Liquidity, Resiliency and Market Quality Around Predictable Trades:

Theory and Evidence*

Hendrik Bessembinder University of Utah

Allen Carrion Lehigh University

Laura Tuttle
Securities and Exchange Commission

Kumar Venkataraman Southern Methodist University

This Draft: June 2015

Keywords: Predatory trading; Sunshine Trading; Resiliency; ETFs; Trading Costs, Commodity trading.

*Earlier versions of this paper were titled "Predatory or Sunshine Trading: Evidence From Crude Oil ETF Rolls." For their comments, we thank Amber Anand, Tarun Chordia, Thierry Foucault, Ruslan Goyenko (Central Bank Conference Discussant), Jeff Harris (WFA Discussant), John Hyland, Steve Mann, Lubos Pastor, Ravi Shukla, Juan Ignacio Pena (CNMV discussant), Nela Richardson, Myron Slovin, Elvira Solji, Wing Wah Tham, Selim Topaloglu and seminar participants at CUNY-Baruch College, Erasmus University, HEC-Paris, University of Notre Dame, University of Washington, Syracuse University, Tinbergen Institute, Texas Christian University, Queens University, Second CNMV International Conference on Securities Markets, the 8th Annual Central Bank Workshop on Market Structure, the US Securities and Exchange Commission, the 2012 Commodity Futures Trading Commission Research Conference, and the 2013 Western Finance Association (WFA) Annual Meeting. We also thank Jeff Harris, Andrei Kirilenko, Scott Mixon and Jim Moser for their help in accessing and interpreting data, and Stephen Meek for research assistance.

Disclaimer: The research presented in this paper was co-authored by Dr. Hendrik Bessembinder, a former CFTC contractor who performed work under contract CFCE 10-CO-0200, Dr. Allen Carrion, a former CFTC contractor who performed work under contract CFCE-10-CO-0161, Dr. Kumar Venkataraman, a former CFTC contractor who performed work under contract CFCE 10-CO-2001. Dr. Laura Tuttle, another co-author, performed work pursuant to an Intergovernmental Personnel Act Agreement and wrote this paper in her official capacity with the CFTC. The Office of the Chief Economist and CFTC economists produce original research on a broad range of topics relevant to the CFTC's mandate to regulate commodity future markets, commodity options markets, and the expanded mandate to regulate the swaps markets pursuant to the Dodd-Frank Wall Street Reform and Consumer Protection Act. These papers are often presented at conferences and many of these papers are later published by peer-review and other scholarly outlets. The analyses and conclusions expressed in this paper are those of the authors and do not reflect the views of other members of the Office of Chief Economist, other Commission staff, or the Commission itself. The Securities and Exchange Commission, as a matter of policy, disclaims responsibility for any private publication or statement by any of its employees. The views expressed herein are those of the author and do not necessarily reflect the views of the Commission or of the author's colleagues on the staff of the Commission.

Liquidity, Resiliency and Market Quality Around Predictable Trades:

Theory and Evidence*

Abstract

We extend the theory of strategic trading around a predictable liquidation by considering the role of market resiliency. Our model predicts that even a monopolist strategic trader improves market quality and increases liquidator proceeds if trades' temporary price impacts are quickly reversed. We provide related empirical evidence by studying prices, liquidity, and individual account trading activity around the large and predictable "roll" trades undertaken by the largest ETF tracking crude oil futures prices. The evidence indicates narrower bid-ask spreads, greater order book depth and improved resiliency on roll dates. We find that a larger number of individual trading accounts provide liquidity on roll dates, and do not find evidence of the systematic use of predatory strategies. On balance, the theory and evidence supports that strategic traders choose to provide liquidity to predictable trades in resilient markets. However, the large volume of trading associated with the roll transactions leads to substantial trade execution costs that average three percent per year.

I. Introduction

A trader who learns that another investor will transact a substantial quantity of a security can potentially profit by trading in the same direction prior to or simultaneous with the investor, before subsequently reversing the trade. Such a practice has been dubbed "predatory trading" by Brunnermeier and Pedersen (2005). Their model implies that the practice degrades market quality, in that it causes prices to temporarily overshoot the longer-term equilibrium, and is harmful in that it causes the investor to realize a less advantageous price.

Admati and Pfleiderer (1991) present an alternative theory of trading around a predictable order. In their "sunshine trading" theory, investors who intend to transact a substantial quantity publicly announce their intention to trade, thereby attracting additional liquidity suppliers as well as natural counterparties to the market. Their model implies that the public announcement of the upcoming trade results in smaller market movement and a more advantageous price to the liquidator.

In this paper, we provide a simple extension of the theory of trading around a predictable order, and present relevant empirical evidence by studying individual account trading, liquidity, and market resiliency around the time of large and predictable monthly trades undertaken by the United States Oil Fund (USO), the largest of the ETFs that are designed to provide returns that track crude oil prices.\(^1\) Rather than holding crude oil inventories, which would entail substantial storage costs, USO gains exposure to crude oil prices by holding positions in New York Mercantile Exchange (NYMEX) crude oil futures contracts. Since individual NYMEX contracts periodically expire, the strategy involves regularly "rolling" positions by selling the expiring contract and purchasing contracts with more distant expiration dates. Data on crude oil ETFs' Assets-Under-Management are publicly available, and USO announces on its web site the dates on which it will roll its positions.\(^2\) The magnitude, direction, and timing of USO roll trades are therefore highly predictable.

¹ During our sample period, USO accounted for 95% of the assets-under-management in crude oil ETFs. Data on ETF's assets-under-management are obtained from ALPS Fund Services.

² USO's investment objective, as well as a calendar schedule of recent and future roll dates, is disseminated on the web site http://www.unitedstatesoilfund.com/.

USO was launched in April 2006, and by early 2009 had more than \$4.2 billion under management, equating at prevailing prices to over 90 million barrels of crude oil. The price of USO shares has lagged the level of crude oil prices, as displayed on Figure 1. Some observers have suggested that predatory trading explains the USO share price record. For example, the Wall Street Journal reported that "Since the fund (USO) is so big, it is unable to switch in and out of contracts....without moving markets and giving speculators an opportunity to make bets on those moves." The article quotes a trader as stating that "It's like taking candy from a baby" and asserts that the "... candy comes out of returns of the investors in the fund."

USO's stated investment objective involves tracking futures settlement prices, which are established daily during a short two-minute interval at the end of the normal trading day.⁴ The magnitude of USO's roll often exceeded NYMEX market volume during the settlement interval, and at times was over fifteen percent of volume on the roll day. Since USO predictably demands a very large quantity of liquidity during a short trading interval, its trades provide an ideal experiment to study the economics of liquidity provision around the execution of large predictable trades.

We employ data on individual orders and trades in crude oil futures made available to us by the Chicago Mercantile Exchange, which owns and operates the NYMEX market. In addition, we use Commodity Futures Trading Commission (CFTC) data that identifies the individual trading accounts associated with each crude oil futures transaction. The former dataset allows us to study posted liquidity in the form of bid and ask quotes, as well as unexecuted displayed depth in the limit order book. The latter dataset allows us to evaluate the strategies used by owners of specific trading accounts around the time of USO's rolls. Our study of individual orders and trades spans the period March 1, 2008 to

³ "U.S. Oil Fund Finds Itself at the Mercy of Traders", by Gregory Meyer and Carolyn Cui, The Wall Street Journal, March 6, 2009, page C1.

⁴ USO's investment objective is stated on the company website http://www.unitedstatesoilfund.com/ as follows: "The investment objective of USO is for the daily changes in percentage terms of its units' net asset value ("NAV") to reflect the daily changes in percentage terms of the spot price of light, sweet crude oil delivered to Cushing, Oklahoma, as measured by the changes in the price of the futures contract for light, sweet crude oil traded on the New York Mercantile Exchange (the "NYMEX") that is the near month contract to expire, except when the near month contract is within two weeks of expiration, in which case it will be measured by the futures contract that is the next month contract to expire (the "Benchmark Oil Futures Contract"), less USO's expenses."

February 28, 2009, and therefore includes twelve monthly rolls.⁵ USO's assets under management reached a peak during this period, implying heightened statistical power to detect the effects of USO's large and predictable trades. We also study daily crude oil settlement price data for the longer time interval January 1990 through December 2013.

In addition to providing empirical evidence, we provide some new analysis of the economics of strategic trading around a known liquidation. Brunnermeier and Pedersen (2005) rely on the assumption that trades have permanent but not transitory effects on prices, and show that the effects of predatory trading are worst when there is a monopolist predator. We analyze the effects of strategic trading when markets are resilient, in the sense that some or all of the immediate price impact of trades is subsequently reversed. Our analysis reveals that the profit maximizing strategy for a monopolist trader who is aware of a pending liquidation is to sell before (and, for some parameters, after) the period where the liquidator sells, while purchasing during the period that the liquidator sells. In other words, the strategic trader essentially chooses to absorb a portion of the liquidator's order imbalance while it occurs, while offloading the resulting inventory before and after the liquidation. The effect of the strategic trades is to increase rather than decrease the liquidator's proceeds and to decrease rather than increase the temporary price impact of the liquidating trade if the market is sufficiently resilient. In addition, we show that competition among multiple strategic traders ultimately benefits the liquidator as compared to the situation when strategic trading is absent, for any level of market resiliency, *except* in the extreme case where all price impacts are fully permanent.

Our empirical analysis reveals several findings. First, the oil futures market is indeed resilient. Using CME order book data, we estimate the permanent and temporary component of trading costs and a resiliency parameter that captures the extent to which temporary price impacts persist beyond the period of the order imbalance. The resulting estimates imply that the temporary price impact of an order

⁵ Note that the apparent underperformance in the USO share price as displayed on Figure 1 accelerates after the March 2008 to February 2009 period covered by our main datasets. We show in the Appendix that the main reason for the apparent underperformance is storage costs, rather than predatory trading or other costs of completing the roll trades. Storage costs, in turn, did not soar until crude oil inventories increased in the wake of the financial crisis.

imbalance is almost entirely reversed within one minute in the expiring contract and within three minutes in the second nearest-to-expiration contract on roll days. We also find that liquidity replenishment in the expiring contract is faster on roll days, as evidenced by a reduction in resiliency parameter, relative to non-roll days.

Second, we find that several measures indicate improved liquidity on roll versus non-roll days. In particular, quoted and effective bid-ask spreads are narrower on roll days, and the quantity of unexecuted orders in the limit order book at prices near the inside quote is greater on roll days. Further, a larger number of distinct accounts provide liquidity through limit orders on ETF roll days relative to non-roll days. These findings are consistent with Admati and Pfleiderer's (1988, 1991) prediction that a preannouncement strategy by a large liquidity trader increases market size by attracting liquidity providers and natural counterparties.⁶

Third, we analyze the CFTC account-level data, and find little or no evidence that individual trading accounts use strategies that would reasonably be considered predatory. Consistent with the simple framework introduced in our paper, we find significant increased usage of a liquidity provision strategy where strategic traders sell the expiring contract before the roll while offloading the resulting position on and after the roll day, thereby absorbing a portion of the ETF sales during the roll day, while shifting a portion of the selling pressure to the preceding day. Our theoretical analysis implies that this strategy mitigates temporary price impacts in the resilient market and improves prices for the rolling ETF.

Fourth, our analysis of daily settlement prices indicates that USO does pay to execute its roll trades – about 25 basis points on average per roll, or 3% per year, in the form of adverse changes in settlement prices in advance of the rolls. While this estimate indicates a substantial roll cost associated with the ETF's large liquidity demand, it needs to be evaluated in light of the high rate of turnover (1200% annually) implied by the monthly roll strategy.

4

⁶ In the limit order book model of Foucault, Kadan, and Kandel (2005), an increase in the proportion of liquidity suppliers narrows the bid ask spread as liquidity suppliers compete by submitting more aggressively priced limit orders. The preannouncement strategy can be viewed as an event that increases the proportion of liquidity suppliers. Henderson, Pearson, and Wang (2014) document that hedging trades in commodity futures that are *not* preannounced have substantial price impacts.

The findings of our study are relevant for portfolio managers who need to rebalance their portfolios for non-informational reasons. For example, revisions to the names of index component stocks are typically disclosed in advance of the index reconstitution date; however participants such as index funds rebalance their portfolios on the reconstitution date to minimize tracking error. Index reconstitution is associated with higher trading activity and significant abnormal returns prior to the event, including both transitory price pressure and the permanent effects of index membership (see Madhavan (2003) and Chen, Noronha and Singal (2004)). To avoid predatory trading, some index funds and transition portfolio managers choose not to disclose the timing of their trades.⁷ Our study shows that predatory trading is unlikely to degrade market quality in the absence of long lived price impacts, implying that the preannouncement of trading intentions is a viable strategy for large non-informed traders in more resilient markets. In contrast, our analysis confirms the Brunnermeier and Pedersen finding that predatory trading can harm liquidators and market quality when trades contain information that permanently alters prices, when markets are not resilient, or when the predatory trades are executed in very close time proximity to the liquidator's trades.

The rest of the paper is organized as follows. Section II discusses the related literature, the structure of the NYMEX market, data sources and summary statistics on USO trading activity on roll days. Section III presents our simple extension of the theory of strategic trading around a known liquidation, while Section IV introduces an empirical method to estimate resiliency, and provides evidence confirming that the NYMEX futures market is indeed resilient. Section V reports on measures of market quality on roll versus non-roll days. Section VI examines account level activity by strategic traders surrounding the roll, while Section VII presents estimates of the effective trading costs paid by USO for the roll trades. Finally, we present in the Appendix an analysis that reconciles USO's share price performance to the level of crude oil prices, as documented on Figure 1.

⁷ Vanguard Emerging Markets Stock Index Fund and ETF recently adopted the FTSE Emerging Index as the new target index, replacing the MSCI Emerging Markets Index. In describing the adjustments to fund investors, Vanguard states that "To protect the fund from the potential for harmful front-running by traders, the exact timing of the index changes will not be disclosed in advance to investors."

II. Related Literature, Data Sources and NYMEX market structure

a. Related Literature

We extend the theory of trading around a predictable order, and provide related empirical evidence from an important commodity market, that for NYMEX crude oil futures. The cornerstone theory of strategic trading when some traders become aware of another trader's need to liquidate a position is provided by Brunnermeier and Pedersen (2005), who introduce the label "predatory trading" to describe the strategies followed. In their model the predators sell ahead of or alongside the liquidating trader, before reversing their positions. The predatory trades damage market quality in that they cause the price to temporarily overshoot its long-term equilibrium. Further, their profits come at the expense of lower proceeds to the liquidating trader. Carlin, Lobo, and Vishwanathan (2007) present a multi-period model in which traders typically provide liquidity to each other. However, in situations where the potential profit from following a predatory strategy is sufficiently large, traders abandon liquidity provision to follow predatory strategies. Their model therefore predicts episodic periods of illiquidity due to predation.

The model presented by Brunnermeier and Pedersen (2005) assumes that trades have permanent price impacts proportional to the size of the order imbalance. In practice large trades can have both temporary and permanent price impacts. Schoneborn and Schied (2007) show that strategic traders may react to known liquidations by trading in the opposite rather than the same direction as the liquidator when price impacts are temporary. Our model extends their work in that we also assess (a) the effect of market "resiliency," i.e. the extent to which trades' temporary price impacts spill over to periods subsequent to trade execution, and (b) whether strategic trading improves or degrades market quality and liquidator proceeds as compared to a scenario where strategic traders are absent. The extension to a resilient market is important in our setting, because although ETF roll trades are unlikely to have permanent price impact, long-lived temporary price impacts could accommodate predatory trading.

Admati and Pfleiderer (1991) consider a trader who needs to complete a large transaction and who is not motivated by private information regarding asset value. They show that a public announcement of the intent to trade, termed "sunshine trading," can attract liquidity suppliers who might not otherwise have been present as well as natural counterparties, and can therefore allow the trader to achieve a more favorable price.

Our empirical analysis of potential price impacts of roll trades in commodity futures is not entirely unprecedented. Stoll and Whaley (2010) and Mou (2011), study commodity trading by index investors. In contrast to specialized ETFs that focus on a single commodity, index investors seek to generate returns that match the performance of multi-commodity indices, such as the Standard and Poor's-Goldman Sachs Commodity Index (SP-GSCI). To the extent that these index investors rely on futures positions to track the indices, they also generate periodic roll trades.⁸ Mou (2011) reports that significant profits can be earned by investors who trade in advance of the dates that the SP-GSCI index begins to track the second-rather than nearest-to-expiration futures contract. In contrast, Stoll and Whaley (2010) find little or no price effects around index shift dates in a broad cross-section of commodity futures prices.

Our study is distinguished from these in that we are able to exploit account-level data on individual trades as well as limit order book updates to assess the relevance of sunshine and predatory trading theories. In addition, we present a model of strategic trading around a known liquidation that extends prior work, estimate trade execution costs and market resiliency for an important commodity market, and explain the apparent underperformance of the largest crude oil ETF.

b. The NYMEX Market

Our empirical analysis focuses on the New York Mercantile Exchange, which trades crude oil and other energy futures contracts. NYMEX prices are widely-used benchmarks for valuing derivative contracts and determining final prices for over-the-counter contracts. Although the NYMEX continues to operate a physical trading floor, the large majority of transactions occur on NYMEX's electronic limit order market, known as Globex. In addition, large traders can negotiate block trades. Though physical delivery is rare, each individual NYMEX crude oil contract calls for delivery of 1,000 barrels of crude oil

⁸ In addition, a number of authors have assessed whether index investors and other passive long-only investors have affected the level and/or the volatility of commodity prices. See, for example, Boyd, Harris and Nowak (2011), Buyuksahin and Harris (2011), Irwin and Sanders (2012), and Kilian (2009).

at Cushing, Oklahoma, during a designated delivery month. Transaction prices reflect prices at which oil is to be delivered in the future, not an amount paid to enter the contract. Trading hours for floor trades are 9:00 AM to 2:30 PM New York time. Globex trading occurs around the clock, except for a 45-minute break from 4:15 PM to 5:00 PM New York time. The weighted average price during the two-minute interval 2:28 to 2:30 PM comprises the contract's "settlement price" for the day, and is used to calculate gains and losses on outstanding positions. In particular, long positions receive and short positions pay the change in the settlement price since the prior day (or since the transaction price if entered the same day).

In addition to outright trades that specify a delivery price, the NYMEX offers "Trade-at-Settlement" (TAS) contracts. The futures price for a TAS trade is the current day settlement price (potentially plus or minus a specified margin), and is generally not known at the transaction time. Press accounts indicate that the USO Oil ETF routinely uses TAS trades to complete its rolls.

c. Data Sources

We employ three main datasets. The Commodity Futures Trading Commission (CFTC) provided data on all completed trades in NYMEX crude oil futures from March 1, 2008 to February 28, 2009. The CFTC data includes floor and block trades, as well as trades completed on the Globex electronic market. In addition to trade type, contract, price, and volume, the CFTC data includes an account identification variable for each trade, which allows us to assess the number of unique trading accounts active on a given day and assess trading strategies used by each account. Although the buy and sell account is identified for each trade, the initiator of the trade is not. We use a modified Lee-Ready algorithm to assign trades as buyer- or seller-initiated.⁹

We also obtain for the same time period from the Chicago Mercantile Exchange (owner of the NYMEX) a 5-level deep representation of the limit order book and a record of completed trades for crude oil futures on the electronic GLOBEX market. The CME dataset allows us to construct a continuous record of best bid and ask quotes, as well as the depth of unexecuted displayed orders at and behind the

8

⁹ Specifically, to sign trades, we compare the transaction price with the contemporaneous quote-midpoint (without a 5-second lag). For midpoint trades we compare the trade price to prices of up to five prior trades to implement the tick-rule.

best quotes.¹⁰ Third, we obtain from the United States Energy Information Agency (EIA) a daily record of settlement prices for NYMEX crude oil contracts traded over the longer time interval January 1999 through December 2013.

d. Descriptive Statistics

Our most detailed analysis focuses on the period March 1, 2008 to February 28, 2009, during which we have data on individual crude oil trades. USO's assets-under-management (AUM) grew significantly during this period, from \$0.47 billion in March 2008 to \$3.92 billion in February 2009. Table 1 reports estimated USO roll activity as a percentage of market volume for each monthly roll date, for the "front" contract and for the "second" nearest-to-expiration contract. Roll activity is estimated based on USO's assets under management and the roll-date settlement price, and shows rapid growth during the sample, with sales of the front contract increasing from 4,445 contracts representing 1% of market volume during the March 2008 roll to 85,055 contracts representing 16% of market volume during the February 2009 roll. Aggregated across the twelve roll dates, USO roll activity comprised 5% of roll-day volume in the front contract. Roll-date purchases accounted for an average of 10% of roll-day volume in the more lightly traded second contract.

Table 1 also reports market volume during the two-minute settlement period. Since performance is benchmarked against changes in settlement price, ETFs that hold futures positions generally try to track the settlement price on the roll day. USO's roll volume on average exceeds market volume during the settlement period, indicating that it would be difficult or impossible for USO to execute its entire roll by use of regular trades during the settlement interval. Crude oil ETFs can employ TAS contracts to ensure a price that closely matches the benchmark settlement price. To the extent that a TAS counterparty to an

¹⁰ In addition to the "outright" book for each contract, the CME maintain a limit order book for calendar spread orders. Each leg of the spread order competes for order flow with the corresponding outright book. Further, the CME data has a finer time resolution (to the 100th of a second) as compared to the CFTC data, for which time stamps are truncated to the second. We use the CME transactions to impute centi-second time stamps for CFTC data transactions through an iterative process of matching unique price-quantity pairings. When transactions cannot be perfectly matched between the two datasets, the latest-possible time stamp is imputed within the CFTC dataset to avoid any possible look-ahead bias of matching trades with LOB information. A limitation is that the CME data does not include floor trades or negotiated block trades.

¹¹ Our estimates of USO's roll date volume may differ slightly from the fund's actual volume if a portion of assets under management have not yet been invested, or if some roll activity takes place in OTC markets.

ETF trade has a 'natural' offsetting position, the TAS trade allows both the ETF and counterparty to offset positions at low cost.¹² If the TAS counterparty is simply providing liquidity, then compensation takes the form of the difference between roll-day settlement prices and prices for the counterparties' offsetting trades.

III. Extending the Theory of Strategic Trading

We present a simple model that relaxes a key assumption of the Brunnermeier and Pedersen (2005) model, that the price impact of trades is entirely permanent. In particular, we allow trades to have both permanent and temporary price impacts, and for the duration of the temporary price impact to depend on the "resiliency" of the market. Allowing for temporary price impacts is particularly important when assessing strategic trading around predictable orders initiated by non-information-motivated traders.

To assess the potential effects of strategic trading we consider two extreme cases. First, we model outcomes when only a single strategic trader is aware of the pending trade. The monopolist strategic trader chooses quantities to maximize profits. Second, we consider outcomes when many strategic traders are aware of the pending trade, and competition leads to zero profits for strategic traders. We refer to a pending liquidation, but the analysis implies equally well to a pending purchase or a pending sale and purchase of related securities, as in a futures roll trade. We find that even a monopolist strategic trader improves market quality and liquidator proceeds when the market is sufficiently resilient, and that competition among strategic traders always benefits the liquidator, except in the extreme case where all price impacts are permanent.

a. The Setting

We assume that the investor will liquidate a known quantity, Q_L . Strategic traders aware of the liquidation can trade before, simultaneous with, or after the liquidation. Strategic trades sum to zero across periods. That is, the limit order book rather than strategic traders ultimately absorb the liquidator's

¹² An example would be an oil producer, who might hedge oil price risk with short futures positions, and roll the short position in expiring contracts. ETF's reliance on TAS contracts has been discussed in popular press articles. See for example, Financial Times article, http://ftalphaville.ft.com/2010/07/15/287061/the-end-of-diversification.

position. For simplicity we abstract from any randomness in outcomes. Assuming risk neutrality, all results would carry through to expected prices if randomness were allowed for.

b. Trade Prices When the Market is Resilient

We assume that the private information conveyed by trades is measured by a permanent price impact parameter, λ , so that security value evolves according to $V_t = V_0 + \lambda Q_t$, where q_i is signed marketable order flow in period i, and $Q_t = \sum_{i=1}^t q_i$ is the accumulated order flow since base period 0. Trades also have temporary price impacts. The immediate temporary price impact, γq_i , is proportional to the signed order flow, reflecting that small orders execute at inside quotes that differ from security value and that larger orders walk up the limit order book. The temporary price impact potentially persists beyond the time of the trade. Specifically, the trade price at time t depends on current and prior order flow according to:

$$P_t = V_t + \gamma A_t = V_0 + \lambda Q_t + \gamma A_t \tag{1}$$

where $A_t = \sum_{j=0}^{t-1} \theta^j q_{t-j}$. Note that the quote midpoint can be stated as $M_t = V_0 + \lambda Q_{t-1} + \gamma \theta A_{t-1}$.

Here, A_t is a weighted sum of orders from time 0 to t, and the parameter θ measures the (inverse) resiliency of the market. If $\theta = 0$, i.e. if the market is completely resilient, the temporary price impact of the order at time t has no effect beyond time t. This requires that the limit order book refill instantaneously after an order is executed. If so, the third term in the expression for the midpoint is zero, implying that the midpoint is equal to security value, and that $P_t = V_{t-1} + (\lambda + \gamma)q_t$. If $0 < \theta < 1$ the limit order book takes time to refill, the temporary effect of the time t order flow extends beyond time t, and the midpoint differs from security value as a function of recent order imbalances. If $\theta = 1$ the temporary impact is never reversed, and thus is indistinguishable from permanent impact.

11

 $^{^{13}}$ Note that Carlin, Lobo, and Vishwanathan (2007) use γ to denote permanent price impact and λ to denote temporary impact.

c. Market Prices and Outcomes

Trading occurs during three periods: before, during, and after the investor's liquidation. Let Q_p , Q_d , and Q_a denote net signed order flow (the sum of liquidator and strategic order flow) during the "pre", "during", and "after" periods, respectively. Then, trade prices during the "pre", "during", and "after" periods are:

$$P_p = V_0 + I_0 Q_p, \text{ where } I_0 = \left[\lambda + \gamma\right]$$
 (2)

$$P_d=V_0+I_1Q_p+I_0Q_d, \ \ \text{where}\ I_1=\left[\lambda+\theta\gamma\right], \text{ and} \eqno(3)$$

$$P_{a} = V_{0} + I_{2}Q_{p} + I_{1}Q_{d} + I_{0}Q_{a}$$
, where $I_{2} = [\lambda + \theta^{2}\gamma]$ (4)

The parameters I_0 , I_1 and I_2 measure the effects on trade prices of same, prior, and second-prior period order imbalances, and depend on market resiliency.

We can describe strategic trader order flow by a pair of proportionality parameters ρ_d and ρ_p , defined so that positive values indicate trading in the same direction as the liquidator. Including the requirement that the strategic trader order flow sums to zero across the three periods, the imbalance absorbed by the limit order book (the sum of liquidator and strategic order flow) in each period is:

$$Q_p = -\rho_p Q_L \,, \tag{5}$$

$$Q_d = -(1 + \rho_d)Q_L, \tag{6}$$

and
$$Q_a = Q_L(\rho_d + \rho_p)$$
. (7)

The liquidator's proceeds depend only on the price in the liquidation period:

$$LP = Q_L P_d, (8)$$

while strategic trading profits depends on price differences across periods:

$$SP = Q_L \left[\rho_p (P_p - P_a) + \rho_d (P_d - P_a) \right]. \tag{9}$$

Using expressions (5) to (7) in expressions (2) to (4) for prices, straightforward calculus reveals that a monopolist strategic trader will choose profit-maximizing quantities according to:

$$\rho_d^* = \frac{\theta - (\lambda/\gamma) - 2}{\theta^2 - 4\theta + 3(\lambda/\gamma) + 6}$$
 and

$$\rho_p^* = -\frac{\theta^3 - 3\theta^2 - [(\lambda/\gamma) - 2]\theta + 2(\lambda/\gamma)^2 + 5(\lambda/\gamma) + 2}{\theta^4 - 4\theta^3 + [2(\lambda/\gamma) + 4]\theta^2 + [4(\lambda/\gamma) + 8]\theta - 3(\lambda/\gamma)^2 - 12(\lambda/\gamma) - 12}.$$
(10)

Note that the monopolist strategic trader's optimal strategy depends only on market resiliency, θ , and on the ratio of permanent to temporary price impacts, λ/γ .

To assess the effect of competition among strategic traders, we assume that each potential strategic trader is identical, implying that each will earn a pro-rata share of profits as given by expression (9). The zero-profit equilibrium therefore requires that prices are equal across periods, $P_p = P_d = P_a$. ¹⁴ Imposing this condition, and using expressions (2) to (7), competitive equilibrium implies that quantities attributable to competitive strategic traders in aggregate are given by:

$$\rho_p = \frac{1 + (\lambda/\gamma)^2 + 2(\lambda/\gamma)}{(\lambda/\gamma)^2 + 3(\lambda/\gamma) + 3 - 2\theta - S\theta^2} \text{ and }$$

$$\rho_d = -\frac{(\lambda/\gamma)^2 + 2(\lambda/\gamma) + 2 - \theta + (\lambda/\gamma)\theta - (\lambda/\gamma)\theta^2}{(\lambda/\gamma)^2 + 3(\lambda/\gamma) + 3 + 2\theta - (\lambda/\gamma)\theta^2}.$$
(11)

d. Illustration of Model Implications

Though the model provides closed form solutions, its implications are not fully transparent. To illustrate the model's implications, Figure 2 provides numerical outcomes. Panel A shows quantities sold by strategic traders in the pre, during, and after periods, as a proportion of the quantity liquidated, while Panel B shows the resulting market prices in each period. The illustration relies on an initial price (V_0) equal to \$100, $Q_L = 20$ units liquidated, and temporary price impact $\gamma = 0.05$, for values of the (inverse)

¹⁴ If prices were not equal, some strategic traders could earn profits by selling less in the period with lower prices and more in the period with higher prices. If the analysis were broadened to incorporate additional pre and post periods the implication would be that prices should be equal across all periods – a result that can be foreseen based on standard efficient market reasoning in the absence of risk aversion, uncertainty, or costs of providing liquidity.

resiliency parameter, θ , ranging from zero to one. The first figure in each Panel focuses on an informationless liquidation, with $\lambda=0$. Since ETFs rolls are unlikely to have a permanent effect on prices, these results are the most relevant to our setting. However, to obtain more general insights we also illustrate outcomes for $\lambda=0.05$ (permanent impact as large as the temporary impact) and $\lambda=0.10$ (permanent impact twice as large as the temporary impact).

Monopolist Strategic Trading

Focusing first on Panel A of Figure 2, a notable result is that is that the profit maximizing strategy for a monopolist strategic trader is to *purchase* during the period when the liquidator sells, a strategy which might reasonably be referred to as providing liquidity to the liquidator. The strategic trader's purchases are in the vicinity of one third of the amount liquidated, for all θ , and are little affected when the permanent price impact parameter is increased.

In addition, for all parameters, the monopolist strategic trader sells in advance of the known liquidation, a strategy that we will loosely refer to as "front running." The quantity that the strategic trader sells in advance of the liquidation increases as the market becomes less resilient, i.e. as temporary price impacts are reversed more slowly, and also increases with λ , the magnitude of the permanent price impact. If the liquidation is informationless ($\lambda = 0$) and the market is quite resilient (θ less than about 0.6) the quantity that the monopolist strategic trader sells ahead of the liquidation is less than the quantity purchased during the liquidation, implying that the strategic trader also sells after the liquidation. When the market is less resilient or when trades have substantial permanent price impacts the quantity sold by the monopolist strategic trader ahead of the liquidation is larger than the quantity purchased during the liquidation, implying that the strategic trader defers some purchases until after the liquidation is complete.

Given a temporary price impact of .05, the price received by the liquidator in the absence of strategic trading, P_d , is \$99 when $\lambda = 0$, \$98 when $\lambda = 0.05$, and \$97 when $\lambda = 0.10$. Panel B of Figure 2 displays prices in each period with and without strategic trading. Importantly, Figure 2 reveals that the liquidator's proceeds are in many cases *greater* (because the price in the "during" period is higher) when the monopolist strategic trader is present. In particular, given an informationless liquidation ($\lambda = 0$) the

liquidator *benefits* from the activities of the monopolist strategic trader when θ is less than about 0.76, i.e. when the market is relatively resilient.

The benefit to the liquidator from being "front run" arises because the strategic trader essentially spreads the price impact of the liquidation into the preceding and subsequent periods. When the market is sufficiently resilient the price impacts of the "front running" trades are largely dissipated by the time of the liquidation, and the liquidator is not harmed.

In contrast, the strategic trader harms the liquidator if the market is not sufficiently resilient, or when permanent price impacts are larger. As noted, decreased resiliency or larger permanent price impacts lead the monopolist strategic trader to sell larger quantities prior to the liquidation and for some parameters to wait until after the liquidation is complete to fully offset the larger sales with purchases. When the market is less resilient or price impacts are permanent the front running trades depress the price received by the liquidator. Note also that market quality is degraded, in the sense that the strategic trader causes the price to overshoot its longer term equilibrium by a larger amount, for the same parameters that the liquidator receives reduced proceeds, and vice versa.

These results underscore the importance of market resilience in assessing whether a predictable liquidator will benefit or be harmed by the trades of a monopolist strategic trader. In particular, there exists a "breakeven" level of market resilience. If the market is more resilient than the breakeven level then the liquidator benefits and the magnitude of temporary price movement is reduced by the presence of the monopolist strategic trader, and vice versa. We calculate breakeven resilience by solving for the value of theta that sets the liquidation price from equation (3) equal to the liquidation price received without strategic trading. On Figure 3 we plot the breakeven level of market resiliency for a range of permanent vs. temporary price impact parameters. The breakeven resiliency parameter declines monotonically as the ratio of permanent price impact to temporary price impact increases. For very large permanent price impacts (over 2.7 times the temporary price impact), the liquidator is harmed by monopolist strategic trading for any positive resiliency parameter.

 15 The expression for break-even theta is complex, and is presented in the Internet Appendix.

15

Competitive Strategic Trading

In Brunnermeier and Pedersen (2005) strategic trading always reduces liquidator proceeds and damages market quality, but competition among strategic traders reduces the degree of harm. That is, trading by a monopolist strategic trader comprises a worst case scenario. In many practical situations, e.g. when leveraged ETFs trade in response to price changes to maintain target return multiples or index investors purchase securities added to an index, numerous potential strategic traders are likely to be aware of a pending trade. In the case of the USO ETF in particular, dates and magnitudes of roll trades are announced publicly in advance, so many strategic traders are potentially present in the market near the time of the USO roll.

Panel A of Figure 2 displays the quantities traded by competitive strategic traders as implied by our model. Notably, competitive strategic trading implies larger quantities sold in advance of the liquidation, i.e. greater quantities of "front running", as compared to outcomes with a monopolist strategic trader. However, competitive strategic trading also implies larger quantities purchased during the liquidation as compared to monopolist strategic trading – that is, competitive strategic traders provide more liquidity simultaneous with the liquidation, reducing the quantity that must be absorbed by the limit order book, as compared to the monopolist strategic trader. Also, for all $\theta < 1$ the quantity purchased in the during period exceeds the quantity sold in the pre period, implying that competitive strategic traders offload their inventory acquired during the liquidation in both the pre and after liquidation periods.

Panel B of Figure 2 displays market prices with competitive strategic trading. Note that the price with competitive strategic trading is always lower in the pre period and higher in the during period, as compared to prices with monopolist strategic trading. Notably, our model implies that the price during the liquidation period with competitive strategic trading is higher than the price in the absence of strategic trading, for all $\theta < 1$ and for all ratios of permanent to temporary price impact. That is, competitive

strategic trading benefits the liquidating trader in all cases *except* in the limiting case where price impacts are entirely permanent, in which case strategic trading has no net effect on liquidation proceeds.¹⁶

It is instructive to compare our results obtained when trades can contain both permanent and temporary price impacts to those obtained by Brunnermeier and Pedersen (2005) in the case where trades' price impacts are strictly permanent. Both models imply that a monopolist strategic trader is the worst case scenario in terms of liquidator proceeds and market quality. In Brunnermeier and Pedersen (2005) strategic trading always reduces liquidator proceeds and causes prices to overshoot, but the damage is reduced by competition among strategic traders. Our results obtained when $\theta = 1$, which also implies only permanent price impacts, are the same. However, our model implies that even monopolist strategic trading can improve market outcomes if the market is sufficiently resilient. Further, competitive strategic trading strictly improves market outcomes as compared to the absence of strategic trading for all resiliency parameters *except* $\theta = 1$. In this sense the Brunnermeier and Pedersen (2005) results are a special case.

This model presented here is based on a number of simplifying assumptions, including either a monopolist strategic trader or perfect competition among strategic traders, that strategic traders incur no costs, that price impacts are linear, and that there are only three trading periods. Nevertheless it delivers important insights. In particular, it implies that even monopolist strategic trading will benefit rather than harm a party whose trades can be predicted if permanent price impacts are not too large and if temporary price impacts are rapidly reversed. Further, competition among strategic traders improves outcomes, and to the extent that a zero-profit outcome is a reasonable approximation, competitive strategic trading is likely to benefit rather than harm the liquidator regardless of price impact parameters. Thus strategic trading in advance of a liquidation is unlikely to be a concern for a liquidator such as an ETF or index fund who can credibly signal that their trade is not motivated by information, if the market is resilient, and if multiple potential strategic traders are aware of the pending trade.

¹⁶ Algebraically, θ =1 is the only positive resiliency parameter that leads to equal liquidator proceeds with competitive as compared to no strategic trading, and therefore defines breakeven resiliency in the competitive case.

More broadly, a key issue in assessing whether strategic trading in advance of a predictable trade will be harmful is the proportion of any "front running" trades that persists into the period when the predictable trade is executed. The proportion of the price impact of a time t trade that persists to period t+n can be summarized by an "Impact Persistence" parameter, defined as:

$$IP_n = \frac{I_n}{I_0} = \frac{\lambda + \gamma \theta^n}{\lambda + \gamma} \tag{12}$$

Impact persistence depends on permanent price impact, temporary impact, market resilience, as well as elapsed time since the trade. An implication is that strategic trades will be more damaging to the liquidator, ceteris paribus, if they occur very shortly before the liquidator trades. Strategic trading will also be less of a concern when permanent price impacts are smaller, implying that predictable trades that are not motivated by private information, including commodity roll trades, are less likely to be harmed by strategic trading. To illustrate, suppose that market resiliency is $\theta = 0.98$, with time is measured in seconds. Recall that breakeven resiliency with a monopolist strategic trader in the case of informationless liquidation (λ =0) is 0.76. These parameters imply that the liquidator would be harmed by monopolist strategic trades occurring immediately or a few seconds prior to liquidation. On the other hand, the liquidator would not be harmed by monopolist strategic trades occurring one minute before liquidation, as only the proportion 0.98⁶⁰ = 0.298 of the price impact would persist into the liquidation period.

As noted, the USO ETF routinely uses "Trade at Settlement" or "TAS" contracts, which are completed at the day's settlement price, to complete its roll trades. The TAS contract allows USO to capture the settlement price for its trades while effectively outsourcing the actual execution of the roll trades to the TAS contract counterparty. Market participants know the date of the USO rolls due to preannouncement, but in general will not know the exact time at which the TAS counterparties will execute trades to offset their TAS commitment. That is, preannouncement of the date of the roll can attract additional counterparties and liquidity providers, as implied by sunshine trading theories, while ambiguity about the precise timing of the roll trades can mitigate the danger of damaging strategic trades, even in the unlikely scenario that only one or a few strategic traders are present.

To summarize, our model confirms the findings of Brunnermeier and Pedersen (2005) that strategic trading can harm the liquidator and degrade market quality if trades have permanent but not temporary price impacts, and the number of strategic traders is limited. However, strategic trading, even by a monopolist strategic trader, can benefit the liquidator and improve market quality if some or all of trades' price impacts are temporary and are reversed quickly. The key issue is the proportion of the strategic trades' price impact that persists during the liquidation period. If this proportion is low the liquidator benefits from having price impacts spread over more time periods. However, even in a resilient market strategic trading can harm the liquidator if a strategic trader can time the liquidator's trades and execute in very close proximity to the time of the liquidation. The model implies that traders should be concerned about potential strategic trading mainly when their trades are motivated by private information that will affect long-run security value, or when the market is non-resilient, or when trade timing can be forecast with such precision that the price impact of prior-period trades persists almost fully into the liquidation period, and in particular when only one or a few strategic traders can anticipate the upcoming trade.

IV. Estimating the Resiliency of the NYMEX Crude Oil Market

The analysis in the preceding section underscores that the effects of a strategic trader who becomes aware of the trading intentions of a large investor depend in an important way on whether the price impact of trades is permanent or temporary, and in the case of temporary price impacts, on the resiliency of the market. To assess the likely effect of strategic traders for crude oil ETF rolls it is useful to obtain estimates of trades' permanent and temporary price impacts, as well as market resiliency, in the NYMEX crude oil markets. Expression (1) implies that these parameters can be estimated by a geometric lag regression of the form:

$$P_{t} - P_{t-1} = \beta_0 q_t + \beta_1 q_{t-1} + \beta_2 q_{t-2} + \dots + \beta_i q_{t-i} + \varepsilon_t$$
 (13)

where P_t is the time t trade price, q_j is the signed order imbalance at time j, and the coefficients are restricted such that: $\beta_0 = \lambda + \gamma$, $\beta_1 = \gamma(\theta-1)$, $\beta_2 = \beta_1 \theta$, $\beta_3 = \beta_2 \theta^2$, $\beta_4 = \beta_3 \theta^3$, and so on. Note that if

 $\theta = 0$, i.e. if the market is completely resilient, then expression (13) reduces to a regression of the price change on q_t and $(q_t - q_{t-1})$, a direct extension of the indicator variable model of Huang and Stoll (1997). We also allow the permanent price impact parameter (λ) to differ for buy vs. sell imbalances, to accommodate uninformed ETF sell orders in the front contract and buy orders in the second contract. Specifically, we specify $\beta_0 = [(\lambda_B s_t + \lambda_B (1 - s_t)) + \gamma]$ where s_t equals 1 if $q_t > 0$, and equals 0 otherwise.

Equation (13) is estimated by aggregating net order imbalance (excess of buyer-initiated over seller-initiated trades in contracts), measured from the CME's order data, within each ten-second window. Estimation is based on j = 30 lags. Following Sadka (2006), we allow the temporary price impact to include a fixed component, γ_0 , that can be interpreted as an order processing cost. Adding this parameter gives the specification:

$$P_t - P_{t-1} = \beta_0 q_t + \beta_1 q_{t-1} + \beta_2 q_{t-2} + \beta_3 q_{t-3} + \dots + \beta_t q_0 + \gamma_0 [D_t - D_{t-1}]$$
 (14)

where D_t is a binary variable that receives a value of (+1) when the last trade in the ten-second window is buyer-initiated and (-1) when the last trade in the ten-second window is seller initiated.

We estimate separately the parameters of resiliency model for roll and non-roll days during the March 2008 to February 2009 period. As discussed previously, during our sample period, USO accomplished its roll in a single day of trading. The Goldman roll (during which time funds that track the Standard and Poor's-Goldman Sachs Commodity Index complete their roll trades) occurs from the fifth through ninth trading day of each month. During our sample, six of USO's roll dates fall before the Goldman roll; five roll dates are concurrent with the start of the Goldman roll; in one month, the Goldman roll begins before the USO roll. Since strategic traders may participate in both rolls, for this analysis we define the roll date as the earliest date upon which either USO or the GSCI rolls. Both the USO roll and the Goldman roll occur prior to what might be termed the "market roll" i.e. when overall trading activity and open interest moves from the front month contract to second month contract. Since the market roll itself can induce changes in market conditions, we compare trading activity during the roll period to a non-roll period that

precedes the roll date. Specifically, we define the period extending from seven to three days prior to the roll as the non-roll period.

Table 2, Panel A presents results for the front contract while Panel B presents analogous results for the second contract. As might be anticipated, the coefficient estimates based on the full sample of roll and non-roll days indicate that the front contract is more liquid than second contract. In particular, relative to the second contract, the front contract has smaller permanent price impact (4.0 cents versus 6.7 cents for buys and 4.3 cents versus 6.5 cents for sells), smaller temporary price impact (1.9 cents versus 5.7 cents) and a smaller resiliency estimate (0.67 versus 0.94), but a marginally higher fixed order processing cost per trade (0.58 versus 0.54).

Overall the parameter estimates on the roll and non-roll days indicate that the futures market absorbs the large imbalance attributable to the ETF roll trade without a significant disruption to market quality. For the front contract, the permanent price impact for sell imbalance on roll days is 0.040 versus .041 on non-roll days (p-value of difference=0.57).¹⁷ Perhaps more important, the estimated resiliency parameter, θ, for the front month contract is smaller (implying greater resiliency) on roll days as compared to nonroll days (0.49 vs. 0.66). Note that these results are observed despite the fact that the ETF roll brings heightened liquidity demand to the market. The fixed order processing cost and the temporary price impact do not differ significantly across roll and non-roll days. We also do not observe differences in parameter estimates for the second contract across roll and non-roll days.

Recall that the resiliency parameter, θ , measures the proportion of the temporary price impact attributable to current period order imbalance that persists to the next period. The model in the preceding section shows that the resiliency parameter is crucial in assessing the trading patterns that will maximize strategic trader profits. Focusing on roll dates in particular, the front month θ estimate of 0.494 implies the estimated proportion of the temporary impact that persists after one minute is $0.494^6 = 0.014$. The corresponding estimate for the second month contract on roll dates is 0.94, which implies that the

non-informed trading. However, while ETF trades are large, they still comprise a minority of trading activity on roll days, and informed traders may be present.

¹⁷ It may be surprising that the estimated permanent price impact on roll days is positive, if ETF rolls comprise

proportion of temporary price impact that persists after one minute is $0.94^6 = 0.69$, while the proportion that persists after three minutes is $0.94^{18} = 0.33$. These estimates indicate that the crude oil futures markets, in particular the nearest to expiration contract, are indeed quite resilient.

The theoretical analysis presented in preceding section indicates that the "breakeven" resiliency parameter for informationless trades is about 0.76, even under the strong assumption of a monopolist strategic trader. The model and empirical estimates provided here therefore imply that strategic trading around the USO roll will benefit rather than harm market quality unless a small number of strategic traders are able to complete same-direction trades within minutes prior to the execution of the USO roll.

V. Additional Evidence Regarding Liquidity on Roll and Non-Roll Days

We provide additional evidence by estimating across roll and non-roll periods several liquidity measures during the March 2008 to February 2009 period. Specifically, we compare roll period and non-roll period measures for each minute between 9:00 AM and 3:00 PM EST, and report results averaged across minutes. To address non-normality attributable to potential time-of-day effects we implement a non-parametric (Wilcoxon signed) test, which requires less stringent distributional assumptions for tests of difference in location.

Results, reported on Table 3, verify that trading volume per minute is substantially greater on roll days, averaging 487 contracts in the front month and 69 contracts in the second month, compared to 402 contracts in front month and 43 contracts in the second month on non-roll days. Panel A of Figure 4 displays average trading volume by minute for roll and non-roll days. Most notable is the spike in trading activity at the time of the daily settlement, particularly on roll days.

We also examine a standardized "trade imbalance" measure for each minute based on the difference between buyer-initiated and seller-initiated trading volume. To account for intraday patterns, we

strategic trading.

22

¹⁸ However, the parameter estimates on Table 2 indicate that permanent price impacts are more than twice as large as temporary price impacts when estimated across all trades, a portion of which are presumable motivated by private information. From Figure 2, the implied breakeven resiliency for a typical mix of informed and uninformed trades trades is therefore 0.2 or less. Of course, this analysis still incorporates the extreme case of monopolist

normalize the measure by subtracting the mean and dividing by the standard deviation of imbalance observed during the same minute on both roll and non-roll periods. For the front contract the results reported on Table 3 show that the net trade imbalance is on average negative on roll days; for the second month contract the result is ambiguous, with a negative mean and a positive median, and no statistically significant difference in trade imbalance between roll and non-roll days.¹⁹

Importantly, the evidence indicates enhanced liquidity provision on roll days. Quoted bid-ask spreads (the difference between the lowest limit price for unexecuted sell orders and the highest limit price for unexecuted buy orders) on roll days decline from an average of 1.17 basis points to 1.13 basis points in the front contract, and from 1.51 basis points to 1.40 basis points in the second contract. Figure 4, Panel B displays average quoted spreads by minute for roll and non-roll days. The patterns indicate smaller intraday quoted spreads for the majority of minutes on roll days. These declines, while modest, are statistically significant, and must be evaluated in light of the substantial increase in liquidity demand attributable to the rolls, which might have been anticipated to widen spreads.

We also assess liquidity supply by computing the "depth" of unexecuted displayed orders in the limit order book. In particular, we determine the total volume of unexecuted sell (ask depth) and buy (bid depth) orders at prices within four ticks of the most competitive prices. Bid depth for the front contract, which is relevant for those seeking to sell, increases from an average of 48 contracts on non-roll days to 53 contracts on roll days (t-statistic = 12.16), while ask depth (relevant for those seeking to buy) for the second contract increases from an average of 19 contracts to 21 contracts (t-statistic = 9.96). Figure 5 displays average bid and ask depths by minute for roll and non-roll days and supports increased liquidity provision in both contracts throughout the day.

Next, we examine effective spreads, defined as the excess of the trade price over the bid-ask midpoint for each buyer-initiated trade and the excess of the midpoint over the trade price for each seller-initiated

¹⁹ Trade imbalance measures are calculated using Globex data only. A higher proportion of trade activity in the second month contract occurs through floor and off-exchange trades, which are not captured in the Globex data. The sign of the imbalance measure in the second month contract is sensitive to the inclusion of these off-exchange trades, but the difference in trade imbalance between roll and non-roll days is not statistically significant under either approach.

trade. Effective spreads differ from quoted spreads because large trades can execute outside the quotes, and because orders can execute against "hidden" orders in the limit order book. We focus on volume-weighted effective spreads for individual trades for each minute. The results reported in Table 3 indicate modest reductions in effective spreads on roll days, from an average of 2.03 basis points to 1.95 basis points (t-statistic = -4.11) for the front contract, and from 2.42 basis points to 2.27 basis points for the second contract (t-statistic = -5.22).

Finally, we use the CFTC data to assess the number of distinct accounts that supply liquidity on roll and non-roll days. An account is deemed to supply liquidity if more than one buy (sell) limit order posted by the account for the front (second) contract is executed on the corresponding day. We find that an average of 10,470 distinct accounts provide liquidity in the front contract on roll days, compared to 9,698 accounts on non-roll days. For the second contract the number of liquidity-supplying increases from 860 accounts on non-roll days to 1,416 accounts on roll days.

These comparisons of liquidity across roll and non-roll days are generally consistent with the sunshine trading theory of Admati and Pfleiderer (1991), who predict that the announcement of an upcoming trade by a credibly uninformed trader will attract additional liquidity suppliers and natural counterparties to market. The results are also generally consistent with our modified theory of strategic trading around known liquidations, which predicts that strategic traders will effectively supply liquidity during the liquidation in a resilient market. In particular, despite large liquidity demand on roll days, quoted spreads and effective spreads (which measure costs of trading) decline, while available liquidity on the relevant side of the limit order book and the number of distinct accounts providing liquidity both increase.

VI. Account-level analysis

We next study the trading strategies followed by individual accounts around the time of USO roll trades, to assess whether some accounts trade strategically as implied by the simple theories outlined in Section III. The CFTC data identifies the trading accounts associated with both the buy and sell side of each transaction. In our analysis, we define the "During" interval as 9 A.M. to 4:15 P.M. New York time

on the ETF roll day, while the "Before" interval covers from midnight on Day -3 (three trading days prior) to 9 A.M. on roll day, and the "After" interval is from the reopening of trading at 5 P.M. on the roll day through Midnight on Day +3 (three trading days after) the roll day. We define "During" period as the full trading day during which the roll occurs as opposed to a shorter interval near the settlement period because market participants know the day of the roll, but they do not know the exact time that USO's TAS counterparties will conduct their offsetting trades. Further, as shown in Table 1, the size of the USO roll often exceeded total market trading volume during the settlement period, implying that roll trades are indeed conducted outside the settlement period.

Our theoretical analysis focuses on a strategic trader who seeks to profit from the predictable liquidation, and who does not ultimately absorb any portion of the liquidation. Since strategic traders might also transact for reasons unrelated to the roll we relax this definition somewhat for the empirical implementation. Specifically, we identify an account as potentially being a strategic trader and retain it in the analysis if the absolute value of the net change in the account's inventory as a fraction of the account's total activity in the roll window (i.e., Days [-3,+3] surrounding the roll) is less than 0.25.20 The ETF's natural counterparties (i.e., accounts that hold or can be induced by price concessions to hold opposite positions as the ETF) are unlikely to be classified as strategic traders, since their inventory change to total trading ratio is likely to exceed the threshold of 0.25. Those accounts with an inventory change to total trading ratio in excess of 0.25 are excluded from the analysis.

We categorize each remaining account as following one of twelve possible trading strategies, as described in Panel A of Table 4. Those traders whose signed position change in the "During" interval is of the opposite sign as the ETF's order flow are labeled as following strategies ST1 to ST5. These accounts can reasonably be described as providing liquidity to the roll. Among these, accounts that trade against the ETF in the "Prior" period are categorized as ST1, while those that do not trade during the

²⁰ As an illustration, suppose an account sells 1,000 contracts before, sells 1,000 contracts during, and buys 1,500 contracts after the roll. The absolute value of net change in account's inventory is 500 contracts, the strategic (round trip) volume is 1,500 contracts and the total trading activity is 3,500 contracts. Since the ratio (500/3500) = 0.14, we classify the account as a strategic trader.

"Prior" period are categorized as ST2. These strategies would be inconsistent with our theory of strategic trading. Strategies ST3, ST4, and ST5 all involve trading with the ETF in the "Prior" period, in addition to trading against the ETF in the "During" period. These strategies are, depending on the degree of competition and market resiliency, all potentially consistent with the theory developed in Section III.

Strategies ST6 and ST7 involve accounts whose trading is close to zero-sum across the three periods, but that do not trade in the "During" period, behavior which also is inconsistent with the theory of strategic trading presented here. Strategies ST8 to ST12 involved trading in the same direction as the ETF in the "During" period, and are distinguished from each other by trading patterns in the "Prior" and "After" periods. These accounts are potentially following predatory strategies along the lines described by Brunnermeier and Pedersen (2005).²¹

For each account, we measure the volume of "strategic trading" during the relevant seven day period as round trip volume completed, i.e. as the minimum of buy volume executed and sell volume executed. We then aggregate strategic trading volume across those accounts that were identified as following each of the twelve strategies identified in Panel A of Table 4. Note that each identified strategy has a complementary strategy involving opposite trading patterns. For example, ST1 and ST12 are complimentary, in that ST1 involves trading against, against, and with the ETF during the three intervals, while ST12 involves the opposite pattern of trading with, with, and against the ETF during the three intervals. Since some strategies might be more common than others for reasons unrelated to the ETF roll, we focus for our analysis on *normalized strategic volume*, which is the strategic volume in a category less the strategic volume in the complementary strategy. To provide a basis for comparison, we also compute normalized strategic volume using the same methods and definitions, for all seven day periods that do not overlap with the seven day intervals that are centered on a roll day.

Panel B of Table 4 reports the key results of this analysis. In particular, we report regression coefficients obtained when normalized strategic volume in categories ST1 to ST6 for all seven day

²¹ As a robustness check, we implement an additional screen where an interval is classified as 'no trade' if the account's trading activity in the interval as a percentage of the account's total trading activity across intervals is lower than the absolute value of 10%. Results are similar to those reported in Table 4.

periods is regressed on an indicator variable, *Roll*, which equals one for the seven day intervals that are centered on the USO roll day, and that equals zero for seven-day periods that do not overlap with the roll.²² The intercept measures average normalized strategic volume by strategy for non-roll intervals, while the coefficient estimate on the indicator variable measures the extent to which the use of the strategy changes during roll periods as compared to non-roll periods.

Notably, the estimated coefficients for the Roll indicator are positive for both the front month and second month contracts for strategies ST3, ST4, and ST5, which are the strategies that our theoretical analysis predicts will be used by strategic traders. Among these, only the coefficient on ST3 is statistically significant. ST3 involves trading in the same direction as the preannounced ETF trades prior to the roll, and trading against the ETF both during and after the roll. Our analysis in Section III implies that ST3 will be the most profitable strategy when trades have some permanent price impact. Other than ST3, the only statistically significant coefficients for the Roll indicator reported in Table 4 Panel B are for ST1, where the coefficient estimate is negative for the both front and second month contracts. This implies a shift away from ST1 during the roll period. This result is also consistent with our analysis, as it implies that ST1 would not be used by strategic traders.

On balance, this analysis of individual account trading activity provides evidence that at least some accounts shift toward strategic trading in the days surrounding the USO roll, but that the strategies used involve the provision of liquidity on the roll day itself, in combination with "frontrunning" trades prior to the USO roll. We fail to find evidence of increased use of strategies that involve trading in the same direction as USO on the roll day itself.

VII. USO's Trade Execution Costs

The analysis reported here provides little evidence that strategic traders engage in predatory strategies around USO rolls and instead documents enhanced liquidity on roll days. However, USO

²² Note that results for strategies ST7 to ST12 would simply be the opposite of results for ST1 to ST6, since they are the complements of the first six strategies.

demands a large amount of liquidity in its rolls, and liquidity suppliers are likely to be compensated in equilibrium for meeting USO's demand for liquidity.

To assess this issue, we compute three daily time series of futures returns from USO's inception date on April 1, 2006 to December 31, 2013, with each futures return defined as the change since the prior day in the log of the futures price for a contract with a fixed maturity. The first series is comprised of returns to the front month contract, colored blue in the illustrative Figure 6, computed based on the current and prior day price of the front month contract, for all days including the last day of trading for the expiring contract. The second series is the return to the second nearest contract, colored red in Figure 6, computed based on the current and prior day price of the second month contract, for all days including the last day of trading for the expiring contract. The third, denoted "Benchmark Return" is based on settlement prices of the contracts that comprise USO's benchmark and that reflect its roll strategies, colored green in Figure 6. The Benchmark Return equals the return on the front month contract in the days up to and including the USO roll date, and equals the return on the second month contract after USO completes its roll.²³

Each mean return is negative during the USO period. Annualized by multiplying by 250, the mean Benchmark futures return is -8.79% per year. By comparison, the mean return to the front month contract is -5.63% per year and the mean return to the second month contract is -5.80% per year. The benchmark return is 3.16% per year less than the return to the front contract and is 2.99% less than the return to the second contract. The latter differential is statistically significant (t-statistic = 2.48). 24

USO's roll trades are executed at the relevant settlement prices due to the use of TAS contracts.

Trade execution costs arise in the form of adverse changes in settlement prices, i.e. increases in the price of the second contract and/or decreases in the price of the front contract, attributable to the roll. Such

28

²³ USO completed its roll on a single day through February 2009, after which it shifted to a four-day roll (see http://www.bloomberg.com/apps/news?pid=newsarchive&sid=aZYy1UXKZRb0). The Benchmark return is equal to the front month contract return on roll days through February 2009. Thereafter, the benchmark return is a weighted average of the return on the front month and second month contracts during the four days of the roll, with the weight on second month return equal to 0, 0.25, 0.50, and 0.75 on the four consecutive days of the roll.

²⁴ That the former is not significant reflects the greater volatility of the Return 1 series, which in turn reflects in part the high volatility in the settlement price of the expiring contract on the final trading day.

costs can potentially arise due to strategic trading, or alternatively because of imperfectly elastic liquidity supply. It is common to measure trade execution costs by comparing the price for a completed trade to either a pre-trade or post-trade benchmark price. In the case of a futures roll, the relevant price is the spread between the prices of the second month and the front month contracts.

Note that the second month return and the benchmark return are identical in the days after USO completes its roll. The statistically significant differential in the second month futures return relative to benchmark returns therefore reflects a widening of the calendar spread between the second- and the front month contract prices in the days before and during the USO roll, which averages 25 basis points per roll. This comprises an estimate of USO's monthly trading costs associated with the roll, analogous to the comparison of a trade price to a pre-trade benchmark.

Similarly, the excess of the front month return over the benchmark return reflects better performance of the front month contract relative to the second month contract, i.e. narrowing of the calendar spread, in the days after the completion of the USO roll. This differential of 3.16% per year equates to 26 basis points per roll, and analogous to estimating costs by comparing execution prices to post-trade benchmark prices, comprises an additional (though noisier) estimate of the roll cost.

To provide additional insights regarding roll trades and price impacts, Table 5 reports mean continuously compounded returns for the nearest to expiration and second nearest to expiration crude oil futures contracts, as well as the difference in mean returns across contracts, based on the number of trading days until the expiration of the near contract. Until February 2009 USO typically rolled its position ten trading days before expiration. Subsequent to that date it typically rolled its position evenly across seven to ten days before expiration.

Several results on Table 5 are noteworthy. First, the cumulative differential in the mean return on the 2nd nearest to expiration contract over that on the nearest to expiration contract is 18 basis points from twenty days before expiration to eleven days before expiration. This widening of the basis between the second and nearest to expiration contract prices is consistent with the interpretation that some traders were "front running" the USO roll. Second, the return differential is an additional 15 basis points from

ten to seven days before expiration, consistent with the reasoning that the USO roll itself impacted prices. Perhaps more surprisingly, the return differential on the fifth and sixth days before expiration (after the USO roll is completed) is 11 basis points, which increases the accumulated return differential since twenty days prior to expiration to a maximum of 44 basis points. This continued widening of the basis between the 2nd nearest and nearest futures prices likely reflects price pressure associated with the Goldman index roll, as described in Section IV above. Finally, during the last five days before expiration the return to the nearest to expiration contract exceeds that on the second nearest contract by 44 basis points, so that the accumulated return differential from twenty days before expiration through the expiration date is zero. This reversal of the earlier widening of the basis is consistent with the reasoning that the price impacts of the roll trades are fully temporary, reflecting that the roll trades are informationless.

Returning to our estimates of the costs incurred by USO, are trading costs of about 25 basis points per month, or about 3% per year, excessive? By comparison, Marshall, Nguyen, and Visaltanachoti (2011, Table 1) report that median effective bid-ask spreads in the NYMEX crude oil market are about 11 basis points. However, each effective bid-ask spread is computed based on a single trade, and USO's ETF rolls are far too large to execute with a single order. As a second point of comparison, Lou, Yan, and Zhang (2013) report that prices of U.S. Treasury securities, which trade in what many consider to be the most liquid financial market in the world, decrease by about nine to 18 basis points in the days ahead of Treasury auctions.

The ETF roll trades we study are large – the monthly roll strategy implies that the ETF turns over its entire portfolio twelve times per year. The individual rolls can be as large (Table 1) as 17% of average daily volume for the front month and 28% for the second month, and often exceed 100% of market volume during the two minute settlement period whose price they seek to match. We view these estimated trade execution costs as moderate in light of the sharp demand for liquidity associated with the monthly roll strategy. Whether the monthly roll strategy itself is superior to alternative methods of

generating returns that track crude oil prices, such as holding physical inventory or using longer-dated futures contracts, is left for future research.²⁵

Finally, it is worth noting that the trade execution costs we estimate, which average about three percent per year, are not sufficient to explain the underperformance of the USO share price relative to the level of crude oil futures prices, as demonstrated on Figure 1. In the Appendix to this paper we demonstrate that the remaining apparent underperformance can be attributed to crude oil storage costs, which have been high on average in recent years.

VIII. Conclusions

This study contributes to our understanding of the economic issues related to the execution of large predictable trades, and also provides estimates of the resiliency of the crude oil markets and of trading costs for a large trader in crude oil futures. More specifically, we study trading strategies, liquidity, and price patterns around the time of large and predictable monthly trades undertaken by the US Oil ETF, which is designed to provide returns that track changes in crude oil futures prices.

USO demands a large amount of liquidity. Aggregated across the twelve roll dates in our sample period, net roll activity comprises approximately 5% of roll-day volume in the front contract and ten percent of roll-day volume in the second contract. Further, USO typically seeks to trade at the daily settlement price, which is established in a two-minute period, and their roll trades on average exceed market wide volume during this interval. We view the large and predictable liquidity demand associated with the USO roll to comprise an ideal experiment for assessing the relevance of two theories: predatory and sunshine trading, in a setting where both potentially apply.

In addition to presenting empirical evidence, we develop a simple model that extends prior work to consider the implications of strategic trading in a resilient market. The key insight is that strategic trading, even by a monopolist strategic trader, necessarily causes prices to overshoot and reduces

²⁵ In unreported results, we observe that longer dated 3-month, 6-month and 12-month crude oil contracts have significantly lower liquidity (wider bid-ask spreads and lower book depth) and trading activity, as compared with the front month and the second month contracts.

liquidator proceeds *only* when trades' price impacts are permanent or long-lasting, and when the number of strategic traders is limited. When markets are resilient, the model predicts that even a monopolist strategic trader will choose to act as a liquidity provider, absorbing a portion of the liquidator's order imbalance on roll day, while offloading the resulting inventory in periods before or after the roll day. Further, for resiliency parameters in line with those estimated from the data, the liquidator's proceeds are *larger* when the monopolist strategic trader is present than when the strategic trader is absent. Further, competition among strategic traders implies that liquidator proceeds are improved by strategic trading in all cases *except* when price impacts are strictly permanent. The extended model is relevant to crude oil ETF rolls, as well as other settings where traders need to make large predictable transactions, such as portfolio rebalancing trades associated with index reconstitution.

In light of the increasing popularity of ETFs in retail and institutional portfolios, regulators are interested in better understanding the impact of ETF activity on market quality. Leveraged ETFs and ETFs that invest in futures contracts pose special challenges because their replication strategies involve frequent large trades. With respect to USO's roll, we find little evidence that strategic traders engage in predatory trading that impairs price discovery or destabilizes the futures market.

We estimate that USO effectively pays about 25 basis points on average to complete its roll trades, which is not much larger than price impacts observed in other liquid markets, including U.S. treasuries (Lou, Yan, and Zhang, 2013). Observing moderate trading costs despite the ETF's large and concentrated demand for liquidity reflect the resiliency of the crude oil futures market, and the effectiveness of the "sunshine trading" strategy where preannouncement attracts liquidity suppliers, including strategic traders, as well as natural counterparties. Still, accumulated trading costs of 3% per year are substantial. These costs arise from the large amount of liquidity demanded by a monthly roll strategy, and highlight the importance of developing innovative strategies to match benchmark returns.

On balance these results are consistent with the theoretical analysis presented here implying that strategic traders have incentives to trade in a benign manner in a resilient market, and are also consistent with the elements of both the sunshine trading theory of Admati and Pfleiderer (1991) and the predatory

trading theory of Brunnermeier and Pedersen. The evidence of improved liquidity and greater resiliency on roll days, as well as the observation that USO announces roll dates on its web site, are strongly consistent with the sunshine trading interpretation. At the same time, the fact that USO completes its roll trades through TAS contracts implies that market participants will not in general know the exact timing of the TAS counterparties offsetting trades. As a consequence it would be difficult for a would-be predator to "front run" the USO roll trades by very short time intervals.

Our model does not imply that predatory trading is never a concern. The two key practical issues are the degree of competition among potential strategic traders and the proportion of the price impact of strategic trades that persists into the period when the predictable trades are executed. This proportion will be higher, and strategic trading more likely to be harmful, when trades are motivated by private information, when there is only one or a few strategic traders, if trades are executed in less resilient markets where price impacts are closer to permanent, or if strategic traders are able to execute their trades in very close time proximity to those of the predictable trades.

Appendix: Explaining Crude Oil ETF Stock Price Performance – The Role of Storage Costs

In this paper, we present evidence that liquidity is improved on USO roll days, that there is little evidence of predatory trading around USO rolls, and that the trade execution costs incurred by USO cannot explain the performance of the USO share price relative to the level of crude oil prices, as documented in Figure 1. What then does explain the divergence?

a. Ex post Return Premia and Storage Costs

We assess this issue by relying on the well-known "cost-of-carry" no-arbitrage relation, stated as:

$$F_t(m) = P_t e^{S_t m} \,, \tag{A1}$$

where P_t denotes the date t spot price, F_t(m) denote the date t futures price for delivery at date t+m, and S_t is the marginal investor's continuously compounded per-period cost of carrying inventory, including forgone interest and other storage costs. Non-interest storage costs include costs of renting storage tanks, insurance, etc., and may at times be offset in part or full by "convenience yields" that reflect the option value of holding inventory. Applying (A1) to futures contracts for delivery at dates t+m and t+n, the per-period cost of carrying inventory can be inferred as:

$$S_t = \frac{\ln(F_t(m)/F_t(n))}{(m-n)} \tag{A2}$$

which implies that the marginal cost of carrying inventory is revealed by the slope of the futures term structure. Using expression (A2) with the daily EIA data, we compute the cost of storage implied by the settlement prices of the first and second nearest-to-expiration crude oil contracts for each trading day from January 1, 1999 to December 31, 2013, and report the results on Table A1, for the full sample and for subsamples. For the full sample the mean implied storage cost (multiplied by 250 to convert to an annual equivalent) is 0.49%. In contrast, during the post-USO subsample the mean implied storage cost was 12.38%. A positive term slope, whereby futures prices rise for more distant delivery dates,

characterizes what practitioners refer to as a "contango" market. The cost-of-carry relation implies that contango will be observed only when net storage costs for the marginal holder of inventory are positive.²⁶

Define an ex post spot return premium as:

$$U_{t+1} = \ln \left[\frac{P_{t+1}}{P_t e^{S_t}} \right]. \tag{A3}$$

The denominator of expression (A3) is the time t spot price adjusted for the cost of storing oil for one period, so U_{t+1} is interpreted as the return in excess of storage costs (analogous to the return in excess of the interest rate often studied in equity markets). We construct a daily time series of spot prices implied by expression (A1), relying on the nearest-to-expiration futures price and the previously computed daily storage cost estimates. From this series we compute the time series of realizations of U_t . Table A1 reports mean outcomes, annualized by multiplying by 250.

For the full sample, the spot return premium for crude oil is 5.46% per year. During the 1990s, the spot return premium was 4.81% per year. In the period before USO was launched, January 2000 through April 9, 2006, the spot return premium surged to 22.80% per year, potentially whetting investor demand for products linked to crude oil prices. In contrast, since USO's April 10, 2006 launch, the mean spot return premium was -7.64% per year. While none of these means are statistically significant (reflecting the high variability of price changes), the accumulated effect is nevertheless economically important. The negative spot return premium for the period when USO was active implies that the appreciation of spot oil prices during the sample period was considerably *less* than sufficient to compensate for the marginal cost of carrying inventory.

 $^{^{26}}$ Pirrong (2011) documents that the collapse in crude oil prices during the recent financial crisis was accompanied by large increases in physical crude oil inventories and in the marginal cost of carrying inventory. To see that the positive term slope represents marginal storage costs, recognize that S_t also represents the pre-storage-cost daily return to a strategy of purchasing crude oil for delivery at date t+n at price $F_t(n)$ and simultaneously selling the same oil for delivery at date t+m at price $F_t(m)$. Positive arbitrage profits are available if oil can be stored from date t+n to date t+m for a per-period cost less than S_t . Of course, the no-arbitrage condition applies to the *marginal* holder of inventory. Those who can store a commodity for lower cost can earn profits. Anecdotal accounts (e.g. http://blogs.reuters.com/great-debate/2010/07/22/contango-and-the-real-cost-of-carry/) indicate entry by non-traditional firms (e.g. hedge funds) into the oil storage business in recent periods.

Applying expression (A1) at dates t and t+1 and denoting $\Delta S = S_{t+1} - S_t$, the one-period return to a long position in a given futures contract can be expressed as:

$$\ln\left[\frac{F_{t+1}(m-1)}{F_t(m)}\right] = U_t + (m-1)\Delta S, \tag{A4}$$

while, by comparison, the continuously compounded growth in the spot price can be written as:

$$\ln\left[\frac{P_{t+1}}{P_t}\right] = U_t + S_t \tag{A5}$$

Comparing expressions (A4) and (A5) yields several insights. First, for a given cost of carry (ΔS = 0), the rate of appreciation in the spot price exceeds that of the futures price by S_t , the cost of carrying inventory. Stated alternately, spot price appreciation will exceed changes in prices of individual futures contracts in contango markets, and vice versa in "backwardated" markets (where the implied cost of carry is negative, presumably due to large "convenience yields"). As noted, the marginal cost of carry was large and significant (12.4% per year) during the USO sample period, implying underperformance of long futures positions relative to spot price changes. Second, the futures return does depend on ΔS . Third, both futures and spot returns are equally affected by U_t, the ex post premium in the spot price. Finally, the cost-of-carry itself has no direct implication for futures returns, as S_t does not appear in expression $(A4).^{27}$

This final insight contrasts with what appears to be a rather wide-spread myth among both academics and the financial press that a roll trade generates an immediate gain or loss attributable to the divergence of the near-to-delivery futures price from that of the more distant delivery price.²⁸ The

²⁷ However, the expression does not rule out covariation between the cost of carry and futures returns, which has in fact been documented in a number of commodity markets. See, for example, Liu and Tang (2010), Szymanowska, De Roon, Nijman, and Van den Goorbergh (2014), and the papers referenced therein. The data reported in Table A1 are consistent with negative covariation, in that futures returns are positive and the cost of storage negative during the January 2000 to April 2006 period, while futures returns are negative and the cost of storage positive during the April 2006 to December 2013 period.

28 For example, Mou (2011) claims (page 13) that the excess of the front month futures price over the more

distant-delivery futures price "is the amount of gain (or loss) per unit of the commodity when rolling futures forward" and asserts (page 2) that futures investors earn a "return called 'roll yield', which refers to the difference between log price of the maturing contract they roll from and the deferred contract they roll into." Similarly, the Wall Street Journal recently claimed ("Winning by Waiting in Commodities: Investors Cash In With a Commodities Trading Strategy," July 15, 2014) that "A fund manager buys a futures contract for delivery next month. Right

confusion seems to arise from a misconception that traders pay or receive the futures price when they transact in futures contracts. In practice, cash flows to or from margin accounts depend only on same-contract price changes. With regard to roll trades in particular, if the roll is completed on date t at the settlement prices, then the date t cash flow depends on the change from date t-1 to date t in the settlement price for the expiring contract, while the date t+1 cash flow depends on the change from date t to date t+1 in the settlement price for the second-nearest-to-expiration contract.

We summarize this analysis as follows. The mean annualized return (daily mean x250) to the USO ETF during the April 10, 2006 to December 31, 2013 period was -8.42%, while the mean annualized rate of change in implied spot prices during the same period was 4.74% (Table A1). This gives rise to the perception, reinforced by Figure 1, that the USO ETF performed very poorly. The perceived underperformance has been attributed to predatory trading or to the effect of "contango" on futures prices. We show that neither of these explanations is particularly relevant.

The most important factor is storage costs, which for the marginal investor averaged 12.38% per year during this period. The actual post-storage-cost return to the marginal investor holding spot crude oil was therefore -7.64% per year. Crude oil futures markets as a whole performed somewhat better, as a long position rolled so as to always remain invested in the second (front) month contract earned -5.80% (-5.63%) per year. The USO benchmark, which reflected the price impacts of the actual USO roll strategies, performed worse, delivering -8.79% per year. We attribute the underperformance of the USO benchmark relative to hypothetical long futures strategies invested in the front or second month contract to trade execution costs that average about 25 basis points per roll. Finally, the actual USO ETF return of -8.42% per year slightly exceeds the benchmark futures return of -8.79%, reflecting the net effect of interest on cash balances versus management fees. We conclude that USO's stock performance can be well explained by the combination of crude oil storage costs and moderately large trade execution costs.

before it expires, the investor sells the contract, buys a cheaper one for delivery at a later date and pockets the difference."

37

References

Admati, A.R., and P. Pfleiderer, 1988, A Theory of Intraday Patterns: Volume and Price Variability, *Review of Financial Studies*, Vol 1 (1), 3-40.

Admati, A.R., and P. Pfleiderer, 1991, Sunshine trading and financial market equilibrium, *Review of Financial Studies*, Vol 4 (3), 443-481.

Boyd, N.E., Harris, J.H., and A. Nowak, 2011, The role of speculators during times of financial distress, *The Journal of Alternative Investments*, Summer 2011, 1-16.

Brunnermeier, M., and L.H. Pedersen, 2005, Predatory trading, *Journal of Finance*, Vol 60 (4), 1825-1863.

Buyuksahin, B., and J.H. Harris, 2011, Do speculators drive crude oil prices?, *The Energy Journal*, 32, 167-202.

Carlin, B., Lobo, M.S. and S. Viswanathan, 2007, Episodic liquidity crises: Cooperative and predatory trading, *Journal of Finance*, 62 (5), 2235-2270.

Erb, C. and C. Harvey, 2006, The strategic and tactical value of commodity futures, *Financial Analysts Journal*, 62, 69-97.

Foucault, T., O. Kadan, and E. Kandel, 2005, Limit order book as a market for liquidity, *Review of Financial Studies*, Vol 18 (4), 1171-1217.

Henderson, B., N. Pearson, and L. Wang, 2015, New evidence on the financialization of commodity markets, Review of Financial Studies, Vol 28(5) 1285-1311.

Honghui, C., Noronha, G., and V. Singal, 2004, The Price Response to S&P 500 Index Additions and Deletions: Evidence of Asymmetry and a New Explanation, *Journal of Finance*, Vol. 59(4), 1901-1929.

Huang, R., and H. Stoll, 1997, The components of the bid-ask spread: a general approach, *Review of Financial Studies*, 10, 995-1034.

Irwin, S., and D. Sanders, 2012, Testing the Masters Hypothesis in commodity futures markets, *Energy Economics*, 34, 256-269.

Kilian, L. 2009, Not all oil price shocks are alike: Disentangling demand and supply shocks in the crude oil market, *American Economic Review* 99, 1053-1069.

Lou, D., H. Yan, and J. Zhang, 2013, Anticipated and repeated shocks in liquid markets, Review of Financial Studies, Vol 26(8) 1891-1912.

Madhavan, A., 2003, The Russell Reconstitution Effect, Financial Analyst Journal, Vol. 59(4), 51-64.

Manaster, S., and S.C. Mann, 1996, Life in the Pits: Competitive Market Making and Inventory Control, *Review of Financial Studies*, Vol. 9(3), 953-975.

Marshall, B.R., Nguyen, N.H., and N. Visaltanachoti, 2012, Commodity liquidity measurement and transaction costs, *Review of Financial Studies*, Vol. 25 (2), 599-638.

Mou, Y., 2011, Limits to Arbitrage and Commodity Index Investment: Front-running the Goldman Roll, working paper, Columbia University.

Pirrong, C., 2010, An evaluation of the performance of oil price benchmarks during the financial crisis, working paper, University of Houston.

Schoneborn, T., and A. Schied, 2009, Liquidity in the face of adversity: Stealth vs. sunshine trading, predatory trading vs. liquidity provision, working paper, Technical University Berlin.

Stoll, H., and R.E. Whaley, 2010, Commodity Index Investing and Commodity Future Prices, *Journal of Applied Finance*, Issue 1, 1-40.

Szymanowska, M., F. De Roon, T. Nijman, and R. Van den Goorbergh, 2014, An Anatomy of Commodity Futures Risk Premia, *Journal of Finance*, 69, 453-482.

Table 1. United States Oil (USO) and Crude Oil Futures Market Trading Activity on Roll Days

Reported are trading volumes (in contracts) in the NYMEX Crude Oil futures market for the full trading day, and during the two-minute settlement period, on USO roll days from March 2008 to February 2009. Also reported are the estimated sell and buy volumes attributable to USO's roll trading on these days. We rely on the Commodity Futures Trading Commission (CFTC) dataset to calculate the market-wide trading volumes for the full day and settlement periods. The CFTC dataset includes all completed trades in NYMEX crude oil futures, including floor and block trades, as well as trades completed on the GLOBEX electronic market. USO roll trading volume is estimated on the basis of USO's Total Net Assets (TNA) on the roll date relative to front and second month settlement prices on the roll day. Settlement prices are obtained from the Energy Information Agency (EIA). TNA values were provided by ALPS on behalf of USO. Roll dates are two weeks prior to the expiration of the nearest-delivery contract. A calendar schedule of USO's recent and future roll dates is available on the website: http://www.unitedstatescommodityfunds.com/fund-details.php?fund=uso&pagetype=roll-dates&page=fund-details.

_		Front Contract o	Second Contract on Roll Date					
Roll date	ETF Selling Activity (contracts)	Market Trading Volume (contracts)	ETF %	Market Trading Volume During Settlement	ETF Buying Activity (contracts)	Market Trading Volume (contracts)	ETF %	Market Trading Volume During Settlement
3/5/08	4,445	414,308	1%	16,756	4,480	205,827	2%	10,449
4/8/08	5,524	307,800	2%	16,338	5,557	165,544	3%	15,775
5/6/08	4,995	331,913	2%	11,933	5,015	129,110	4%	6,632
6/6/08	8,582	508,749	2%	18,139	8,572	231,984	4%	11,112
7/8/08	7,205	382,404	2%	15,378	7,169	154,453	5%	13,299
8/6/08	6,289	307,994	2%	16,189	6,296	140,471	4%	13,489
9/8/08	11,960	317,923	4%	18,581	11,929	142,644	8%	14,791
10/7/08	9,116	342,917	3%	21,235	9,254	193,234	5%	15,414
11/6/08	13,025	292,018	4%	6,756	12,877	87,869	15%	3,578
12/5/08	23,723	327,140	7%	27,508	22,552	157,572	14%	22,765
1/6/09	55,688	331,307	17%	9,145	50,919	183,802	28%	7,659
2/6/09	85,055	518,382	16%	32,674	74,033	318,960	23%	29,187
Sum	235,607	4,382,855	5%	210,632	218,653	2,111,470	10%	164,150

Table 2. Regression estimates of Permanent and Temporary Price Impact and the Resiliency of the Crude Oil Futures Market

Reported are estimates of the permanent price impact (λ), the temporary price impact (γ) and the resiliency of the market (θ) in the NYMEX crude oil futures contracts for the full sample, and separately on USO roll and non-roll days. Non-roll days are defined as Days [-7,-3] before the earliest of the USO roll or GSCI Index roll days in the month. The analysis relies on trades and limit order book data from Chicago Mercantile Exchange's Datamine database on GLOBEX electronic market. USO's and GSCI Index roll dates are identified based on their publicly stated roll strategy. We estimate these parameters with geometric lag regressions of the form:

$$P_t - P_{t-1} = \beta_0 q_t + \beta_1 q_{t-1} + \beta_2 q_{t-2} + \beta_3 q_{t-3} + \dots + \beta_t q_0 + \gamma_0 [D_t - D_{t-1}]$$
 (14)

where P_t is the time t trade price, q_j is the signed order imbalance at time j, and the coefficients are restricted such that: $\beta_1 = \gamma(\theta-1)$, $\beta_2 = \beta_1\theta$, $\beta_3 = \beta_2\theta^2$, $\beta_4 = \beta_3\theta^3$, and so on. The number of lags (j) is set to 30. To allow for asymmetry permanent price impact of buy and sell imbalance, we estimate $\beta_0 = [(\lambda_B s_t + \lambda_B (1 - s_t)) + \gamma]$ where s_t equals 1 if s_t qt > 0, and equals 0 otherwise.

Panel A presents the regression coefficients (with p-values in parenthesis) for the front month contract and Panel B presents the results for the second month contract. Results are based on estimating equation (14) by aggregating the net order imbalance (excess of buyer-initiated over seller-initiated trades, measured in contracts) on a ten-second window. For time periods with multiple trades, P_t is measured as the last trade in the ten second window and the binary variable D_t receives a value of (+1) when the last trade is buyer-initiated and (-1) when the last trade is seller initiated. The geometric lag expression (12) is estimated by Generalized Method of Moments (GMM), using SAS Proc Model with a Bartlett Kernel set equal to the lag length plus one. Reported are Hansen's J statistic test (and the corresponding p-value) of the null hypothesis that the over identifying restrictions of the model are valid.

Table 2 (continued)

	Number of Observations	Gamma (γ)	Theta (θ)	Gamma0 (γ ₀)	L	ambda-Sell (λ _s)	La	mbda-Buy (λ _Β)	R ²	Hansen's J Statistic
Panel A: Front Contract										
Full sample	137613	0.019 (0.00)	0.667 (0.00)	0.577 (0.00)		0.040 (0.00)		0.043 (0.00)	41.41%	(0.00)
Non-Roll Days		0.019 (0.00)	0.662 (0.00)	0.580 (0.00)		0.041 (0.00)		0.045 (0.00)	41.48%	(0.00)
Roll Days		0.017 (0.00)	0.494 (0.00)	0.553 (0.00)	,	0.040 (0.00)	•	0.042 (0.00)		
Difference p-value		-0.002 (0.23)	-0.168 (0.00)	-0.027 (0.19)	•	-0.001 (0.57)	r	-0.003 (0.31)		
Panel B: Second Contract	:									
Full sample	114508	0.057 (0.00)	0.946 (0.00)	0.541 (0.00)	,	0.067 (0.00)	•	0.065 (0.00)	11.33%	(0.62)
Non-Roll Days		0.054 (0.00)	0.935 (0.00)	0.538 (0.00)	,	0.074 (0.00)	•	0.076 (0.00)	11.39%	(0.32)
Roll Days		0.050 (0.00)	0.940 (0.00)	0.538 (0.00)		0.064 (0.00)	•	0.054 (0.00)		
Difference p-value		-0.005 (0.82)	0.004 (0.91)	0.001 (0.98)	,	-0.010 (0.67)	r	-0.022 (0.32)		

Table 3. Average Market Quality Measures on USO Roll and Non-roll days

Reported are market quality measures on roll days and non-roll days in the NYMEX Oil Futures market. Roll dates are defined as the earliest of either the Goldman roll or the USO roll based on their publicly stated roll strategy during the period March 1, 2008 to February 28, 2009. Non-roll days are defined as Days [-7,-3] before the roll day. Market quality is calculated each minute of the day and then averaged across roll and non-roll days. We rely on Commodity Futures Trading Commission (CFTC) data for calculating the number of liquidity providing accounts. Trading volume includes all trades completed on GLOBEX. Trade imbalance is the signed difference between buyer and seller initiated volume standardized by subtracting the mean and dividing by the standard deviation of imbalance during the same minute (across roll and non-roll days). Quoted bid-ask spread (in basis points) is the difference between the lowest limit price for unexecuted sell orders and the highest limit price for unexecuted buy orders. Depth is the total volume of unexecuted sell (ask depth) and buy (bid depth) orders at prices within four ticks of the most competitive prices. Effective spread (in basis points) for a buyer (seller) initiated trade is twice the excess of trade price (quote midpoint) over the quote midpoint (trade price). We report Wilcoxon signed rank t-statistics and p-values with the null hypothesis of zero difference in median.

	Roll P	eriod	Non-Ro	ll Period	Differenc	Wilcoxon Signed	
	Mean	Median	Mean	Median	e of	T-Stat	P-Value
A. Front Month							
Trading Volume per Minute (contracts)	486.6	473.2	402.3	386.5	84.32	10.13	(0.00)
Standardized Trade Imbalance	-0.0196	-0.0294	0.0150	0.0209	-0.03	-2.12	(0.02)
Effective Spread	1.95	1.80	2.03	1.96	-0.08	-4.11	(0.00)
Quoted Spread	1.13	1.12	1.17	1.15	-0.04	-4.84	(0.00)
Near-inside Bid Depth (contracts)	53	52	48	47	5.39	12.16	(0.00)
Near-inside Ask Depth (contracts)	50	49	45	45	4.49	12.54	(0.00)
Liquidity Supplying accounts (N) *	10,470	10,541	9,698	9,787	772	0.82	(0.41)
B. Second Month							
Trading Volume per Minute (contracts)	69.1	62.4	43.1	40.0	26.07	14.69	(0.00)
Standardized Trade Imbalance	-0.0119	0.0008	0.0106	0.0086	-0.02	-1.33	(0.09)
Effective Spread	2.27	2.03	2.42	2.29	-0.16	-5.22	(0.00)
Quoted Spread	1.40	1.37	1.51	1.47	-0.11	-8.17	(0.00)
Near-inside Bid Depth (contracts)	24	24	22	21	2.76	11.17	(0.00)
Near-inside Ask Depth (contracts)	21	20	19	18	2.13	9.96	(0.00)
Liquidity Supplying accounts (N) #	1,416	1,198	860	835	556	3	(0.00)

^{#-} non-roll period is based on Days[-5,-2] relative to roll date due to CFTC data availability contraint

Table 4: Strategic Trading Surrounding the USO Roll

Reported in Panel A are patterns associated with twelve strategic trading strategies associated with the USO roll. To be identified as a strategic trader, the absolute value of net change in the (non-ETF) account's inventory to the account's total activity surrounding the roll must be less than 0.25. The "During" period is defined as between 9 A.M. and 5 P.M. EST on the USO roll day, the "Before" period is defined from Midnight on Day -3 (three trading days prior) to 9 A.M. on roll day, and the "After" period is defined from 5 P.M. on roll day to Midnight on Day +3 (three trading days after) relative to the roll day. A strategic trader whose signed position change on roll day moves *against* USO's inventory change is deemed a liquidity provider (Strategies ST1 to ST5) while a strategic trader whose signed position change on roll day moves *with* USO's inventory change is deemed a predatory trader (Strategies ST8 to ST12). Categories ST6 and ST7 correspond to trading patterns with no trading activity on the roll day. Strategic traders are further classified into one of five sub-strategies within liquidity provision (ST1-ST5) and predatory trading (ST8-ST12) based on the account's change in net positions in the Before and After period. Also identified below is the complementary strategy where strategic traders pursue an opposite trading pattern surrounding the USO roll. Panel A reports the direction of USO activity and those for each strategy for expiring (front) contract and next-to-expiring (second) contract on USO roll days.

Panel A: Direction of ETF and Strategic Trading Surrounding the USO Roll

Strategy	Trading Pattern (relative to ETF)			Fro	Front month			ond mor	Complement strategy	
	Before	During	After	Before	During	After	Before	During	After	
ETF*				none	sell	none	none	buy	none	
ST 1	against	against	with	buy	buy	sell	sell	sell	buy	ST 12
ST 2	none	against	with	none	buy	sell	none	sell	buy	ST 11
ST 3	with	against	against	sell	buy	buy	buy	sell	sell	ST 10
ST 4	with	against	none	sell	buy	none	buy	sell	none	ST 9
ST 5	with	against	with	sell	buy	sell	buy	sell	buy	ST 8
ST 6	against	none	with	buy	none	sell	sell	none	buy	ST 7
ST 7	with	none	against	sell	none	buy	buy	none	sell	ST 6
ST 8	against	with	against	buy	sell	buy	sell	buy	sell	ST 5
ST 9	against	with	none	buy	sell	none	sell	buy	none	ST 4
ST 10	against	with	with	buy	sell	sell	sell	buy	buy	ST 3
ST 11	none	with	against	none	sell	buy	none	buy	sell	ST 2
ST 12	with	with	against	sell	sell	buy	buy	buy	sell	ST 1

Table 4, Panel B presents regressions coefficients of strategic trading surrounding the ETF roll. The definition of 'Before'', "During" and "After" periods are described in the legend to Panel A. We identify an account as potentially being a strategic trader and retain it in the analysis if the absolute value of the net change in the account's inventory as a fraction of the account's total activity in the roll window (i.e., Days [-3,+3] surrounding the roll) is less than 0.25.

For each account, we measure the volume of "strategic trading" during the relevant seven day period as round trip volume completed, i.e. as the minimum of buy volume executed and sell volume executed. We then aggregate strategic trading volume across those accounts that were identified as following each of the twelve strategies identified in Panel A of Table 4. *Normalized strategic volume* is the strategic volume in a strategy less the strategic volume in the complementary strategy. To provide a basis for comparison, we also compute normalized strategic volume using the same methods and definitions, for all seven day periods that do not overlap with the seven day intervals that are centered on a roll day.

We report regression coefficients obtained when normalized strategic volume in categories ST1 to ST6 for all seven day periods is regressed on an indicator variable, *Roll*, which equals one for the seven day intervals that are centered on the USO roll day, and that equals zero for seven-day periods that do not overlap with the roll. Reported are the regression coefficients and the associated t-statistics in the front month and the second month crude oil futures contract.

Panel B: Normalized Strategic Volume Regressions											
	ST1	ST2	ST3	ST4	ST5	ST6					
B.1. Front Month Contract											
Intercept	-306	-52	-851	368	3	-244					
t-stat	(-0.86)	(-0.77)	(-2.14)	(4.09)	(0.01)	(-1.44)					
Roll Indicator	-2187	200	3178	219	9	-807					
t-stat	(-2.06)	(0.98)	(2.66)	(0.81)	(0.01)	(-1.60)					
B.2. Second Month Contract											
Intercept	-89	30	-396	-79	102	-83					
t-stat	(-0.42)	(0.43)	(-1.52)	(-0.87)	(0.59)	(-0.59)					
Roll Indicator	-1453	179	2028	67	2	56					
t-stat	(-2.30)	(0.88)	(2.60)	(0.25)	(0.00)	(0.13)					

Table 5: Crude Oil Futures Returns by Number of Days to Expiration of the Nearest Contract

The table reports the mean continuously compounded returns to the nearest (front month) and second Nearest to Expiration (second month) NYMEX Crude Oil Contract, by number of trading days to expiration of the nearest contract. Futures return is defined as the change since the prior day in the log of the futures price for a contract with a fixed maturity. Also reported are the differences in mean returns across contracts and T-statistics for the hypothesis that the mean difference is zero. The sample is based on daily settlement prices from April 1, 2006 to December 31, 2013.

Crude Oil Fu	itures Returns by Da	ys to Contract Expiration	on		
	Mean Log	Return (%)			
Days to Expiration	Nearest to Expiry (Front Month)	Second to Nearest (Second Month)	Difference	T-statistic	Cumulative Difference (%)
20	-0.56	-0.56	0.00	-0.02	0.00
19	0.36	0.39	0.03	1.13	0.03
18	-0.02	-0.02	0.00	-0.01	0.03
17	0.44	0.46	0.02	0.56	0.05
16	-0.02	0.02	0.04	2.07	0.09
15	0.06	0.08	0.02	0.96	0.11
14	0.06	0.09	0.03	1.20	0.14
13	-0.12	-0.09	0.03	1.08	0.17
12	0.38	0.34	-0.03	-1.29	0.14
11	-0.47	-0.42	0.05	1.92	0.18
10	-0.12	-0.09	0.03	1.20	0.21
9	0.21	0.27	0.06	1.14	0.28
8	-0.27	-0.23	0.04	1.54	0.32
7	-0.09	-0.09	0.01	0.30	0.33
6	-0.10	-0.08	0.02	0.62	0.35
5	-0.22	-0.13	0.10	1.50	0.44
4	-0.12	-0.26	-0.13	-1.15	0.31
3	-0.12	-0.16	-0.05	-1.62	0.27
2	-0.06	-0.05	0.01	0.18	0.28
1	0.08	-0.01	-0.09	-0.86	0.19
0	-0.07	-0.27	-0.19	-1.09	0.00

Table A1: Understanding the Performance of USO ETF's Stock Price

The table reports the performance of various futures benchmarks, based on daily data from January 1990 to December 2013. All data except USO ETF share prices are obtained from the United States Energy Information Agency (EIA). Each mean has been annualized by multiplying by 250. The cost of storage is the futures term slope implied by the settlement prices of the nearest (front month) and second nearest (second month) crude oil futures contracts. The spot price return is the change in the implied (by the nearest futures price and the cost of carry relation) spot price, while the ex-post Spot Premium is the excess of the spot return over the cost of storage. The front month contract return is based on price changes in the nearest-to-expiration contract, for all days including the last day of trading for the expiring contract. The second month return is based on price changes in the second-nearest-to-expiration contract, for all days including the last day of trading for the expiring contract. The Futures "Benchmark Return" series is based on the settlement price changes that track the USO roll. The divergence of benchmark returns from returns based on alternative roll dates provide estimates of the cost of executing trades at USO benchmark prices.

	ETF Period 4/10/06 to 12/31/13 1946		2000s, pre ETF 1/1/00 to 4/9/06 1564		1990s 1/1/90 to 12		Full Sample 1/1/90 to 12/31/13		
Number of Days					2510		6020		
Variable	Mean (x250)	T-stat	Mean (x250)	T-stat	Mean (x250)	T-stat	Mean (x250)	T-stat	
Spot Price Return (S + U)	4.74%	0.33	15.11%	0.94	1.18%	0.09	5.95%	0.70	
Cost of Storage S	12.38%	22.22	-7.69%	-12.04	-3.63%	-6.46	0.49%	1.39	
Expost Spot Premium U	-7.64%	-0.53	22.80%	1.41	4.81%	0.35	5.46%	0.64	
Return, Front Month Contract	-5.63%	-0.43	26.02%	1.79	2.41%	0.21	5.95%	0.79	
Return, Second Month Contract	-5.80%	-0.47	25.37%	1.86	5.24%	0.52	6.90%	1.02	
Futures Benchmark Return	-8.79%	-0.70	20.80%	1.49	4.84%	0.45	4.59%	0.64	
Front Month - Benchmark	3.16%	1.02	5.22%	2.17	-2.43%	-0.90	1.36%	0.83	
Second Month - Benchmark	2.99%	2.48	4.57%	2.82	0.40%	0.15	2.31%	1.88	
USO ETF Return	-8.42%	-0.69							

Figure 1: United States Oil Fund (USO) Share Price and Front Month NYMEX Crude Oil Price

The figure presents daily USO share prices and front month NYMEX crude oil prices over the period April 12, 2006 to December 31, 2013. The USO share price is obtained from Bloomberg while the NYMEX crude oil price is obtained from the United States Energy Information Agency.

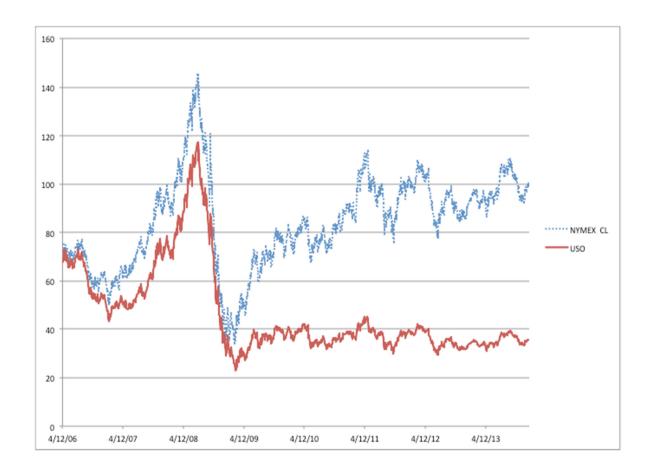
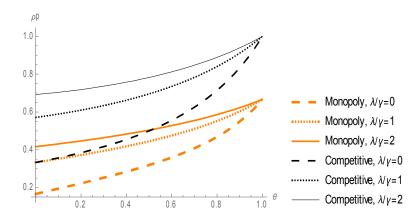


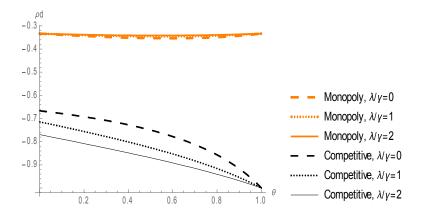
Figure 2: Theoretical Outcomes of Strategic Trading Around a Known Liquidation

The figure presents outcomes from a predictable liquidation accompanied by strategic trading based on the theoretical model presented in Section IIII. Panel A shows the quantities sold by strategic traders expressed as proportions of the quantity liquidated (ρ) in the periods prior to, during, and after the liquidation as a function of the market's resiliency (θ). Subpanels present the outcomes by period. Each curve specifies a ratio of permanent to temporary price impact (λ/γ) and whether strategic traders are monopolistic or competitive. Panel B shows the price in each period as a function of θ for various λ/γ ratios, either with monopolistic strategic trading or competitive strategic trading. The price received by the liquidator without strategic trading is also shown for comparison (during period without strategic trading). Subpanels present the outcomes for different levels of λ/γ . In each subpanel, the competitive case is presented as a single curve because prices are identical in all three periods. The results in Panel A are general and the results in Panel B are parameterized with an initial price of \$100, a liquidation quantity of 20 units, and a γ of .05.

Panel A.1: Quantities Traded Prior to Liquidation

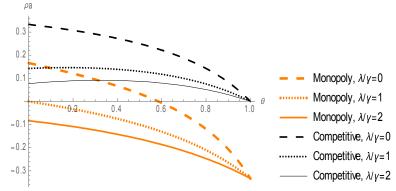


Panel A.2: Quantities Traded During Liquidation

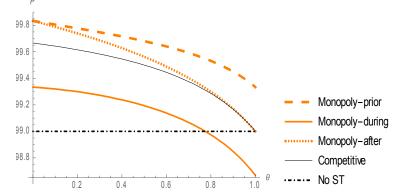


49

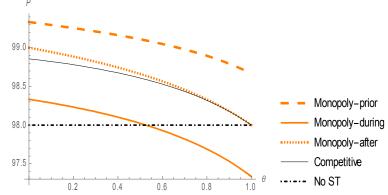
Panel A.3: Quantities Traded After Liquidation



Panel B.1: Prices Around Liquidation for $\lambda/\gamma=0$



Panel B.2: Prices Around Liquidation for $\lambda \gamma = 1$



Panel B.3: Prices Around Liquidation for $\lambda/\gamma=2$

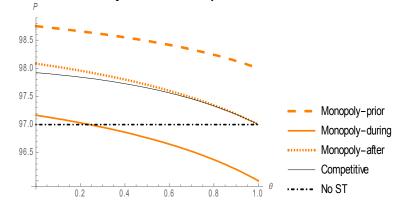


Figure 3: Breakeven Resilience with a Monopolist Strategic Trader

The figure plots the market resilience parameter that provides the liquidator with the same revenue when there is a monopolist strategic trader as when there is no strategic trading, for variations in the ratio of permanent price impact (λ) to temporary price impact(γ). A smaller resiliency parameter (a more resilient market) than the breakeven implies larger liquidator revenues with strategic trading than without, and vice versa. The shaded area below the curve indicates the region where strategic trading increases the liquidator's revenues.

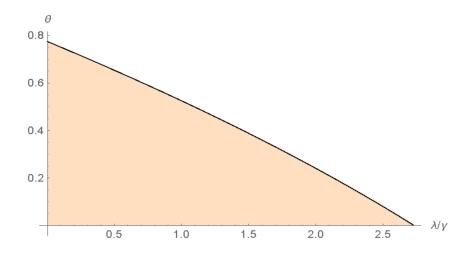
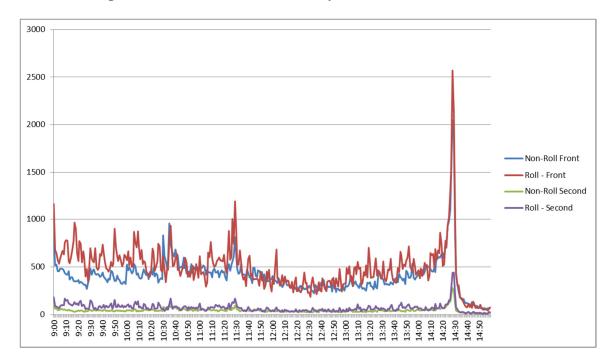


Figure 4: Trading Volume and Bid-Ask Spreads on USO Roll and Non-roll days.

Reported are intra-day patterns in trading volume (Panel A) and quoted and effective spreads (Panel B) on roll days and non-roll days, in the NYMEX Oil Futures market. Data is obtained from the Chicago Mercantile Exchange's Datamine database, which pertains to electronic trading on the GLOBEX system. Roll dates are defined as the earliest of either the Goldman roll or the publicly stated USO roll, during the period March 1, 2008 to February 28, 2009. Non-roll days are defined as Days [-7,-3] before the roll day. Market quality measures are calculated each minute of the day and then averaged across roll and non-roll days. Quoted bid-ask spread (in basis points) is the difference between the lowest limit price for unexecuted sell orders and the highest limit price for unexecuted buy orders. Effective spread (in basis points) for a buyer (seller) initiated trade is twice the excess of trade price (quote midpoint) over the quote midpoint (trade price).

Panel A: Trading volume on roll and non-roll days



52

Panel B: Front month contract – quoted and effective spread

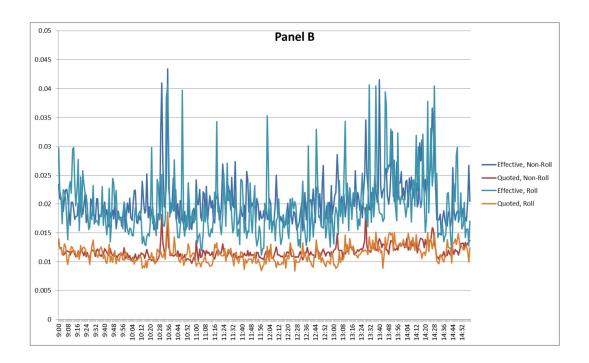
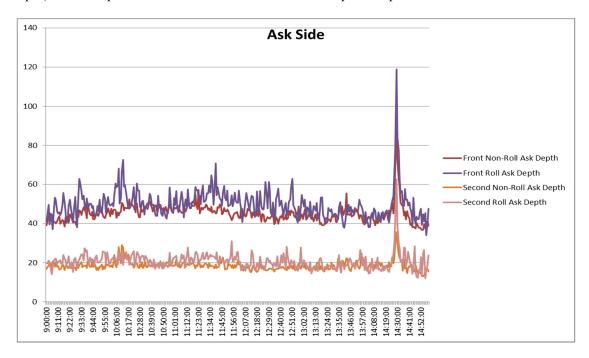
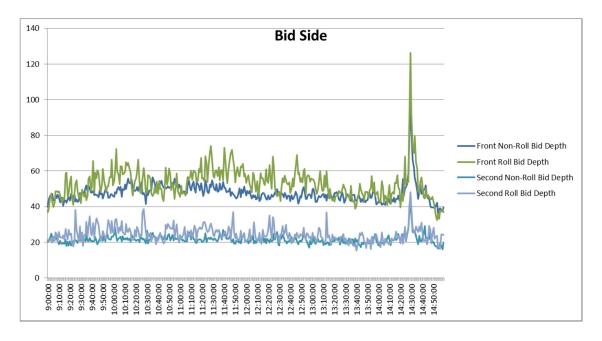


Figure 5: Intraday Limit Order Book Depth on USO Roll and Non-roll Days

Reported are limit order book depth on roll days and non-roll days in the NYMEX Oil Futures market. Data is obtained from the Chicago Mercantile Exchange's Datamine database, which pertains to electronic trading on the GLOBEX system. Roll dates are defined as the earliest of either the Goldman roll or the publicly stated USO roll, during the period March 1, 2008 to February 28, 2009. Non-roll days are defined as Days [-7,-3] before the roll day. Depth is calculated each minute of the day and then averaged across roll and non-roll days. Depth is the total volume of unexecuted sell (ask depth) and buy (bid depth) orders at prices within four ticks of the most competitive prices.

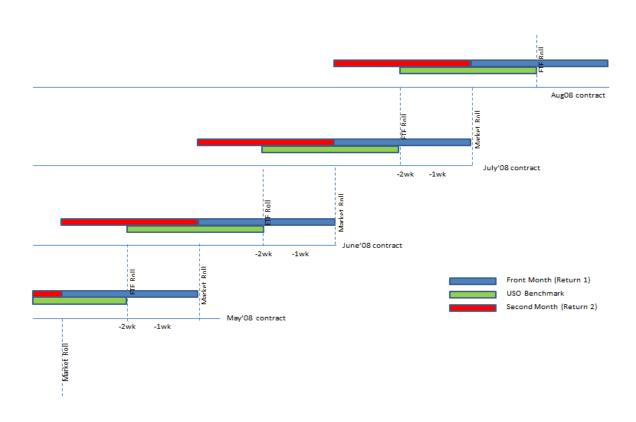




54

Figure 6: Identifying Front Month contract, Second Month contract, and the USO Roll Strategy

Trading activity in the futures market shifts from nearest-to-expiration (or front) contract to next-nearest-to-expiration (or second) contract a few days before the expiration of the front contract. We refer to the shift in overall trading activity as the Market Roll. USO's roll dates are identified based on their publicly stated investment objective, by which the fund tracks the price of the front NYMEX contract until two weeks before expiration, after which the fund tracks the second contract price. During the March 2008 to February 2009 sample period, USO's roll trades occurred on a single day in the month, approximately two weeks before the market roll.



Internet Appendix: Breakeven Resiliency Calculation

This appendix details the calculation of breakeven resiliency described in Section III and presented in Figure 2. Breakeven resiliency is the value of the resiliency parameter θ for which the liquidator's revenues will be the same with or without the presence of a monopolistic profit-maximizing strategic trader following the strategy described in Section III. The liquidator's revenues will be increased when resiliency is lower than this threshold, and decreased when it is higher.

The liquidator's revenues are given by expression (8), reproduced here:

$$LP = Q_L P_d \tag{IA1}$$

where Q_L is the quantity liquidated and P_d is the price during the liquidation period. The price during the liquidation period is given by expression (3), reproduced here:

$$P_D = V_0 + I_1 Q_p + I_0 Q_d (IA2)$$

where V_0 is the initial value of the security, Q_p is the net signed order flow in the "pre" period, Q_d is the net signed order flow in the "during" period, and I_0 and I_1 are transformations of the parameters of the price impact function that are described below. Without the presence of the strategic trader, $Q_p = 0$ and $Q_d = -Q_L$, giving:

$$P_{DN} = V_0 - I_0 Q_L \tag{IA3}$$

With the strategic trader, $Q_p = -\rho_p^* Q_L$ and $Q_d = -(1 + \rho_d^*) Q_L$, giving:

$$P_{DS} = V_0 - I_1 \rho_n^* Q_L - I_0 (1 + \rho_d^*) Q_L \tag{IA4}$$

where ρ_p^* and ρ_d^* are proportionality parameters that specify the strategic trader's profit maximizing order flow in the "pre" and "during" periods relative to the liquidation quantity. Setting $P_{DN} = P_{DS}$ yields:

$$V_0 - I_0 Q_L = V_0 - I_1 \rho_p^* Q_L - I_0 (1 + \rho_d^*) Q_L \tag{IA5}$$

Which simplifies to:

$$I_1 \rho_p^* + I_0 \rho_d^* = 0 \tag{IA6}$$

 I_1 , ρ_p^* , and ρ_d^* are all functions of the resiliency parameter θ . Further, ρ_p^* and ρ_d^* depend on I_0 and I_2 , and I_2 is also a function of θ . We use the definitions of I_0 , I_1 , and I_2 from expressions (2)-(4), and make the substitution $S = \frac{\lambda}{\nu}$ to yield:

$$I_0 = \lambda + \gamma = \gamma(S+1) \tag{IA7}$$

$$I_1 = \lambda + \theta \gamma = \gamma (S + \theta) \tag{IA8}$$

$$I_2 = \lambda + \theta^2 \gamma = \gamma (S + \theta^2) \tag{IA9}$$

We also repeat the solutions for ρ_d^* and ρ_p^* from expression (10), and use the same substitution as above to give:

$$\rho_d^* = \frac{\theta - (\lambda/\gamma) - 2}{\theta^2 - 4\theta + 3(\lambda/\gamma) + 6} = \frac{\theta - S - 2}{\theta^2 - 4\theta + 3S + 6}$$
(IA10)

$$\rho_p^* = -\frac{\theta^3 - 3\theta^2 - [(\lambda/\gamma) - 2]\theta + 2(\lambda/\gamma)^2 + 5(\lambda/\gamma) + 2}{\theta^4 - 4\theta^3 + [2(\lambda/\gamma) + 4]\theta^2 + [4(\lambda/\gamma) + 8]\theta - 3(\lambda/\gamma)^2 - 12(\lambda/\gamma) - 12}$$

$$= -\frac{\theta^3 - 3\theta^2 - (S - 2)\theta + 2S^2 + 5S + 2}{\theta^4 - 4\theta^3 + (2S + 4)\theta^2 + (4S + 8)\theta - 3S^2 - 12S - 12}$$
(IA11)

Substituting expressions (IA7) – (IA11) into expression (IA6) and simplifying yields the following condition for breakeven θ :

$$\frac{\theta^4 - 4\theta^3 + (S^2 - S + 4)\theta^2 + (2S^2 + 10S + 4)\theta + S^3 - 6S - 4}{-\theta^4 + 4\theta^3 - (2S + 4)\theta^2 - (4S + 8)\theta + 3S^2 + 12S + 12} = 0$$
(IA12)

This expression has four roots:

```
\left\{\left\{\Theta \to 1 - \frac{1}{2}\sqrt{\left(S - S^2 + \frac{1}{3}\left(4 - S + S^2\right) + \left(2^{1/3}\left(4 + 5S + S^2\right)^2\right)\right\}}\right\}
                                                                           \left(3 \left(560 + 2208 \text{ S} + 3180 \text{ S}^2 + 1894 \text{ S}^3 + 282 \text{ S}^4 - 78 \text{ S}^5 + 2 \text{ S}^6 + \sqrt{297216 + 2350080 \text{ S}} + 2828 \text{ S}^4 + 2828 \text{ S}^4 + 2828 \text{ S}^6 + \sqrt{297216 + 2350080 \text{ S}} + 28288 \text{ S}^4 + 28288 \text{ S}^4 + 28288 \text{ S}^4 + 28288 \text{ S}^6 + \sqrt{297216 + 2350080 \text{ S}} + 282888 \text{ S}^4 + 28288 \text{ S}^6 + \sqrt{297216 + 2350080 \text{ S}} + 282888 \text{ S}^6 + \sqrt{297216 + 2350080 \text{ S}} + 282888 \text{ S}^6 + \sqrt{297216 + 2350080 \text{ S}} + 282888 \text{ S}^6 + \sqrt{297216 + 2350080 \text{ S}} + 282888 \text{ S}^6 + \sqrt{297216 + 2350080 \text{ S}} + 282888 \text{ S}^6 + \sqrt{297216 + 2350080 \text{ S}} + 282888 \text{ S}^6 + \sqrt{297216 + 2350080 \text{ S}} + 282888 \text{ S}^6 + \sqrt{297216 + 2350080 \text{ S}} + 282888 \text{ S}^6 + \sqrt{297216 + 2350080 \text{ S}} + 2828888 \text{ S}^6 + \sqrt{297216 + 2350080 \text{ S}} + 2828888 \text{ S}^6 + \sqrt{297216 + 2350080 \text{ S}} + 2828888 \text{ S}^6 + \sqrt{297216 + 2350080 \text{ S}} + 2828888 \text{ S}^6 + \sqrt{297216 + 2350080 \text{ S}} + 2828888 \text{ S}^6 + \sqrt{297216 + 2350080 \text{ S}} + 2828888 \text{ S}^6 + \sqrt{297216 + 2350080 \text{ S}} + 2828888 \text{ S}^6 + \sqrt{297216 + 2350080 \text{ S}} + 2828888 \text{ S}^6 + \sqrt{297216 + 2350080 \text{ S}} + 28288888 \text{ S}^6 + \sqrt{297216 + 2350080 \text{ S}} + 28288888 \text{ S}^6 + \sqrt{297216 + 2350080 \text{ S}} + 28288888 \text{ S}^6 + \sqrt{297216 + 2350080 \text{ S}} + 28288888 \text{ S}^6 + \sqrt{297216 + 2350080 \text{ S}} + 28288888 \text{ S}^6 + \sqrt{297216 + 2350080 \text{ S}} + 28288888 \text{ S}^6 + \sqrt{297216 + 2350080 \text{ S}} + 28288888 \text{ S}^6 + \sqrt{297216 + 2350080 \text{ S}} + 28288888 \text{ S}^6 + \sqrt{297216 + 2350080 \text{ S}} + 28288888 \text{ S}^6 + \sqrt{297216 + 2350080 \text{ S}} + 28288888 \text{ S}^6 + \sqrt{297216 + 2350080 \text{ S}} + 28288888 \text{ S}^6 + \sqrt{297216 + 2350080 \text{ S}} + 28288888 \text{ S}^6 + \sqrt{297216 + 2350080 \text{ S}} + 28288888 \text{ S}^6 + \sqrt{297216 + 2350080 \text{ S}} + 28288888 \text{ S}^6 + \sqrt{297216 + 2350080 \text{ S}} + 282888888 \text{ S}^6 + \sqrt{297216 + 2350080 \text{ S}} + 282888888 \text{ S}^6 + \sqrt{297216 + 2350080 \text{ S}} + 2828888888 \text{ S}^6 + \sqrt{297216 + 2350080 \text{ S}} + 282888888 \text{ S}^6 + 28288888 \text{ S}^6 + 28288888 \text{ S}^6 + 282888888 \text{ S}^6 + 282888888 \text{ S}^6 + 28288888 \text{ S}^6 + 2828888888 \text{ S}^6 + 2828888888 \text{ S}^6 + 282888
                                                                                                                                              8 028 288 52 + 15 370 560 53 + 17 792 784 54 + 12 346 992 55 + 4 526 928 56 +
                                                                                                                                             366768 \text{ s}^7 - 265680 \text{ s}^8 - 48816 \text{ s}^9 + 5616 \text{ s}^{10} - 432 \text{ s}^{11}))^{1/3} + \frac{1}{3 \times 2^{1/3}}
                                                                     (560 + 2208 \text{ S} + 3180 \text{ S}^2 + 1894 \text{ S}^3 + 282 \text{ S}^4 - 78 \text{ S}^5 + 2 \text{ S}^6 + \sqrt{(297216 + 2350080)}
                                                                                                                             S + 8 028 288 S<sup>2</sup> + 15 370 560 S<sup>3</sup> + 17 792 784 S<sup>4</sup> + 12 346 992 S<sup>5</sup> + 4 526 928
                                                                                                                            S^6 + 366768 S^7 - 265680 S^8 - 48816 S^9 + 5616 S^{10} - 432 S^{11}))^{1/3}
                                \frac{1}{2} \sqrt{\left[4 + S - S^2 + \frac{1}{3} \left(-4 + S - S^2\right) - \left(2^{1/3} \left(4 + 5 S + S^2\right)^2\right)\right]}
                                                                           \left(3 \, \left(560 + 2208 \, \mathrm{S} + 3180 \, \mathrm{S}^2 + 1894 \, \mathrm{S}^3 + 282 \, \mathrm{S}^4 - 78 \, \mathrm{S}^5 + 2 \, \mathrm{S}^6 + \sqrt{\left(297 \, 216 + 2350 \, 080 \, \mathrm{S} + 1894 \, \mathrm{S}^4 + 282 \, \mathrm{S}^4 + 282 \, \mathrm{S}^6 + 284 \, \mathrm{S}^6 + 
                                                                                                                                             8 028 288 S<sup>2</sup> + 15 370 560 S<sup>2</sup> + 17 792 784 S<sup>4</sup> + 12 346 992 S<sup>5</sup> + 4 526 928 S<sup>6</sup> +
                                                                                                                                            366768 \text{ s}^7 - 265680 \text{ s}^8 - 48816 \text{ s}^9 + 5616 \text{ s}^{10} - 432 \text{ s}^{11}))^{1/3} - \frac{1}{2 \times 2^{1/3}}
                                                                     (560 + 2208 + 3180 + 3180 + 1894 + 282 + 282 + 282 + 282 + 286 + 286 + 3180 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 + 288 
                                                                                                                             S + 8 028 288 S2 + 15 370 560 S3 + 17 792 784 S4 + 12 346 992 S5 + 4 526 928
                                                                                                                            S^6 + 366768 S^7 - 265680 S^8 - 48816 S^9 + 5616 S^{10} - 432 S^{11})
                                                                    (64-16(4-S+S^2)-16(2+5S+S^2)) \left(4\sqrt{(S-S^2+\frac{1}{3}(4-S+S^2)+S^2)}\right)
                                                                                                                      \left(2^{1/3}\left(4+5\text{ S}+\text{S}^{2}\right)^{2}\right) / \left(3\left(560+2208\text{ S}+3180\text{ S}^{2}+1894\text{ S}^{3}+282\text{ S}^{4}-78\text{ S}^{5}+\right)^{2}\right)
                                                                                                                                                                      2 S^6 + \sqrt{(297216 + 2350080 S + 8028288 S^2 + 15370560 S^3 +
                                                                                                                                                                                             17 792 784 S<sup>4</sup> + 12 346 992 S<sup>5</sup> + 4 526 928 S<sup>6</sup> + 366 768 S<sup>7</sup> - 265 680
                                                                                                                                                                                                     S^8 - 48816 S^9 + 5616 S^{10} - 432 S^{11}))^{1/3} + \frac{1}{3 \times 2^{1/3}} (560 + 2208 S +
                                                                                                                                              3180 \text{ S}^2 + 1894 \text{ S}^3 + 282 \text{ S}^4 - 78 \text{ S}^5 + 2 \text{ S}^6 + \sqrt{(297216 + 2350080 \text{ S} + 2350080 \text{ S})}
                                                                                                                                                                      8 028 288 S2 + 15 370 560 S3 + 17 792 784 S4 + 12 346 992 S5 + 4 526 928
                                                                                                                                                                              S^6 + 366768 S^7 - 265680 S^8 - 48816 S^9 + 5616 S^{10} - 432 S^{11}))^{1/3}
```

```
\left\{\theta \to 1 - \frac{1}{2} \sqrt{\left(S - S^2 + \frac{1}{3} \left(4 - S + S^2\right) + \left(2^{1/3} \left(4 + 5 S + S^2\right)^2\right)\right)}\right\}
                                                                                                                  \left(3 \left(560 + 2208 \text{ s} + 3180 \text{ s}^2 + 1894 \text{ s}^3 + 282 \text{ s}^4 - 78 \text{ s}^5 + 2 \text{ s}^6 + \sqrt{\left(297216 + 2350080 \text{ s} + 1894 \text{ s}^3 + 282 \text{ s}^4 - 78 \text{ s}^6 + 288 \text{ s}^6 + 1894 \text{ s}^4 + 1894 
                                                                                                                                                                                                                       8028288 \text{ s}^2 + 15370560 \text{ s}^3 + 17792784 \text{ s}^4 + 12346992 \text{ s}^5 + 4526928 \text{ s}^6 +
                                                                                                                                                                                                                    366768 \text{ s}^7 - 265680 \text{ s}^8 - 48816 \text{ s}^9 + 5616 \text{ s}^{10} - 432 \text{ s}^{11}))^{1/3} +
                                                                                                    \frac{1}{3\times3^{1/3}} (560 + 2208 s + 3180 s<sup>2</sup> + 1894 s<sup>3</sup> + 282 s<sup>4</sup> - 78 s<sup>5</sup> + 2 s<sup>6</sup> + \sqrt{(297216 + 3180)^2}
                                                                                                                                                                              2\ 350\ 080\ s + 8\ 028\ 288\ s^2 + 15\ 370\ 560\ s^3 + 17\ 792\ 784\ s^4 + 12\ 346\ 992\ s^5 +
                                                                                                                                                                               4526928 \text{ s}^6 + 366768 \text{ s}^7 - 265680 \text{ s}^8 - 48816 \text{ s}^9 + 5616 \text{ s}^{10} - 432 \text{ s}^{11}))^{1/3} +
                                                  \frac{1}{2}\sqrt{\left[4+S-S^2+\frac{1}{3}\left(-4+S-S^2\right)-\left(2^{1/3}\left(4+5S+S^2\right)^2\right)\right]}
                                                                                                                  \left(3 \left(560 + 2208 \text{ s} + 3180 \text{ s}^2 + 1894 \text{ s}^3 + 282 \text{ s}^4 - 78 \text{ s}^5 + 2 \text{ s}^6 + \sqrt{\left(297216 + 2350080 \text{ s} + 1894 \text{ s}^3 + 282 \text{ s}^4 - 78 \text{ s}^6 + 288 \text{ s}^6 + 1894 \text{ s}^4 + 1894 
                                                                                                                                                                                                                     8\,028\,288\,{\rm S}^2+15\,370\,560\,{\rm S}^3+17\,792\,784\,{\rm S}^4+12\,346\,992\,{\rm S}^5+4\,526\,928\,{\rm S}^6+
                                                                                                                                                                                                                    366768 \text{ s}^7 - 265680 \text{ s}^8 - 48816 \text{ s}^9 + 5616 \text{ s}^{10} - 432 \text{ s}^{11}))^{1/3}
                                                                                                       \frac{1}{3 \times 2^{1/3}} \left( 560 + 2208 \text{ s} + 3180 \text{ s}^2 + 1894 \text{ s}^3 + 282 \text{ s}^4 - 78 \text{ s}^5 + 2 \text{ s}^6 + \sqrt{297216} + 282 \text{ s}^4 + 282 \text{ s}^6 + \sqrt{297216} + 282 \text{ s}^6
                                                                                                                                                                              2\,350\,080\,\mathrm{S} + 8\,028\,288\,\mathrm{S}^2 + 15\,370\,560\,\mathrm{S}^3 + 17\,792\,784\,\mathrm{S}^4 + 12\,346\,992\,\mathrm{S}^5 +
                                                                                                                                                                               4526928 \text{ s}^6 + 366768 \text{ s}^7 - 265680 \text{ s}^8 - 48816 \text{ s}^9 + 5616 \text{ s}^{10} - 432 \text{ s}^{11}))^{1/3} -
                                                                                                       (64-16(4-s+s^2)-16(2+5s+s^2)) \left(4\sqrt{(s-s^2+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s
                                                                                                                                                                                 \left(2^{1/3}\left(4+5\text{ s}+\text{s}^2\right)^2\right) / \left(3\left(560+2208\text{ s}+3180\text{ s}^2+1894\text{ s}^3+282\text{ s}^4-78\text{ s}^5+\right)^2\right)
                                                                                                                                                                                                                                                           2 S^6 + \sqrt{(297216 + 2350080 S + 8028288 S^2 + 15370560 S^3 + 153
                                                                                                                                                                                                                                                                                               17792784 \text{ s}^4 + 12346992 \text{ s}^5 + 4526928 \text{ s}^6 + 366768 \text{ s}^7 - 265680
                                                                                                                                                                                                                                                                                                          s^8 - 48816 s^9 + 5616 s^{10} - 432 s^{11}))^{1/3} + \frac{1}{2 \times 3^{1/3}} (560 + 2208 s +
                                                                                                                                                                                                                    3180 \text{ s}^2 + 1894 \text{ s}^3 + 282 \text{ s}^4 - 78 \text{ s}^5 + 2 \text{ s}^6 + \sqrt{(297216 + 2350080 \text{ s} + 1894)^2}
                                                                                                                                                                                                                                                          8\,028\,288\,\text{s}^2 + 15\,370\,560\,\text{s}^3 + 17\,792\,784\,\text{s}^4 + 12\,346\,992\,\text{s}^5 + 4\,526\,928
                                                                                                                                                                                                                                                                    s^6 + 366768 s^7 - 265680 s^8 - 48816 s^9 + 5616 s^{10} - 432 s^{11}))^{1/3}
```

```
\left\{\theta \to 1 + \frac{1}{2}\sqrt{\left(S - S^2 + \frac{1}{3}\left(4 - S + S^2\right) + \left(2^{1/3}\left(4 + 5S + S^2\right)^2\right)\right)}\right\}
                                                                                           (3(560 + 2208 s + 3180 s^2 + 1894 s^3 + 282 s^4 - 78 s^5 + 2 s^6 + \sqrt{(297216 + 2350080 s + 3180 s^2 + 1894 s^3 + 282 s^4 - 78 s^5 + 2 s^6 + \sqrt{(297216 + 2350080 s + 3180 s^2 + 1894 s^3 + 282 s^4 - 78 s^5 + 2 s^6 + \sqrt{(297216 + 2350080 s + 3180 s^2 + 1894 s^3 + 282 s^4 - 78 s^5 + 2 s^6 + \sqrt{(297216 + 2350080 s + 3180 s^2 + 1894 s^3 + 282 s^4 - 78 s^5 + 2 s^6 + \sqrt{(297216 + 2350080 s + 3180 s^2 + 1894 s^3 + 282 s^4 - 78 s^5 + 2 s^6 + \sqrt{(297216 + 2350080 s + 3180 s^4 + 282 s^4 + 3180 s^4 + 282 s^4 + 3180 s
                                                                                                                                                                         8028288 \text{ s}^2 + 15370560 \text{ s}^3 + 17792784 \text{ s}^4 + 12346992 \text{ s}^5 + 4526928 \text{ s}^6 +
                                                                                                                                                                        366768 \text{ s}^7 - 265680 \text{ s}^8 - 48816 \text{ s}^9 + 5616 \text{ s}^{10} - 432 \text{ s}^{11}))^{1/3} +
                                                                               \frac{1}{3 \times 2^{1/3}} (560 + 2208 S + 3180 S<sup>2</sup> + 1894 S<sup>3</sup> + 282 S<sup>4</sup> - 78 S<sup>5</sup> + 2 S<sup>6</sup> + \sqrt{(297216 + 3)^2}
                                                                                                                                           2\,350\,080\,\mathrm{s} + 8\,028\,288\,\mathrm{s}^2 + 15\,370\,560\,\mathrm{s}^3 + 17\,792\,784\,\mathrm{s}^4 + 12\,346\,992\,\mathrm{s}^5 +
                                                                                                                                           4526928 \text{ s}^6 + 366768 \text{ s}^7 - 265680 \text{ s}^8 - 48816 \text{ s}^9 + 5616 \text{ s}^{10} - 432 \text{ s}^{11})
                                        \frac{1}{2} \sqrt{\left[4 + S - S^2 + \frac{1}{3} \left(-4 + S - S^2\right) - \left(2^{1/3} \left(4 + 5 S + S^2\right)^2\right)\right]}
                                                                                           \left(3 \left(560 + 2208 \text{ s} + 3180 \text{ s}^2 + 1894 \text{ s}^3 + 282 \text{ s}^4 - 78 \text{ s}^5 + 2 \text{ s}^6 + \sqrt{\left(297216 + 2350080 \text{ s} + 1894 \text{ s}^3 + 282 \text{ s}^4 - 78 \text{ s}^6 + 288 \text{ s}^6 + 1894 \text{ s}^4 + 1894 
                                                                                                                                                                         8\,028\,288\,{\rm S}^2+15\,370\,560\,{\rm S}^3+17\,792\,784\,{\rm S}^4+12\,346\,992\,{\rm S}^5+4\,526\,928\,{\rm S}^6+
                                                                                                                                                                        366768 \text{ s}^7 - 265680 \text{ s}^8 - 48816 \text{ s}^9 + 5616 \text{ s}^{10} - 432 \text{ s}^{11}))^{1/3}
                                                                                \frac{1}{3 \times 9^{1/3}} (560 + 2208 s + 3180 s<sup>2</sup> + 1894 s<sup>3</sup> + 282 s<sup>4</sup> - 78 s<sup>5</sup> + 2 s<sup>6</sup> + \sqrt{(297216 + 1894)^3}
                                                                                                                                          2\,350\,080\,\,\mathrm{s} + 8\,028\,288\,\,\mathrm{s}^2 + 15\,370\,560\,\,\mathrm{s}^3 + 17\,792\,784\,\,\mathrm{s}^4 + 12\,346\,992\,\,\mathrm{s}^5 +
                                                                                                                                           4526928 \text{ s}^6 + 366768 \text{ s}^7 - 265680 \text{ s}^8 - 48816 \text{ s}^9 + 5616 \text{ s}^{10} - 432 \text{ s}^{11}))^{1/3} +
                                                                                  (64-16(4-s+s^2)-16(2+5s+s^2)) \left(4\sqrt{(s-s^2+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s+s^2)+\frac{1}{3}(4-s
                                                                                                                                            \left(2^{1/3}\left(4+5\text{ s}+\text{s}^2\right)^2\right) / \left(3\left(560+2208\text{ s}+3180\text{ s}^2+1894\text{ s}^3+282\text{ s}^4-78\text{ s}^5+\right)^2\right)
                                                                                                                                                                                                       2 S^6 + \sqrt{(297216 + 2350080 S + 8028288 S^2 + 15370560 S^3 + 153
                                                                                                                                                                                                                                    17792784 \text{ s}^4 + 12346992 \text{ s}^5 + 4526928 \text{ s}^6 + 366768 \text{ s}^7 - 265680
                                                                                                                                                                                                                                            s^8 - 48816 s^9 + 5616 s^{10} - 432 s^{11}))^{1/3} + \frac{1}{3 \times 3^{1/3}} (560 + 2208 s +
                                                                                                                                                                        3180 \text{ s}^2 + 1894 \text{ s}^3 + 282 \text{ s}^4 - 78 \text{ s}^5 + 2 \text{ s}^6 + \sqrt{(297216 + 2350080 \text{ s} + 2350080 \text{ s})}
                                                                                                                                                                                                      8\ 028\ 288\ {
m s}^2+15\ 370\ 560\ {
m s}^3+17\ 792\ 784\ {
m s}^4+12\ 346\ 992\ {
m s}^5+4\ 526\ 928
                                                                                                                                                                                                              s^6 + 366768 s^7 - 265680 s^8 - 48816 s^9 + 5616 s^{10} - 432 s^{11}))^{1/3}
```

$$\left\{ \theta \to 1 + \frac{1}{2} \sqrt{\left(s - s^2 + \frac{1}{3} \left(4 - s + s^2\right) + \left(2^{1/3} \left(4 + 5 s + s^2\right)^2\right) / \right. \right. \\ \left. \left(3 \left(560 + 2208 \, s + 3180 \, s^2 + 1894 \, s^3 + 282 \, s^4 - 78 \, s^5 + 2 \, s^6 + \sqrt{\left(297216 + 2350\, 080 \, s + 8028\, 288 \, s^2 + 15370\, 560 \, s^3 + 17792\, 784 \, s^4 + 12\, 346\, 992 \, s^5 + 4\, 526\, 928 \, s^6 + 366768 \, s^7 - 265\, 680 \, s^8 - 48\, 816 \, s^9 + 5616 \, s^{10} - 432 \, s^{11}\right)\right]^{1/3} \right\} + \\ \frac{1}{3 \times 2^{1/3}} \left(560 + 2208 \, s + 3180 \, s^2 + 1894 \, s^3 + 282 \, s^4 - 78 \, s^5 + 2 \, s^6 + \sqrt{\left(297216 + 2350\, 080 \, s + 8\, 028\, 288 \, s^2 + 15\, 370\, 560 \, s^3 + 17\, 792\, 784 \, s^4 + 12\, 346\, 992 \, s^5 + 4\, 526\, 928 \, s^6 + 366\, 768 \, s^7 - 265\, 680 \, s^8 - 48\, 816 \, s^9 + 5616 \, s^{10} - 432 \, s^{11}\right)\right)^{1/3} \right) + \\ \frac{1}{2} \sqrt{\left(4 + s - s^2 + \frac{1}{3} \left(-4 + s - s^2\right) - \left(2^{1/3} \left(4 + 5\, s + s^2\right)^2\right) / \left(3 \left(560 + 2208 \, s + 3180 \, s^2 + 1894 \, s^3 + 282 \, s^4 - 78 \, s^5 + 2\, s^6 + \sqrt{\left(297216 + 2\, 350\, 080 \, s + 8\, 028\, 288 \, s^2 + 15\, 370\, 560 \, s^3 + 17\, 792\, 784 \, s^4 + 12\, 346\, 992 \, s^5 + 4\, 526\, 928\, s^6 + 366\, 768\, s^7 - 265\, 680\, s^8 - 48\, 816\, s^9 + 5616\, s^{10} - 432\, s^{11}\right)\right)^{1/3} \right) - \\ \frac{1}{3 \times 2^{1/3}} \left(560 + 2208 \, s + 3180\, s^2 + 1894 \, s^3 + 282\, s^4 - 78\, s^5 + 2\, s^6 + \sqrt{\left(297216 + 2\, 350\, 080\, s + 8\, 028\, 288\, s^2 + 15\, 370\, 560\, s^3 + 17\, 792\, 784\, s^4 + 12\, 346\, 992\, s^5 + 4\, 526\, 928\, s^6 + 366\, 768\, s^7 - 265\, 680\, s^8 - 48\, 816\, s^9 + 5616\, s^{10} - 432\, s^{11}\right)\right)^{1/3} \right) + \\ \left(64 - 16 \left(4 - s + s^2\right) - 16 \left(2 + 5\, s + s^2\right)\right) / \left(4 \sqrt{\left(s - s^2 + \frac{1}{3} \left(4 - s + s^2\right) + 2\, s^6 + \sqrt{\left(297216 + 2\, 350\, 080\, s + 8\, 028\, 288\, s^2 + 15\, 370\, 560\, s^3 + 17\, 792\, 784\, s^4 + 12\, 346\, 992\, s^5 + 4\, 526\, 928\, s^6 + 366\, 768\, s^7 - 265\, 680\, s^8 - 48\, 816\, s^9 + 5616\, s^{10} - 432\, s^{11}\right)\right)^{1/3} + \\ \left(2^{1/3} \left(4 + 5\, s + s^2\right)^2\right) / \left(3 \left(560 + 2208\, s + 3180\, s^2 + 1894\, s^3 + 282\, s^4 - 78\, s^5 + 2\, s^6 + 3\, 56\, 6768\, s^7 - 265\, 680\, s^8 + 366\, 768\, s^$$

Only the second of the four roots is an economically sensible solution for breakeven θ . This solution is graphed against $\frac{\lambda}{\nu}$ in Figure 2. The other roots are imaginary or imply values of θ outside of its

theoretical range of [0,1]. We have validated this analytical solution by implementing a numerical solution that yields the same values of breakeven θ for a large range of values of $\frac{\lambda}{\gamma}$.