 0 and carry inventory. In addition, the price protection offered by holding the resource in the ground (as in the case of no storage) implies that an increase in uncertainty will make the delayed extraction choice more profitable.

The plant level optimization is very similar to the case without storage, albeit with different equilibrium resource prices. The objective function of the plant still satisfies (3) and its optimal extraction policy is determined as in (4). Given this, the profit at the firm level is

$$\pi_0 = \max_{I_0>0} \max_{x_0^e \in [0,\bar{x}]} \max_{Z_1 \in [0, \frac{x_0^e}{\bar{x}} R_0 + Z_0]} S_0 \left[\frac{x_0^e}{\bar{x}} R_0 + Z_0 - Z_1 \right] - 0.5 \frac{(x_0^e)^2}{\bar{x}} g(K_0) R_0 - P_0 I_0 + E^Q \left[e^{-(r+u)} \tilde{S}_1 Z_1 \right] + \left(\int_{x_0^e}^{\bar{x}} C(x\, g(K_1)|Y_0)\, dx \right) \frac{R_0}{\bar{x}}, \tag{14}$$

where Y_0 denotes the vector of state variables: $Y_0 = (x_0^e, K_0, I_0, Z_0, Z_1)$. Conditional on extraction, investment, and storage policy, the optimality conditions for the extraction policy are similar to the case without storage, but now building in the impact of storage on prices at both dates:

$$S_0 - x_0^e\, g(K_0) = C(x_0^e\, e^{g(K_1)}|Y_0), \text{ if } 0 < x_0^e < \bar{x}, \tag{15}$$

$$x_0^e = 0 \text{ if } s(0)\, g(K_0) < C(0|Y_0), \tag{16}$$

$$x_0^e = \bar{x} \text{ if } s(\bar{x})\, g(K_0) - \bar{x} > C(\bar{x}|Y_0), \tag{17}$$

where for parsimony we have written the date 0 price $s(x_0^e)$, only as a function of the choice of the extensive margin x_0^e, even though it depends on the entire vector Y_0.

We can similarly formulate the firm's optimal storage policy conditional on the investment and extraction decisions. Given our assumption on the inverse demand function we can write the price at date 1 as

$$\tilde{S}_1 = s(Q_1 + Z_1\, e^{-u}; \tilde{\epsilon}). \tag{18}$$

Continuing to assume that the firm is a price taker, the first order condition with respect to inventory, Z_1 is

$$-S_0 + e^{-(r+u)}\, E^Q[S_1] = 0 \text{ if } 0 < Z_1 < \frac{x^e}{\bar{x}} R_0 + Z_0, \tag{19}$$

$$< 0 \text{ if } Z_1 = 0, \tag{20}$$

$$> 0 \quad \text{if } Z_1 = \frac{x^e}{\bar{x}} R_0 + Z_0. \tag{21}$$

The interior case in (19) determines the regular textbook equation for the value of a forward contract, while (20) occurs in "stockouts", when all available resource is consumed, and hence no inventory is carried. Finally, (21) occurs in periods when nothing in consumed at date 0, and all produced resource is stored for future consumption. We will discuss explicitly below how Z_0 and x_0^e are determined. The investment policy is maximized numerically over a grid of values similar to the case without storage.

Specializing again to the linear demand case: $q_0 = a - b\,S_0$, and $q_1 = a \cdot e^{\mu+\sigma\,\epsilon} - b\,S_1$, enables us to solve for resource prices and extraction options in closed form. Equilibrium at date 1 now requires:

$$\frac{1}{\bar{x}}\left(S_1/g(K_1) - x_0^e\right)\, R_0 + Z_1\, e^{-u} = a\, e^{\mu+\sigma\,\epsilon} - b\, S_1.$$

Solving for prices, we now have

$$S_0 \;=\; 1/b\left(a + Z_1 - Z_0 - \frac{x_0^e}{\bar{x}} R_0\right), \tag{22}$$

$$S_1 \;=\; \frac{a\, e^{\mu+\sigma\,\epsilon} + \frac{x_0^e}{\bar{x}}\, R_0 - Z_1\, e^{-u}}{b + \frac{R_0}{\bar{x}\, g(K_1)}}. \tag{23}$$

Similar to Proposition 1 we solve for the extraction option value in closed form conditional on all firm level decisions.

Proposition 2 *The value of the extraction call option at date 0 in the presence of a storage technology with proportional storage costs of u, given installed capital K_1, cut-off resource quality $x_0^e \in [0, \bar{x}]$ and resource storage amount of Z_1 for a resource with current extraction cost of x is*

$$\begin{aligned}
C(x\, g(K_1)|Y_0) &= \frac{a\, e^{-r}}{D}\left[e^{(\mu-\sigma_M\,\sigma+0.5\,\sigma^2)} N(-d_1^s) - kN(-d_2^s)\right],\\
d_1^s &= \frac{\log(k^s) - m - \sigma_M\,\sigma - \sigma^2}{\sigma}; \qquad d_2^s = \frac{\log(k^s) - m - \sigma_M\,\sigma}{\sigma};\\
k^s &= \frac{1}{a}\left(D\, x\, g(K_1) - \frac{x_0^e}{\bar{x}}\, R_0 \;+\; Z_1\, e^{-u}\right);\\
D^s &= b + \frac{R_0}{\bar{x}\, g(K_1)}.
\end{aligned}$$

The value of a put option is

$$P(x|Y_0) \;\; = \;\; \frac{a\,e^{-r}}{D^s}\left[k^s\,N(d_2^s) - e^{(\mu-\sigma_M\,\sigma+0.5\,\sigma^2)}N(d_1^s)\right].$$

The proof is similar to that of Proposition 1.

Using the stock price in (23) implies that the forward price for the linear demand case satisfies:

$$F_0 = E^Q[s(Q_1;\epsilon)] = \frac{a\,e^{\mu-\sigma_M\,\sigma+0.5\sigma^2} + \frac{x_0^e}{\bar{x}}\,R_0 - Z_1\,e^{-u}}{b + \frac{R_0}{\bar{x}\,g(K_1)}}. \tag{24}$$

The partial relation between investment and the futures basis is essentially the same as for the case without storage. In addition, higher inventories, ceteris paribus, imply lower futures prices due to an in increase in supply in period 1.

Combing the futures and expected spot price we have:

$$\frac{E[S_T]-F_0}{F_0} = \frac{a\,e^{\mu+0.5\,\sigma^2} - e^{\mu-\sigma_m\,\sigma+0.5\,\sigma^2}}{a\,e^{\mu-\sigma_m\,\sigma+0.5\,\sigma^2} + \frac{x_0^e}{\bar{x}}\,R_0 - Z_1\,e^{-u}} \tag{25}$$

which is the expected return from the long futures position. This quantity is often called the “risk premium” (see e.g. Gorton and Rouwenhourst (2006)). Keynes observed that speculators mostly take the short side of futures contracts and therefore require a risk premium for holding the commodity risk. Therefore, the futures price that the producers (hedgers) would sell at should be lower than the expected spot price that they could obtain by holding the commodity and selling in the future. However, here we notice that the risk premium for the commodity is not only the standard $\sigma_M\,\sigma$, but also depends on the firm’s investment policy through its effect on the firm’s production and inventories, each of which is endogenous and related to the firm’s investment policy.

We now provide a description on how the extensive margin and inventories are jointly determined in the storage version of the model. For a given choice of the extensive margin x, we use (19) - (21) and (22) - (23) to determine inventory as

$$
\begin{aligned}
Z_1(x_0^e) &= \frac{e^{r+u}(-a + \frac{x^e}{\bar{x}} R_0 + Z_0)(b\, g(K_1)\bar{x} + R_0) + b\, g(K_1)\bar{x}(a\, e^{\mu - \sigma_M\, \sigma + 0.5\sigma^2} + \frac{x_0^e}{\bar{x}} R_0)}{e^{r+u}(b\, g(K_1)\bar{x} + R_0) \,+\, e^{-u}\, b\, g(K_1)\bar{x}} \\
&= 0 \;\text{ if }\; s(x_0^e | Z_1 = 0) > e^{-(r+u)} F(x_0^e | Z_1 = 0) \\
&= \frac{x_0^e}{\bar{x}} R_0 + Z_0 \;\text{ if }\; s(x_0^e | Z_1 = \frac{x_0^e}{\bar{x}} R_0 + Z_0) < e^{-(r+u)} F(x_0^e | Z_1 = \frac{x_0^e}{\bar{x}} R_0 + Z_0),
\end{aligned}
$$

for interior, and boundary choices, respectively. In particular, optimal inventory for the interior case is the solution to the equation $s(x_0^e) = e^{-(r+u)} F(x_0^e)$. We then use this optimal inventory function in the first order conditions for the extensive margin in (15) – (17).

One immediate implication of introducing storage possibilities into the model is that it makes the futures basis less variable. In particular, whenever an interior level of inventories is chosen, we have by construction that $S_0 = e^{-(r+u)} F$, so that the futures relative basis identically equals $(e^{-r}F - S_0)/S_0 = e^u - 1$. Departures from this constant slope therefore occur only when inventory is constrained, either from becoming negative or from it exceeding the sum of incoming inventory and current output. In the former case, $S_0 > e^{-(r+u)} F$ leading to a backwardated futures curve, while the latter case leads to a futures curve in contango.

1.3 Comparative Statics of Optimal Firm's Decisions, Futures Basis, and the Risk Premium with Respect to Mean Demand Shocks

As discussed above, it is hard to analytically determine the relation between the optimal firm's decisions and asset prices since the extensive margin is endogenous and affects prices. We consider the comparative statics for a specific numerical example in Figure 4 with respect to alternative levels of the capital stock. In particular, in this figure, we examine the extensive margin, investment, inventory, consumption at dates 0 and 1, the weak relative basis, the risk premium for alternative levels of the capital stock. All optimal decisions are calculated for the model with storage in Section 1.2.

As seen in the first row, the extensive margin is increasing in the level of the capital stock. With higher capital, extraction costs are lower and and the firm chooses a more aggressive extraction policy. The pattern for investment is quite interesting as it is V-shaped in the capital stock, with a minimum at around $K = 0.7$. We discuss this further below. In the second row, we see that inventory is zero, except when capital is $K = 1.3$, the highest level plotted. When capital is very high, firms extract the most and carry forward inventory for future consumption. The right panel shows consumption at dates 0 and 1. We see that until about $K = 0.7$, consumption at the two dates are negatively related, i.e. they are substitutes. In this region, with a higher capital stock, which will depreciate to some extent in the following period, consumption in date 0 is boosted at the expense of date 1 consumption. With lower planned consumption at date 1, investment declines in the capital stock. For capital stock higher than 0.7, consumption at dates 0 and 1 become complements, with both increasing in the level of the capital stock. However, to ensure greater production at date 1, investment must increase with the level of the capital stock at date 1.

The third row shows the weak relative basis and the risk premium. The weak basis increases in the level of the capital stock, specially at low levels of capital. In this region, with greater capital, extraction costs are low at date 0, leading to higher date 0 consumption and lower date 1 consumption (as discussed above). With the given inverse demand function, expected future prices are higher under the Q-measure, leading to a greater futures basis. For higher levels of the capital stock, the futures basis remains slightly positive even as consumption at both dates increase at about the same rate, as the extensive margin continues to increase and has a positive effect on the basis (see the discussion below Proposition 1). The risk premium (in the right panel) declines in the level of the capital stock again due to the impact of the increasing extensive margin as shown in equation (25).

Taken together, these comparative statics suggest a negative relation between the risk premium and the weak basis consistent with the findings of Fama and French (1987) and several other papers. They are also at least qualitatively consistent with the positive relation between the weak basis and the capital stock and the negative relation between the risk premium and the capital stock noted in the stylized facts in the introduction. Still, these are only comparative statics in one dimension, the capital stock, and even in the model, the variables are affected by the fluctuations in demand shocks as well. We discuss the dynamic relation between these variables in the context of the infinite horizon model below.

2 The Infinite Horizon Model with Production, Exploration, and Storage

We preserve much of the structure of the 2-period model. The one additional assumption that we make here is that there are adjustment costs to investment, an assumption that is standard in the investment literature to reduce the volatility of the investment process (see e.g. Kogan, Livdan, and Yaron (2009) in the context of a production model for commodities). This will help us provide a more empirically realistic model relationship between investment and the futures basis.

The demand function for the resource at time t is once again given by $q_t = f(S_t, \epsilon_t)$, where ϵ_t is a demand shock realization for the resource at date t. Conditional on a realization of ϵ_t, the inverse demand function is $s = f^{-1}(q_t; \epsilon_t)$. In addition, we assume the same form for the pricing kernel as for the 2-period model as specified in (1). We assume that the demand shock follows a mean-reverting Ornstein-Uhlenbeck (OU) process:

$$\epsilon_{t+1} - \epsilon_t = -k_\epsilon \, \epsilon_t + \sigma_\epsilon \, (1 + |\epsilon_t|) \, e_t, \tag{26}$$

where $e_t \sim N(0,1)$. The use of mean-reverting demand shocks is standard in the commodity pricing literature (e.g. Carlson, Khokher, and Titman (2007) and Pirrong (2012)). The process exhibits time varying volatility, which has a V-shaped relation with the demand shock. Therefore, volatility is high when demand is extremely low or extremely high. This feature captures the essence of time varying uncertainty of fundamentals that is now standard in macroeconomics and finance (see e.g. Bansal and Yaron (2004) and Bloom (2009)). We model this feature to potentially generate a time varying risk premium of the resource.

Let x_t^e be the extensive margin at the start of time t. The plant level decisions determine the increase in the extensive margin, i_t, so that $x_{t+1}^e = x_t^e + i_t$. Then at date t, the total production equals

$$Q_t = R_0 \cdot \int_{x_t^e}^{x_t^e + i_t} \frac{1}{\bar{x}} dx = R_0 \, \frac{i_t}{\bar{x}}. \tag{27}$$

The total extraction costs incurred by the firm at date t are

$$C_t = g(K_t) \cdot R_0 \cdot \int_{x_t^e}^{i_t + x_t^e} \frac{x}{\bar{x}} dx = \frac{1}{2}\, g(K_t)\, R_0\, \frac{(x_t^e + i_t)^2 - (x_t^e)^2}{\bar{x}}. \tag{28}$$

It is useful to note that extraction costs are not simply proportional to i_t^2, but instead are proportional to $i_t^2 + 2\, x_t\, i_t$. This is because an increase in the extensive margin leads to higher resource extraction costs as lower quality wells are accessed. An interesting implication is that the industry will have to maintain a higher level of capital stock over time to maintain a constant level of extraction costs.

We assume that the firm also has a costly storage technology. It is able to place a non-negative quantity Z_t in storage at time t. Storage costs are a proportion u of the quantity stored so an amount Z_t placed in storage at t, will make available an amount $Z_{t+1} = Z_t\,(1-u)$ at $t+1$. The firm behaves competitively in production markets, and we assume here that its storage decision has no price impact either. We will extend the analysis for the case of a non-negligible storage decision in future versions of the paper. For the competitive case, we alternatively could assume that inventory decisions are made by a risk neutral speculator. However, with complete markets, the equilibrium will be identical with storage by either the firm or speculators. Combining production as in (27) and inventory, the total amount available for consumption in period t is

$$q_t = Q_t + Z_t - Z_{t+1}. \tag{29}$$

If there is a stockout, then $Z_{t+1} = 0$, that is, all available resource is consumed in period t.

To solve for equilibrium prices and quantities, we solve the related problem of a social planner who maximizes the discounted expected consumer plus producer surplus (see e.g. Weinstein and Zeckhauser (1975) and Carlson, Khokher, and Titman (2007)). The social surplus at time t is therefore,

$$\begin{aligned} \mathrm{SS}_t &= \int_0^{q_t} s(q;\epsilon_t)dq - C_t - P_t\, I_t, && (30)\\ &= \int_0^{\frac{i_t}{\bar{x}} R_0 + Z_t - Z_{t+1}} s(q;\epsilon_t)dq - \frac{1}{2}\, g(K_t)\, R_0\, \frac{(x_t^2 + i_t)^2 - (x_t^e)^2}{\bar{x}} - P_t\, I_t, && (31)\end{aligned}$$

where total production, costs of production, and consumption, are given in (27), (28), and (29), respectively, and P_t is the price of capital goods in units of consumption goods at date t. We hold $P_t = 1$ for all t.

The social planning problem can be solved by standard dynamic programming methods. The Hamilton-Jacobi-Bellman equation is

$$J(x_t^e, Z_t, K_t, \epsilon_t) = \max_{i_t \in [0, \bar{x} - x_t^e],\ Z_{t+1} \in [0, \frac{i}{\bar{x}} R_0 + Z_t],\ 0 \le I_t \le \bar{I}(K_t)} SS_t + e^{-r} E^Q [J(x_t^e + i_t, e^{-u} Z_{t+1}, e^{-\delta} K_t + I_t, \epsilon_{t+1})]. \quad (32)$$

Note that we have placed an upper bound on investment, potentially as a function of the capital stock, to capture the essence of adjustment costs. To economize on notation below, we suppress the arguments of the J function and write $J_t = J(x_t^e, Z_t, K_t, \epsilon_t)$ and $J_{t+1} = J(x_t^e + i_t, e^{-u} Z_{t+1}, e^{-\delta} K_t + I_t, \epsilon_{t+1})$.

The first order conditions for this problem are:

$$\frac{R_0\ (s(q_t\ \epsilon_t) - (x_t^e + i_t)\, g(K_t))}{\bar{x}} + e^{-r} E^Q [J_{x,t+1}] \le 0; = 0 \quad \text{if} \quad i_t > 0 \quad (33)$$

$$\frac{R_0\ (s(q_t\ \epsilon_t) - (x_t^e + i_t)\, g(K_t))}{\bar{x}} + e^{-r} E^Q [J_{x,t+1}] \ge 0 \quad \text{if} \quad i_t = \bar{x} - x_t^e \quad (34)$$

$$-s(q_t; \epsilon_t) + e^{-(r+u)} E^Q [J_{Z,t+1}] \le 0; = 0 \quad \text{if} \quad Z_{t+1} > 0, \quad (35)$$

$$-s(q_t; \epsilon_t) + e^{-(r+u)} E^Q [J_{Z,t+1}] \ge 0 \quad \text{if} \quad Z_{t+1} < \frac{i_t}{\bar{x}} R_0 + Z_t, \quad (36)$$

$$-P_I + e^{-r} E^Q [J_{K,t+1}] \le 0; = 0 \quad \text{if} \quad 0 < I_t < \bar{I}(K_t), \quad (37)$$

$$-P_I + e^{-r} E^Q [J_{K,t+1}] \ge 0 \quad \text{if} \quad I_t = \bar{I}(K_t), \quad (38)$$

It is worth noting that the optimality of the extensive margin, inventory and investment must be checked at both lower and upper boundaries.

We solve the HJB equation using projection methods as described in Judd (1999). Using the policy functions written in polynomial form, we can calculate expected future production in each state, and hence using the inverse demand function and the Markovian shocks, we can compute the forward prices as the expected value of the future spot price under the risk-neutral measure. We note that all expecations are calculated using Gaussian Quadrature. Details of the approximation method are provided in Appendix 2.

3 Explaining the Stylized Facts

In this section we provide simulation results from the infinite horizon model in Section 2. Before doing so, we need to specify choices made on the demand function and the cost function in the model. We use an inverse demand function of the form: $s(q_t, \epsilon_t) = e^{b+\epsilon_t}/(EQ + q_t^{\alpha})$, where $\alpha > 1$. The term $EQ > 0$ is some exogenous consumption of a substitute to oil. We use $\alpha > 1$ since most empirical estimates of the elasticity of oil consumption are smaller than 1. We note that this inverse demand function does not satisfy the Inada condition at zero oil consumption. In unreported results, we also conduct simulations of the inverse demand $s(q_t, \epsilon_t) = e^{b+\epsilon_t}/q_t^{\alpha}$. The consumer surplus for this latter inverse demand is finite when $0 < \alpha < 1$. The equilibrium responses of firms for this latter inverse demand function provide results that are similar in many dimensions to that for the former inverse demand, once the technology parameters are chosen in conjunction with this specification (see below). The demand shock process $\{\epsilon_t\}$ follows the mean reverting process in (26). The former inverse demand function is similar to that in Carlson, Khokher, and Titman (2007), who in addition, model a time varying price of the substitute. The extraction cost function that we use is of the form $g(K) = \frac{\gamma}{K^p}$, where $p > 0$. This implies that extraction costs decline as capital accumulates, but explode as capital tends to zero, so that positive capital is required to ensure the supply of the resource.

We next discuss a set or results for some parameter values assumed for the model.

3.1 Parameter Values for Model

For the former inverse demand function we choose $\alpha = 2$, $EQ = .05$, and $b = 0$. The technology parameters we choose are $p = 2$, and $\gamma = 200$. The parameters for the inverse demand function determine the level of prices, while the technology parameters determine extraction costs. Together, the parameters chosen determine the gross margin of resource production. The parameters are chosen jointly to match the historical average gross margin of oil producing firms. Using data from Compustat, we find that the historical average of the gross margin of oil firms is about 30%. We set the rate of capital depreciation at 10 percent a year, a standard rate assumed in the real business cycle literature. We set proportional storage costs of 5 percent a year, similar to that in Routledge, Seppi, and Spatt (2000) and Pirrong (2012).

The parameters for the demand shock process in equation (26) that we use are $k_\epsilon = -0.3$, and $\sigma_\epsilon = 0.2$. The drift parameter governing the speed of mean reversion is the same as that in Carlson, Khokher, and Titman (2007), while we choose a lower volatility, since we scale up the volatility by the amount $(1 + |\epsilon_t|)$.

We assume that the price of risk is $\sigma_m = 0.3$. This is around the standard level used in asset pricing models to justify an aggregate Sharpe ratio of 30 percent on stocks, close to its historical average. Finally, we assume that the price of capital is constant and set equal to one. Essentially this means that the numeraire good can either be consumed or converted for investment one-for-one. In addition, assuming a constant price of capital implies that none of the investment dynamics in the model arise from it.

We also make some choices on the scale of the problem. We assume that $\bar{x} = 50$, and the total reserves of the resource, $R = 10$. These we believe do not affect the results of the paper.

3.2 Results from a Single Simulation

We highlight in this section that this model displays inventory and capital cycles that help explain the variation in the relative basis and the risk premium. We start by showing a typical sample path of 200 years of the model's real and financial variables in Figures 5 and 6 for the first specification of the inverse demand function.

The top left panel of Figure 5 shows the simulated demand shock process, which fluctuates around 0. The demand process exhibits some persistent booms and busts, some of which can last up to 10 years, but there are shorter episodes as well. The top right panel shows how the extensive margin expands over time. In this model, production can only occur by expanding at the margin, so in the remaining periods, positive consumption can only be attained by storing the resource. As seen, after 60 years resources with an extraction cost of 14 are extracted, while the maximum is 50, i.e., 28 percent of the resource is extracted. The bottom left panel shows the optimal consumption of the resource. In the simulation shown, consumption peaks at about 13 years, and continues to fluctuate until year 40. It then declines to a trickle, and after year 60 is always zero. In the next subsection, we will see that the point at which consumption declines to zero is highly zero, and in some simulations, the year of stopping consumption is as high as 250-300 years. The bottom right panel shows a positive trend in the price of oil as the resource gets increasingly exhausted.

Figure 6 shows the main variables of interest of the paper. The top left panel shows the path of optimal inventory and the weak relative basis. The panel conveys much of the intuition behind the relation between inventory and the basis from the storage literature. The two series are positively related, specially when the basis is positive. In such periods, the positive basis provides incentive to carry the resource forward into time. The inventory has limited explanatory power when the basis is negative, since it is constrained to be nonnegative. We see some large downward spikes (backwardation) in the panel when inventory becomes zero. In such periods, the agents would like to transfer resources from the future to the current, but since they are unable to, spot prices spike, and the futures become backwardated. The right panel shows the basis and a scaled series of optimally accumulated capital. Again the series are positively related, and the capital process cycles look fairly stable even as the resource gets depleted. Inventory and capital have a correlation of about 60 percent in the shown sample path, while it is about 38 percent in the data. The positive correlation shows that in the model during periods of a positive basis, firms also have the incentive to accumulate capital to lower their future extraction costs. Putting both inventory and capital as explanatory variables for the basis we get (t-statistics adjusted for heteroskedasticity and autocorrelation are shown in parenthesis):

$$\begin{aligned} \text{Weak Relative Basis} &= -0.084 + 0.466\,\text{Inventory} + 0.017\,\text{Capital}; \qquad R^2 = 0.176 \\ &= [-6.779] \quad [2.044] + \quad [4.391] \end{aligned} \tag{39}$$

The plots also suggests that the basis declines, going into backwardation as consumption drops off after year 40 of the simulation. In this subsample, both inventory and the capital stock decline as well.

The bottom panels show the risk premium plotted with inventory and the capital stock, respectively. The risk premium is noisier than the basis. Still, the right panel shows a positive risk premium and its fitted value from the linear regression only on capital. In the later part of the shown time series, as the resource becomes increasingly costly to extract, the capital stock declines smoothly so that the risk premium trends upwards.

Regressing the risk premium on the two variables we get:

$$\begin{aligned}\text{Risk Premium} &= 0.053 + 0.135\,\text{Inventory} - 0.011\,\text{Capital}; \qquad R^2 = 0.124 \\ &= [8.022] \quad [1.232] \quad [-7.133]\end{aligned}$$

Therefore, the risk premium is lower when capital is higher, while the relationship with the inventory is not significant. These are the same relationships we found in the data.

The plots also suggests that the risk premium increases, as consumption drops off after year 40 of the simulation, and as noted above, both inventory and the capital stock decline as well in this subsample.

How does the variance of the simulated spot prices at alternative frequencies compare with that in the data? We look again at Figure 3, where the right panels show the decomposition of model real oil prices in trend and cycle (top panel) and the frequency decomposition (bottom panel) from the simulated model path. As seen, the model oil price has a positive trend in oil prices that lasted nearly 50 years, and displayed fluctuations around this trend as well. Similar to the data, the bottom panel shows that the variance frequency decomposition has large mass both at low frequencies (occur once in 50-60 years) as well as at the business cycle frequency (around 3 years). The positive trend in the model arises from our assumption of exhaustibility of the resource and the exploitation of resources that increasingly costly to extract, while the business cycle component arises from the demand shocks, which follow a mean reverting process.

3.3 Results from Multiple Simulations

While the results from the single simulation shed light on the observed relations between real and financial variables, due to non-renewability, we are limited in the length of the simulation. In addition, the exact sequence of demand shocks will determine the time of stopping consumption and potentially the statistical significance of our results. In this section, we simulation multiple times and examine the long run distribution of several statistics.

The results are shown in Figure 7. The top left panel shows the distribution of the time at which consumption stops. As can be seen the mean time to non-renewability is most frequently between 50 and 100 years although there is considerable variation, and there is significant probability that consumption continues to year 250-300. The top right panel shows the distribution of the year to maximum consumption. The histogram shows that the time of peak consumption has the largest mass in the first ten, but also a high likelihood of a maximum up to about 30 years.

The middle panels shows the distribution of the R^2 of the regression of the basis on inventory and the capital stock, respectively, in each sample path. For each regression, the R^2 varies between 0 and 30 percent, with a mean of 15 percent. This is lower than the data R^2 of about 24 percent (see Table 1), but we must recall that in our data sample, there has been a sustained period of rapidly accumulating capital in the 2000s, which the model predicts will cool off when demand weakens. The bottom panels show the R^2 of the risk premium on inventory and the capital stock, respectively. The R^2 of inventory is mostly less than 8 percent, while the R^2 on capital stock is most frequently in the 10-20 percent range.

It is finally noteworthy, that the regression results in the model are similar to that in the data despite our explicit modeling of exhaustibility/non-renewability. While peak consumption is a debatable hypothesis, it is useful to note that its implications are not at odds with the data. Still, as noted in the previous subsection, the basis declines and the risk premium increases once consumption declines.

4 Conclusion

In this paper, we provide a new model of exhaustible resource extraction, inventory accumulation, and accumulation of E&D capital, to address stylized facts on real and financial variables of the oil industry. The model shows that such evidence is consistent with the peak consumption view and an increasing trend of real oil prices.

The model shows that even though consumption peaks, and real prices of the resource trend upwards, the cycles of inventory and capital accumulation are quite stable, as increasingly costlier resources are extracted. While the resource is not completely exhausted even after more that a hundred years, consumption drops significantly after about half the resource base is exhausted, while capital fluctuates with the demand cycle. Therefore, the positive relationship between E&D capital and the futures basis, and the negative relationship between capital and the risk premium, are consistent with a model of near exhaustion of the resource, or the peak consumption view. The model generates a frequency decomposition of the futures basis with significant non business cycle components and relatedly, an increasing trend in spot prices similar to that in the data. Overall, our model shows that simulatenously modeling the long run (E&D) and short run (inventory) decisions of firms, is important in understanding the relationship between real and financial variables of the oil industry.

Data Appendix

We obtain historical crude oil futures contracts prices from July 1986 to November 2014 from the *Chicago Mercantile Exchange* (CME). The data series provided summarize the prices from all public traded exchanges. We obtain the series of constant maturity Treasury yields from the *Federal Reserve Board*, which are required for calculating the weak relative basis. We filter the series and use only prices for contracts with positive volume. We obtain the core CPI (to deflate spot oil prices) from the St. Louis Fed.

"Inventory" is denoted as the total US stock of crude oil and petroleum products ex-strategic oil reserves. We obtain these data from the US Energy Information Administration (EIA). "Capital Stock' is the sum of the "Property Plant and Equipment" variable in Compustat of firms in oil and gas field exploration services (SIC code 1382). Consumption of petroleum producs is provided by the EIA.

Appendix 1

Proof of Proposition 1.

Using the equilibrium stock price at date 1 in (11), we have that the call option value is simply

$$\begin{aligned}
C(x|x_0^e, K_1) &= e^{-r}\, E^Q[\left(\frac{a\, e^{\mu+\sigma\,\epsilon} + \frac{x_0^e}{\bar{x}}\, R_0}{b + \frac{R_0}{\bar{x}\, g(K_1)}} - x\, g(K_1) \right)^+] \\
&= \frac{e^{-r}}{D} E^Q[\left(a\, e^{\mu+\sigma\epsilon} - (D\, x\, g(K_1) - \frac{x_0^e}{\bar{x}}\, R_0) \right)^+] \\
&= \frac{a\, e^{-r}}{D} \left[E[e^{\mu-\sigma_M\,\sigma+\sigma\epsilon^*} | \mu - \sigma_M\,\sigma + \sigma\epsilon^* > \log(k)] - k\,\text{Prob}[\mu - \sigma_M\,\sigma + \sigma\,\epsilon^* > \log(k)] \right] \\
&= \frac{a\, e^{-r}}{D} \left[e^{(\mu-\sigma_M\sigma+0.5\,\sigma^2)} N(-d_1) - k N(-d_2) \right],
\end{aligned}$$

as stated. We note that in the third line we use the definition of the 'risk-neutral shock' $\epsilon^* = \epsilon + \sigma_M$, while in the fourth line we use the conditional expectation for log normal variables (see e.g. Proposition 2.29 in Nielsen (1999)). The proof for the put is similar. ■

Appendix 2

We proceed by formulating an 'approximate' solution to the Hamilton-Jacobi-Bellman equation in 32 using projection methods (Judd 1999, Chapter 11). The value function is denoted as $J(x_t^e, Z_t, K_t, \epsilon_t)$.

STEP 1. Choice of individual basis functions. I choose the Chebyshev polynomials in each of the 4 dimensions: The Chebysev polynomials on $[-1, 1]$ for the basis for each dimension are given by

$$q_m(x) \;=\; \cos(m \cos^{-1} x),$$

for $m = 1, 2, \cdots$, which satisfy the recursive scheme

$$q_{m+1}(x) = 2\, x q_m(x) - q_{m-1}(x). \tag{40}$$

These polynomials are restricted for the interval $[a, b]$ using the transformation

$$p_m, (x) = \frac{q_m(\frac{2x-a-b}{b-a})}{||q_m(\frac{2x-a-b}{b-a})||}.$$

We solve the value function on bounded spaces in each dimension: $[0, \bar{x}] \times [0, \bar{Z}] \times [0, \bar{K}] \times [-\bar{\epsilon}, \bar{\epsilon}]$. The family $\{p_m(x)\}_{m=1,2,\cdots}\}$ are orthonormal polynomials over the chosen intervals.

STEP 2. Choose a basis of 'complete' polynomials over the space

The basis of degree M over the 4 dimensions is given by

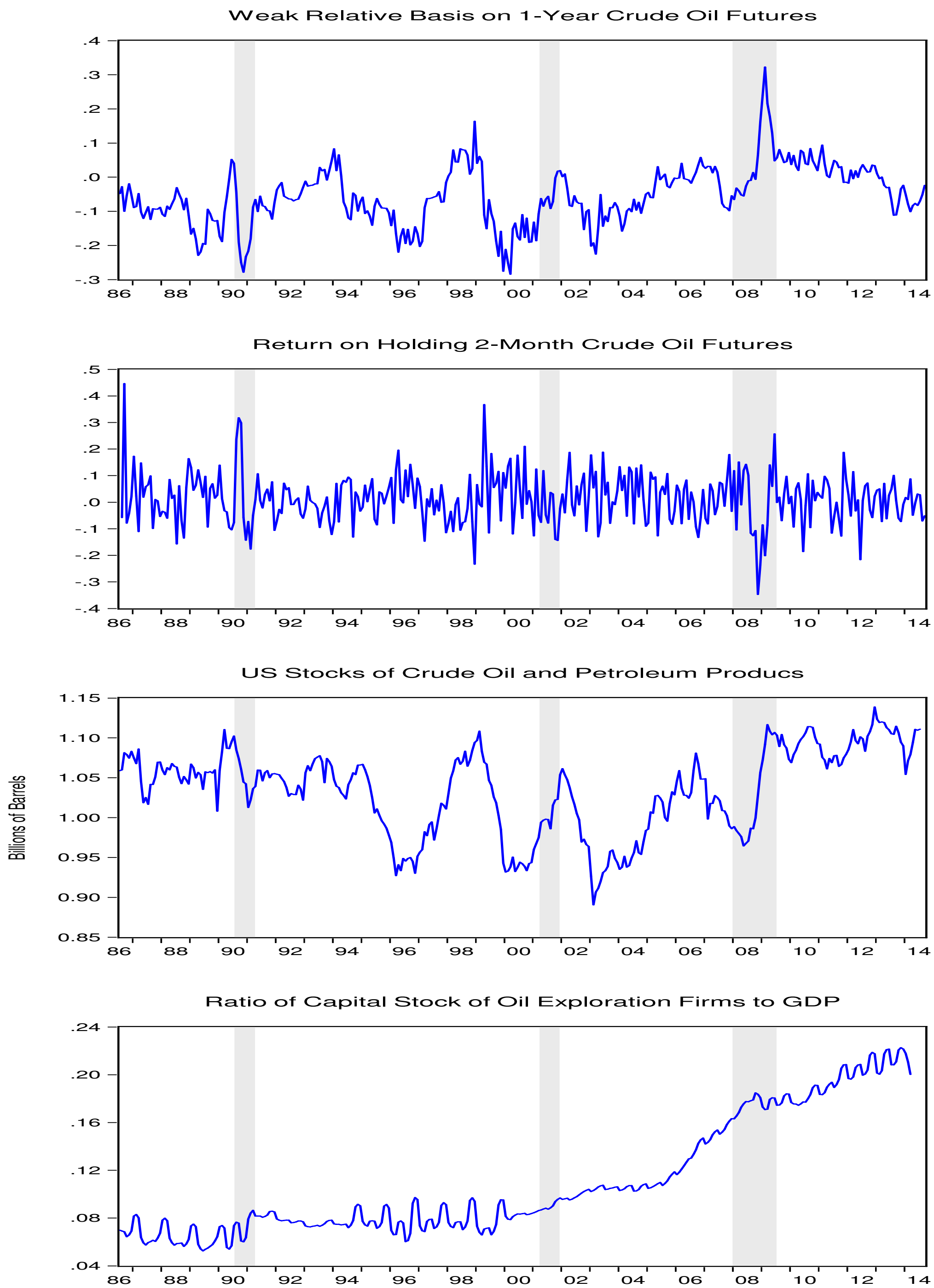
$$\mathcal{P}_M = \{p_{1,i_1}(X) \cdot p_{2,i_2}(Z) \cdot p_{3,i_3}(K) \cdot p_{3,i_3}(\epsilon) | \sum_{n=1}^{4} i_n \leq M, 0 \leq i_1, \cdots, i_3\}$$

We write the generic element of $\mathcal{P}_M$ as $\phi_m(X, Z, K, \epsilon)$, $m = 1, 2, \cdots M^c$, where M^c is the length of the complete polynomial basis. The set of complete polynomials for a 4 dimensional problem grows polynomially in 4, as opposed to the tensor product basis which would use every possible product of the degree-M individual basis functions, and hence would grow at the rate of M^4 (see, e.g., pp. 239 in Judd 1999). The complete polynomials asymptotically, as M becomes large, provide as good an approximation as the tensor product, but with far fewer elements. . Extending the L^2 norm over the 4-dimensional space as the 4-fold integral, it can be verified that the basis of complete polynomials is orthonormal on the bounded Cartesian product space.

STEP 3 Let $b^{(n)}$ be the nth guess on the coefficients of the polynomial, i.e $J^{(n)}(x_t^e, Z_t, K_t, \epsilon_t) = \sum_{m=1}^{M^c} b^{(n)} \cdot \phi_m(X, Z, K, \epsilon)$. Then we solve 32 for the n+1th guess as $J^{(n+1)}(x_t^e, Z_t, K_t, \epsilon_t)$, using the first order conditions (33) – (38). Note that we are able to take partial derivatives of the nth guess value function, which is just a polynomial sum. The first order conditions are solved on a discrete grid of values for inventory, investment, and the extensive margin.

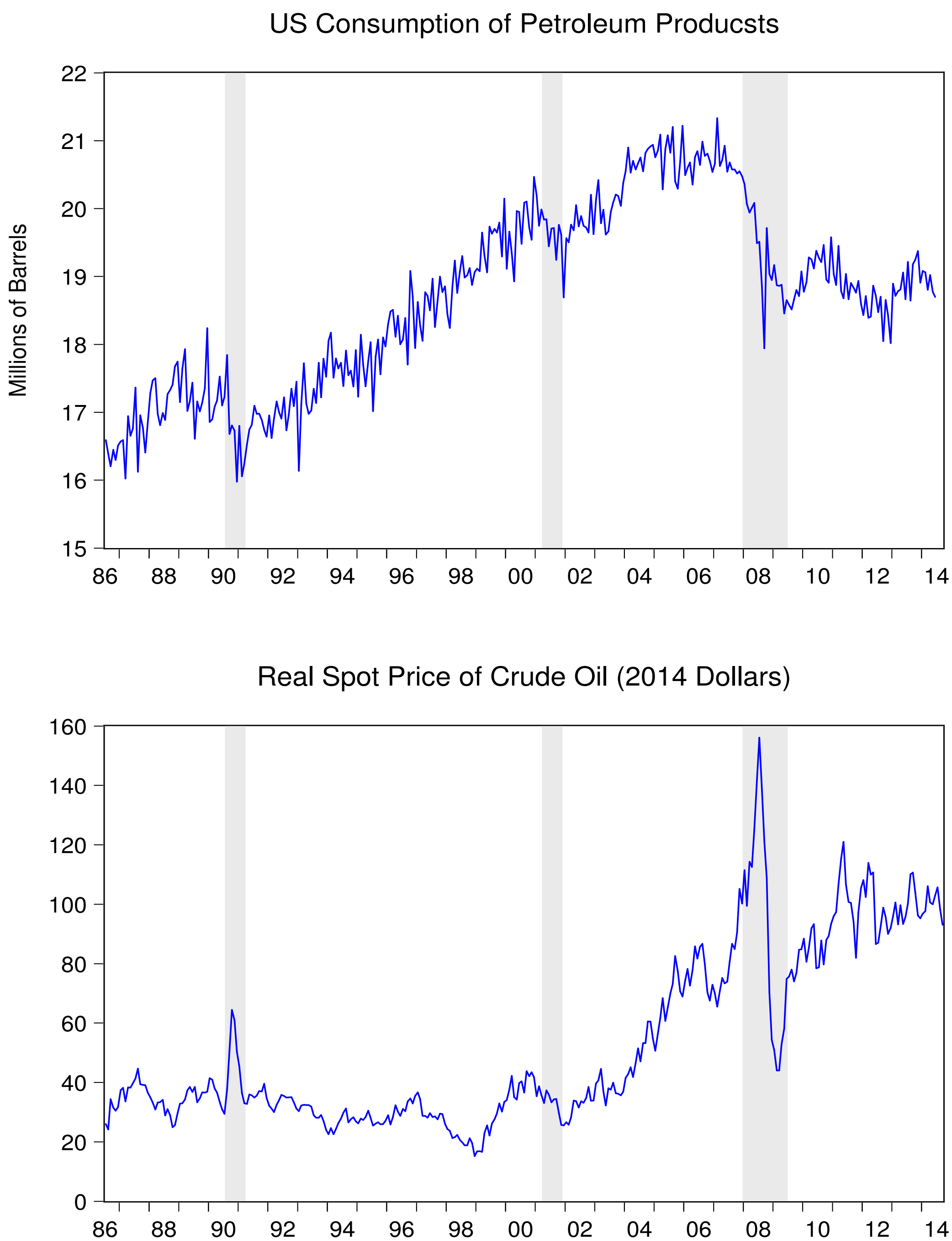
STEP 4 We now appeal to the Chebyshev Interpolation Theorem (see Judd (1999)) to find an approximate solution to the Bellman equation. Denote $Y = (x^e, Z, K, \epsilon)$. The approximation is made by evaluating the $J^{(n+1)}(Y)$ at the Chebyshev zeros in the Cartesian space, given the coefficients $b^{(n)}$. Each interpolation point therefore provides us a linear equation in the coefficients $(b_m)^{(n+1)}$. With M^I interpolation points, we have an overidentified system of equations in M^c unknown coefficients, and we solve for $(b_m)^{(n+1)}$ using linear regression. We then repeat until convergence.

Figure 1: Crude Oil Futures Basis and Returns, Oil Inventory and Capital Stocks of Oil Exploration Firms (1986:7-2014:9)



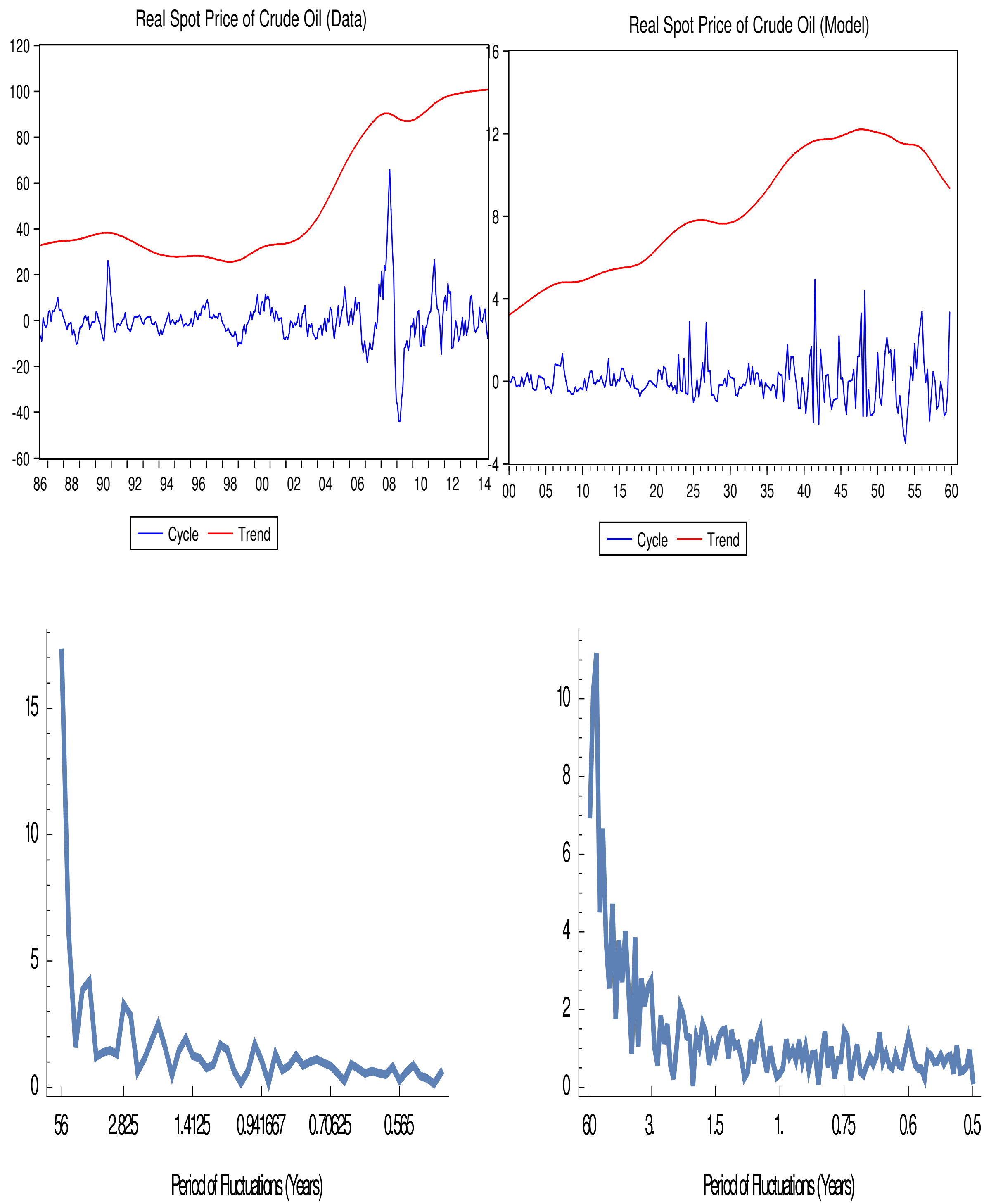
The **top panel** shows the seasonally adjusted weak relative basis on 1-year crude oil futures contracts, which in month t is $[e^{-r(t)}F(t) - S(t)]/S(t)$, where $F(t)$ is the 1-year futures prices at the beginning of each quarter and $S(t)$ is the spot price of WTI oil in Cushing, Oklahoma. Seasonal adjustment is done using the X-12 procedure (used by the US Department of Commerce). The **second panel** reports the return from month $t-1$ to month t of a 2-months WTI futures contract. The **third panel** shows the seasonally adjusted "Inventory" of total US stock of crude oil and petroleum products (in billions of barrels) excluding special purpose reserves at the end of each month. The **bottom panel** shows the "Capital Stock", which is the sum of the "Property Plant and Equipment"

Figure 2: Consumption of Petroleum Products and Real Spot Price of Crude Oil (1986:7-2014:6)



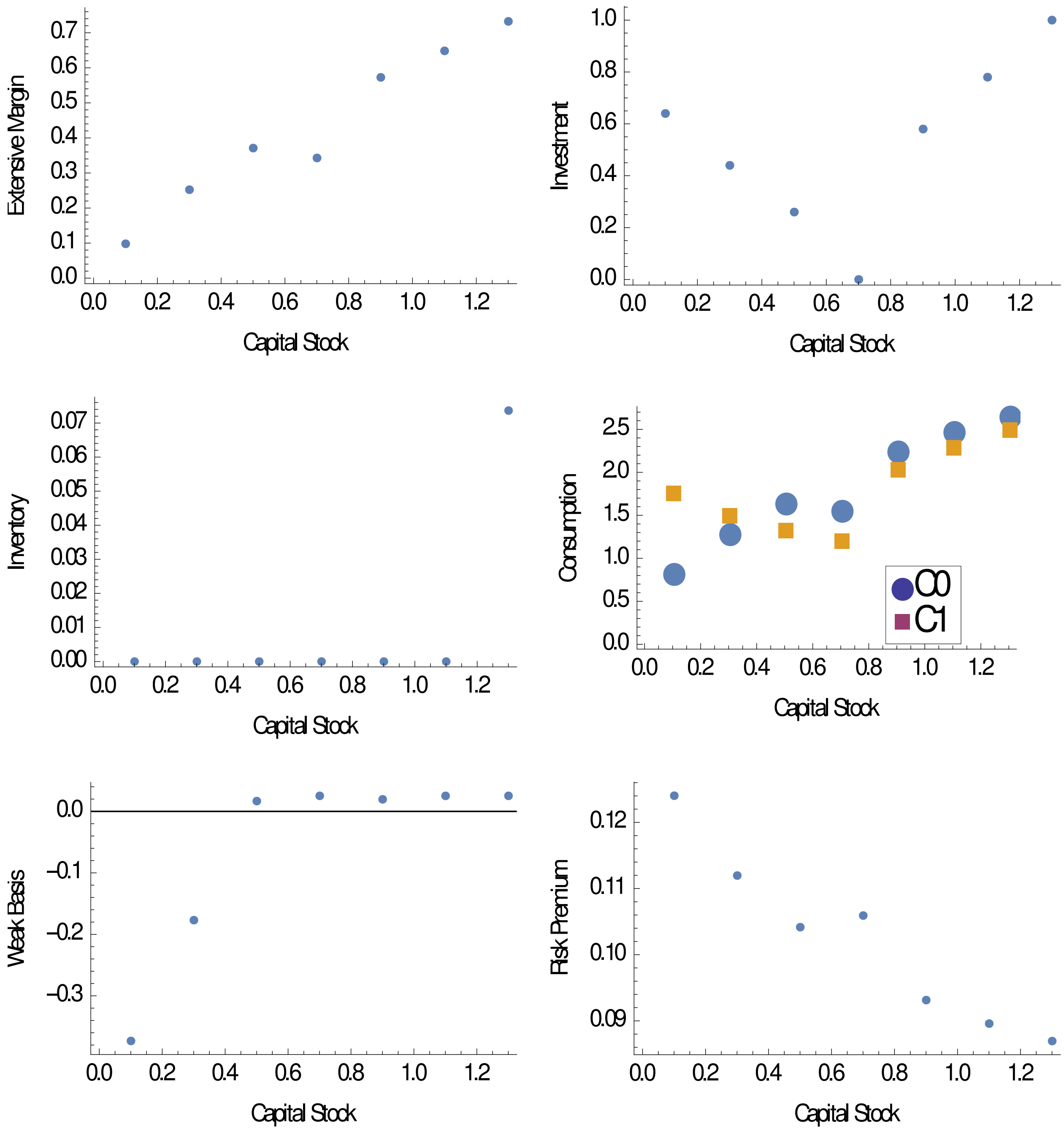
US consumption of petroleum products is reported by the EIA. The spot price is the price of a futures contract with less than one month to maturity reported at the beginning of the month. The real spot price is the spot price divided by core CPI (excluding food and energy).

Figure 3: Trend, Cycles, and Variance Frequency Decomposition of Real Oil Price



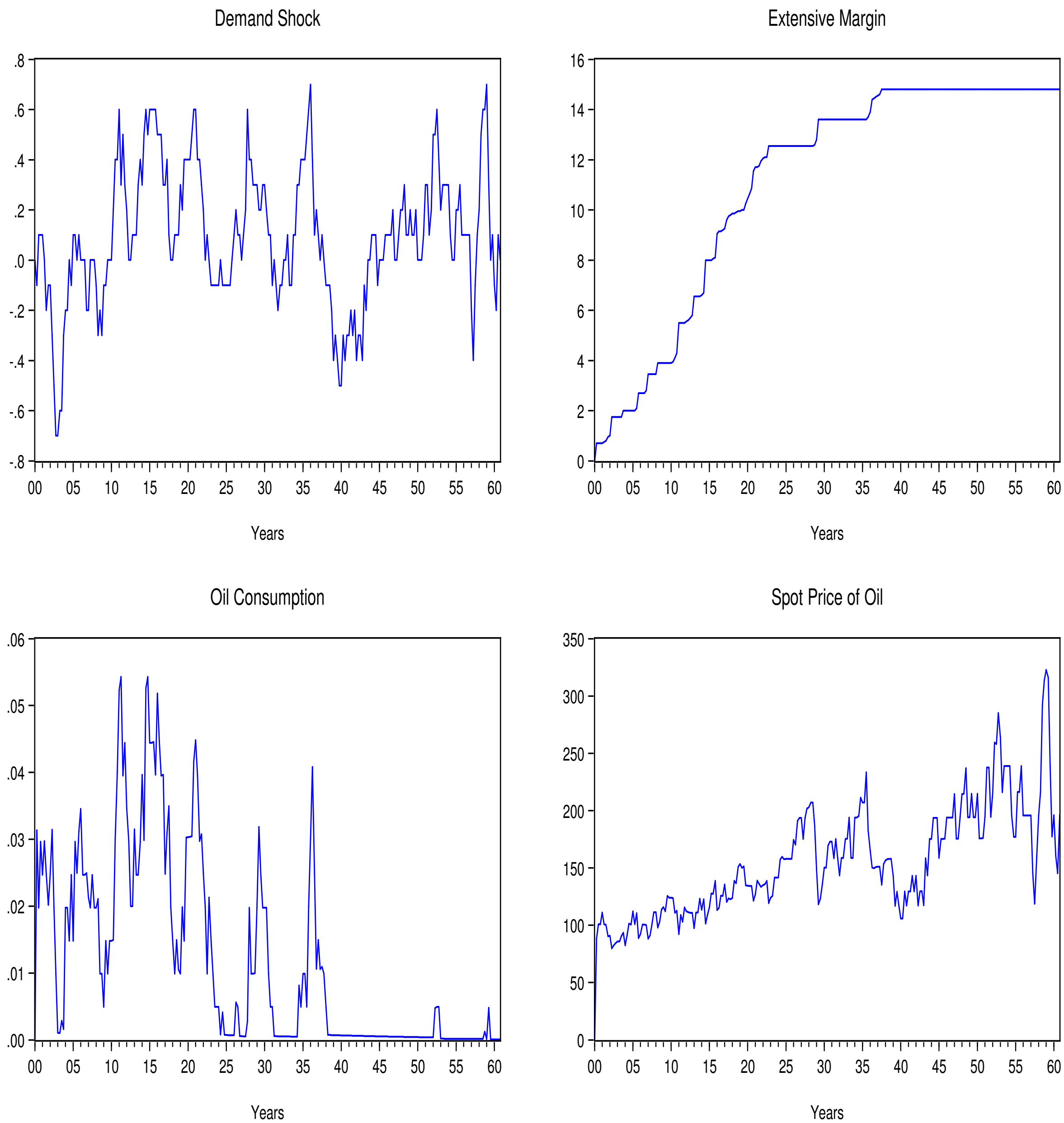
The top panel shows the trend and cycle components of the real spot price of oil obtained using the Hodrick-Prescott procedure on the historical data (left panel) and model simulation (right panel). The bottom panels show the variance frequency decomposition (Fourier Transform or spectrum) for the historical data (left panel) and model simulation (right panel). The horizontal axis on the spectrum plot is the period of the fluctuation in years rather than the frequency, which is often shown.

Figure 4: Comparative Statics of Optimal Firm's Decisions, Futures Basis, and the Risk Premium with Respect to the Capital Stock in the 2-Period Model with a Linear Demand Function



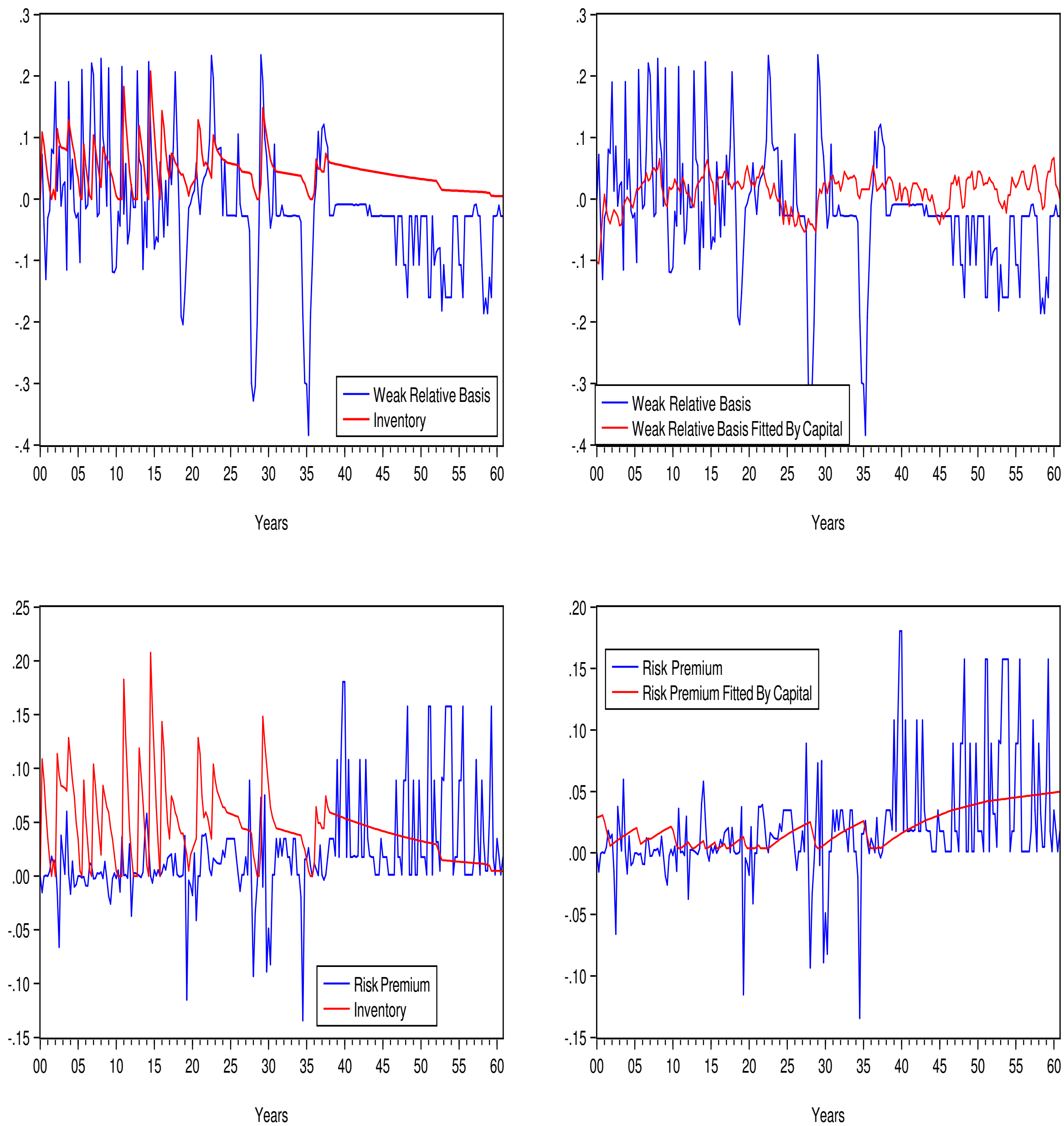
We report the comparative statics of inventory, investment, the weak relative basis, and the risk premium in the 2-period model with respect to alternative levels of the capital stock. The parameters of the model are $r = 0.02$, $a = 4$, $b = 1$, $R = 3$, $Z_0 = 0.5$, $\bar{x} = 1$, $\mu = 0$, $\sigma = 0.5$, $\gamma_0 = 0.2$, $\gamma_1 = 0.1$, $p_I = 0.1$,$\delta = 0.1$, $\sigma_m = 0.25$, $u = 0.025$

Figure 5: Optimal Firm's Decisions from a Single Simulation of Infinite Horizon Model



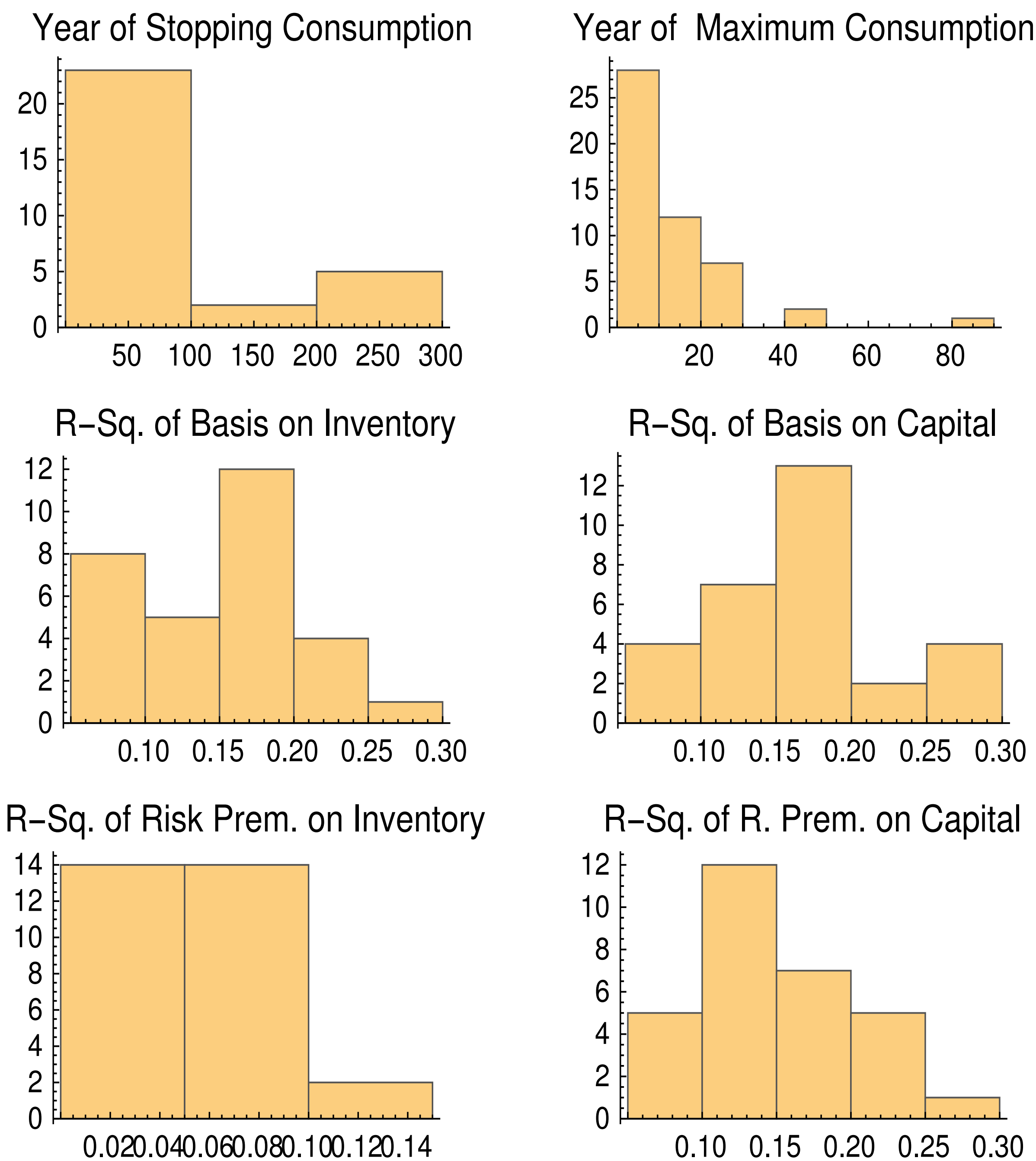
We report the simulated time series of the model in Section 2. The parameter choices are in Section 3.1.

Figure 6: Weak Relative Basis and Risk Premium From a Single Simulation of Infinite Horizon Model



We report the simulated time series of the model in Section 2. The parameter choices are in Section 3.1.

Figure 7: Distribution of Consumption Properties and Financial Variable Regressions from Multiple Simulations.



We report the simulated time series of the model in Section 2. The parameter choices are in Section 3.1.

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