Expected Spot Prices and the Dynamics of Commodity Risk Premia*

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Abstract

We investigate the dynamics of the ex-ante risk premia for different commodities and maturities through the lens of a model of adaptive learning in which expected future spot prices are revised based on past prediction errors and changes in aggregate economic growth. The main results show that time-varying risk premia are predominantly driven by market activity and financial risks. More generally, we provide evidence of heterogeneity in the dynamics of factor loadings, both across commodities and time horizons. Finally, we show that the expectations generated by adaptive learning are consistent with the cross-sectional average of Bloomberg professional analysts' forecasts.

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1 Introduction

The way in which investors form expectations about future commodity prices is of great interest to economists and market participants at least since Keynes (1930). Forward prices have been used extensively in economic models as an approximation of market beliefs.[1] However, the forward curve includes not only investors' expectations for the future, but also a component reflecting the compensation required by market participants for bearing the risk of uncertain fluctuations in spot prices, i.e. a risk premium.[2] Whether this risk premium is positive, negative, or time-varying and driven by changes in economic fundamentals has been controversial in the literature.[3] This controversy stems from the fact that investors' expectations are not directly observable.

In this paper, we first examine to what extent investor expectations in commodity markets are the result of a belief updating scheme in which expected future spot prices are revised in line with past prediction errors and changes in aggregate demand. We assume that suppliers, buyers and inventory holders hedge their commodity positions by trading on futures, such that we explicitly consider the effect of hedging in the decision-making process that leads to the investors' expectations formation mechanism. Such model of adaptive expectations, allows us to approximate the time-varying *ex-ante* risk premia – calculated as the spread between the futures price as of date t with maturity $t + h$ and expectations at time t on future spot prices over the same time-horizon – for a reasonably long sample period. Thus, we investigate the determinants of risk premia across investment horizons and commodities by using a dynamic linear regression framework, which features random-walk betas on a set of widely discussed economic risk factors.

Our main results show that risk premia are time-varying, both across commodities and time-horizons, and their dynamics is predominantly driven by risks sharing mechanisms and the changing nature of market activity, as proxied by Open Interest (OI henceforth), Hedging Pressure (HP

[1]For instance, futures-based forecasts for the Oil price play a role in the policy decision making process at the ECB, see e.g. Svensson (2005), at the Federal Reserve Board, see e.g. Bernanke (2004), and at the International Monetary Fund, see e.g. IMF World Economic Outlook 2005.

[2]Throughout the paper we use the terms *risk premium* and *expected payoff* interchangeably. In fact, all these terms identify a payoff expected at time t as a compensation for a risk which materializes at maturity $t + h$. Differently, a realized payoff, or realized risk premium, couples the risk premium with any unanticipated deviation of the future spot price from the expected future spot price (see Section 2 for a more detailed discussion).

[3]See, e.g. Keynes (1930), Hicks (1939), Kaldor (1939), Working (1949), Brennan (1958), Hsieh and Kulatilaka (1982), Fama and French (1987), Fama and French (1988), Gorton et al. (2013), Singleton (2014), Szymanowska et al. (2014) and Bakshi et al. (2015) just to cite a few.

henceforth) and time-series Momentum (TSMOM henceforth). These results hold after controlling for a variety of other commonly used proxies for risk factors, e.g. changes in inventories and realized volatility. Yet, we show that emerging markets, as proxied by the MSCI Emerging Market Index (MXEF), plays a sensible role for both WTI Oil and Copper, which is coherent with the increasing weight of emerging economies in the global economic growth and the presence of potential spillover effects to be associated with concerns about a worldwide economic slowdown.[4] More generally, we provide evidence of heterogeneity in the dynamics of factor loadings in the time series of commodity risk premia across both products and maturities.

Also, we compare the expected future spot prices obtained from adaptive learning with the cross-sectional average of survey forecasts provided by Bloomberg. This survey contains point predictions on future spot prices at multiple quarterly horizons from professional analysts specialized in commodity markets. We show that, although with differences across commodities, a model of adaptive learning generates conditional expectations which are broadly consistent with the average survey forecast from two to four quarters ahead

Finally, we show that our model of adaptive learning compares favorably against alternative specifications for forecasting future spot prices. More precisely, an out-of-sample comparison of mean squared prediction errors against models in which expectations are based on either futures or current spot prices, or a spread of the two, demonstrates that adaptive expectations reaches a statistically significance 1% higher predictive R^2 on average across commodities and maturities. This result, possibly, rules out the concern that the model-implied ex-ante risk premia merely represent forecast errors which have nothing to do with investors preferences or the actual expectations formation process. As a matter of fact, a further analysis clarifies that the expected payoffs extracted from adaptive expectations are highly correlated to the actual, realized, excess rolling returns in the same-maturity generic futures contract.

This paper builds on a number of existing works such as Nerlove (1958), Evans and Honkapohja (2001), Sargent (2002), Sargent and Williams (2005), and Malmendier and Nagel (2015), who consider a model of adaptive learning to explain the dynamics of expectations on inflation and more general macroeconomic outcomes. Also, our work is related to recent research that posits

[4]China itself is the second largest economy and the second largest importer of both goods and commercial services.

trading activity is the result of an adaptive process in which hedgers and speculators learns about economic fundamentals, both from public information and market prices (see, e.g. Singleton 2014). Along these lines, we formally postulate an adaptive learning scheme for future commodity spot prices which is consistent with a "learning from past errors" scheme. Finally, our paper connects to a recent literature that aims at understanding the origins of unconditional realized commodity risk premia such as Carter et al. (1983), Bessembinder (1992), De Roon et al. (2000), Acharya et al. (2010), Hong and Yogo (2012), Asness et al. (2013), Basu and Miffre (2013), Hamilton and Wu (2014), Szymanowska et al. (2014), and Bakshi et al. (2015). Different from them, we exploit a model of adaptive learning and implement a full-scale dynamic regression model to investigate the time-varying sensitivity of the *ex-ante* risk premia to a set of commonly used observable factors.

The rest of the paper is organized as follows. Section 2 discusses the motivation of the paper, while Section 3 introduces the model of adaptive learning as well as compares the implied expectations with the cross-sectional average of the Bloomberg's individual analysts forecasts. Section 4 represents the core of the paper and reports the empirical results. Section 5 concludes. We leave the details of the model derivation and further results to the Appendix.

2 Motivation

Let S_t denote the spot price of a given commodity at time t, and $F_t^{(h)}$ the price of a futures at time t with maturity $t+h$. The basis $F_t^{(h)} - S_t$ can be decomposed in two main components,

$$F_t^{(h)} - S_t = E_t\left[\Delta S_{t+h}\right] \; + \; \underbrace{F_t^{(h)} - E_t\left[S_{t+h}\right]}_{y_t^{(h)}} \tag{1}$$

with $E_t\left[S_{t+h}\right]$ the market aggregate expected spot price for time $t+h$, $y_t^{(h)}$ a risk premium component in dollar terms, and $E_t\left[\Delta S_{t+h}\right]$ the expected change in spot valuations between t and $t+h$. To the extent that one wants to investigate the origins of risk premia, equation (1) offers an ideal setting since directly isolates risk-related components in futures prices conditioning on investors' expectations about the spot commodity.

One may argue that the ex-ante and realized payoff of a futures position are equivalent, such that we can indifferently use the spread between the spot price at maturity S_{t+h} and the futures price $F_t^{(h)}$ as a reliable proxy for risk premia. Unconditionally, this is indeed the case. Suppose S_t evolves according to a simple AR(1) process $S_{t+h} = \phi S_t + \nu_{t+1}$, the expectation at time t for the spot price at time $t+h$ is $E_t[S_{t+h}] = \phi^h S_t$, and the realized forecast error would be

$$S_{t+h} - E_t S_{t+h} = \sum_{i=0}^{h-1} \phi^i \nu_{t+h-i},$$

Note that by definition the forecast error is autocorrelated. Now, the *realized* payoff of a futures contract held until maturity can be decomposed as

$$F_t^{(h)} - S_{t+h} = y_t^{(h)} - \sum_{i=0}^{h-1} \phi^i \nu_{t+h-i}, \tag{2}$$

If expectations are unbiased the unconditional average of the forecasting error is zero. However, the persistence of price dynamics can make the conditional expectation errors sizable for finite samples and horizons. Figures 1 makes this case in point; the expectation errors $E_t[S_{t+h}] - S_{t+h}$ for two different horizons, i.e. $h = 2, 4$ quarters ahead, and two alternative commodities, i.e. WTI Crude Oil and Silver, tend to be time-varying and quite persistent.[5]

[Insert Figure 1 about here]

Unsurprisingly, unexpected depreciation for crude oil occurred over the great financial crisis of 2008/2009 and the recent collapse of late 2014/beginning of 2015. Similarly, unexpected appreciation of Silver occurred in the recovery of financial markets after 2009, consistent with the idea that the value of precious metals tend to be negatively correlated with the business cycle.

As a whole, the assumption of either small or constant conditional unexpected price change turns out to be fairly restrictive. Figure 2 makes a case in point, where to the extent that investors' misjudge the level of future spot prices over time, the ex-ante and realized risk premia differ.

[Insert Figure 2 about here]

[5]The aggregate forecast $E_t[S_{t+h}]$ is proxied by the cross-sectional average of the Bloomberg's survey individuals forecasts. A complete discussion on how the survey is collected and structured, as well as a description of the data, is provided below.

For instance, let us consider a simple situation in which the price of the commodity at time t is equal to 50\$ and market expectations for the future spot price at time $t+h$ are equal to 47\$, i.e $E_t[S_{t+h}] = 47$. Also, let us assume that in order to make investors willing to enter the market the current price of a futures contract at time t for delivery at time $t+h$ is equal to 43\$, which means futures are sold at a discount. The difference between the futures price and $E_t[S_{t+h}]$ at time t implies that the expected payoff of a long position is equal to 4\$.

The top panel of Figure 2 shows the case in which the commodity is indeed traded at 47\$ at maturity. Under no-arbitrage and given there are no unexpected price changes, the ex-ante and the realized payoffs are equivalent. Consider instead a situation in which investors make errors in forecasting future spot prices (see, e.g. Alquist and Kilian 2010 for a complete discussion on the predictability of nominal spot prices). More specifically, let assume that the commodity is traded at a lower price of 45\$ at time $t+h$ on the spot market, which implies a forecast error equal to -2 (bottom panel). The realized payoff is now 2\$; the ex-ante and the realized risk premia differ by the amount of the unexpected price change. Figures 1 and 2 coupled, make clear that although expectations error can be zero asymptotically, they might have sizable effects on investigating the origins of risk premia for reasonable sample sizes. In the following, we propose a reduced-form model of adaptive learning which allows to disentangle the actual, ex-ante, risk premium $y_t^{(h)}$.

3 Adaptive Learning and Expectations

To set up an analytical framework, we start from an extended Muth (1961)'s market model with the addition of both predictable changes in aggregate demand and the presence of a futures market (see, e.g. Turnovsky 1983, Kawai 1983, and Beck 1993). The market is characterized as an infinite horizon, discrete time model with both spot and futures market clearing conditions that hold in each time period. By including a futures market we assume that suppliers, buyers and inventory holders hedge their positions by trading on futures, and so we explicitly consider the effect of hedging in the decision-making process that leads to the Perceived Law of Motion (PLM henceforth) of spot prices. By allowing demand shocks to be predictable and possibly persistent we make explicit the effect of changes in aggregate demand in the dynamics of equilibrium spot prices.[6] A unique reduced-form rational expectations equilibrium is defined as (see Appendix A)

$$S_{t+1} = \phi_0 + \phi_1 S_t + \phi_2 z_t + \eta_{t+1}, \tag{3}$$

with S_{t+1} the commodity price at date $t+1$, z_t the change in aggregate demand at time t, and η_{t+1} an unobservable random shock.[7] Notably, a similar solution would be obtained by assuming market segmentation between spot and futures as originally proposed in Muth (1961)'s model.

We do not take a stand on the marginal relevance of supply vs. demand shocks in the dynamics of commodity stock prices, and assume that changes in aggregate supply are conditionally i.i.d. This assumption can be relaxed at the cost of having some reliable empirical proxy for aggregate supply shocks for agriculturals, e.g. Corn, and precious metals, e.g. Silver, to be used as exogenous variables in the adaptive learning dynamics. Also, while the i.i.d. assumption for supply shocks can be restrictive for energy or industrial commodities, the same assumption possibly represents a fair approximation of supply shocks in agriculturals and precious metals, e.g. "harvest" can be thought as i.i.d. and storage of, say, corn is temporally limited.

A visual inspection of the relationship between (a proxy for) economic growth and spot prices confirms that changes in aggregate demand represent an important source of fluctuations in commodity prices. Figure 3 shows the year-on-year changes in the (log of) commodity spot prices (blue line) and aggregate demand as proxied by an index of world industrial production published by the Netherlands Bureau for Economic and Policy Analysis (magenta line).[8]

[Insert Figure 3 about here]

[6] In the original Muth (1961) framework demand shocks that induce changes in inventories quickly revert to their long-run equilibrium values. In this respect, inventories adjustments are perceived to have a stabilizing effect on prices. However, as recently showed by Dvir and Rogoff (2010) quick adjustments in inventories to demand shocks cannot explain the persistence in the time series of commodity prices and volatilities.

[7] One may also specify a model in which expectations of future changes in aggregate demand rather than current values enter in the equilibrium outcome. As far the unique reduced-form solution in Eq. (3) is concerned, the two things are virtually equivalent. Aggregate demand is specified as an AR(1), i.e. $z_{t+1} = bz_t + e_{t+1}$. This implies that $E_t z_{t+1} = bz_t$, which means that the structural coefficient b of the actual law of motion, although cannot be identified, is embedded in the reduced-form parameter ϕ_2 of the perceived law of motion.

[8] The index of world industrial production is published by the Netherlands Bureau for Economic and Policy Analysis and aggregate information from 81 countries worldwide, which account for about 97% of the global industrial production. The aggregate series starts in January 1991 and relate to import-weighted, seasonally adjusted industrial production.

With the only partial exception of Corn (bottom-left panel), which is less sensitive to business cycles, changes in spot prices tend to align with changes in aggregate demand, especially after the beginning of the 2000s. Similarly, Kilian and Hicks (2013) show that unexpected economic growth sensibly affects the dynamics of spot prices in the Oil market. In our adaptive expectations framework, beliefs are revised in line with past prediction errors based on available information, i.e. aggregate demand shocks affect investors' expectations as well. This is consistent with Singleton (2014), who argue that differences in beliefs can generate persistence in the dynamics of commodity spot prices.[9]

Learning is introduced by assuming that agents do not know true values of the parameters of the PLM $\phi = (\phi_0, \phi_1, \phi_2)$ and expectations are instead formed on the basis of a weaker form of rational expectations that allow for model instability, uncertainty, and learning (see, e.g. Hsieh and Kulatilaka 1982, Frenkel and Froot 1987, Marcet and Sargent 1989, Evans and Honkapohja 2001, and Sargent 2002, and Sockin and Xiong 2015 relatively to commodity markets). Aggregate beliefs on the parameters are updated over time conditioning on current observations plus a constant $X_t = (1, S_t, z_t)$. More specifically, we follow Cho et al. (2002), Sargent (2002), and Sargent and Williams (2005) and model the agents' recursive estimates in terms of a Bayesian prior that describes how coefficients drift at each time t;[10]

$$\begin{aligned} S_{t+1} &= \phi'_{t+1} X_t + \eta_{t+1}, \qquad && \text{with} \qquad \omega_{t+1} \sim N\left(0, \sigma^2\right), \\ \phi_{t+1} &= \phi_t + \xi_{t+1} \qquad && \text{with} \qquad \xi_{t+1} \sim N\left(0, \Omega\right), \end{aligned} \tag{4}$$

with $\phi_t = (\phi_{0,t}, \phi_{1,t}, \phi_{2,t})'$ and $X_t = (1, S_t, z_t)'$. The shock ω_{t+1} is uncorrelated with ξ_{t+1}, and $\Omega << \sigma^2 I$. The innovation covariance matrix Ω governs the perceived volatility of increments to the parameters (see, Sargent and Williams 2005). Agents' recursive optimal estimate of ϕ_{t+1}

[9]Related to the Oil market, Singleton (2014) pointed out that "Perhaps more plausible is the assumption that participants [...] learn about the true mapping between changes in fundamentals and prices by conditioning on past fundamentals and prices".

[10]This random walk specification for the evolution of the parameters is widely used in applied work in macroeconomics and finance, e.g. Frühwirth-Schnatter (1994), West and Harrison (1997), Stock and Watson (1998), Primiceri (2005), Hansen (2007), and Leduc et al. (2015).

conditional on information available at time t, $\gamma_{t+1} = \hat{\phi}_{t+1|t}$ are provided by a standard recursion;

$$\begin{aligned}\gamma_{t+1} &= \gamma_t + K_t \left(S_{t+1} - \gamma_t' X_t\right), \\ R_{t+1} &= R_t - \frac{R_t X_t X_t' R_t}{X_t' R_t X_t + 1} + \sigma^{-2}\Omega,\end{aligned} \tag{5}$$

where $K_t = R_t X_t \left(X_t' R_t X_t + \sigma^2\right)^{-1}$ determines the degree of updating of agents' beliefs when faced when an unexpected commodity spot price $S_t - \gamma_t' X_t$. This beliefs updating dynamics represents a generalization of recursive learning with constant gain. The recursive estimates (5) imply perpetual learning as they converge to a steady-state solution for a given initial condition of the state covariance matrix Ω (see, Hamilton 1994 Proposition 13.1, pag. 390). We use the subscript $t+h|t$ to indicate a forecast for the $h > 0$ horizon made using information available to agents' at time t. The market price expected to prevail at time $t+1$ given the information available through the t-th period is obtained as

$$\hat{E}_t \left[S_{t+1}\right] = \gamma_{t+1}' X_t, \tag{6}$$

Multi-period forecasts $\hat{E}_t \left[S_{t+h}\right]$ are obtained by iterating forward the time-t estimates of the model parameters. Learning schemes as (5) are widely motivated in the macroeconomics literature by the fact that agents face constraints in cognitive abilities that limit their possibility to observe the true equilibrium parameters and use optimal forecasting rules (see, e.g. Carceles-Poveda and Giannitsarou 2008, Adam and Marcet 2011 and Malmendier and Nagel 2015). Conditional forecasts from Eq. (5) allows to extract risk premia across predictive horizons and commodities. More specifically, let $\hat{E}_t \left[S_{t+h}\right]$ be the model-implied expected future spot price of a given commodity at time t for the horizon $t+h$. The dollar value risk premium can be extracted from the price of a future contract at time t for delivery at time $t+h$, $F_t^{(h)}$, as;

$$\hat{y}_t^{(h)} = F_t^{(h)} - \hat{E}_t \left[S_{t+h}\right], \tag{7}$$

Eq. (7) implies that it is not necessary for the investors to have private information for their actions to affect commodity risk premia. As a consequence, the latter may depend on the nature of agents' learning mechanism based on common signals.

3.1 Comparison with Survey Expectations

We now compare the time series of monthly expected future spot prices obtained from our model with the average forecast by professional analysts that operate in commodity markets. Individual price forecasts for different commodities and horizons are obtained from the Bloomberg's commodity price forecasts database. This database contains analysts' price expectations at multiple quarterly forecasting horizons and across diverse commodities from 2006 to 2016. The survey includes only operators highly specialized in commodity markets mainly from banks and consulting firms. Participants are asked to provide a point forecast on the average quarterly commodity price for a specified futures contract.

A deep knowledge of the peculiarities of commodity markets from the survey respondent, coupled with a clear objective of the survey, allows to reduce the effect of potential biases, quality homogeneity issues, and limited information processing, which generally characterizes directional forecasts of non-specialized, or retail, cross-markets investors (see, e.g. Cutler et al. 1990, Greenwood and Shleifer 2014 and Koijen et al. 2015).[11] There are two main objections on the use of survey expectations in empirical studies; first, the respondent may misunderstand the question which, for instance, can be posed in a simple directional way, e.g. do you expect prices increase, decrease or stay roughly constant. Second, a respondent may intentionally hide their true expectations for strategic purposes. Our survey mitigates the effect of both of these sources of error as (1) the question is about giving a clear point estimate for future spot prices, and (2) survey participants are professional market participants who possibly have payoffs that directly depends on the precision of their estimates.[12] One comment is in order; the use of the Survey does not represent on itself the core of the paper, which is instead based on a model of adaptive expectations. In this respect, we use the survey as an instrument to "validate" our model. As a matter of fact, although the survey represents the closest possible approximation of observable expectations, it still suffers from potential strategic biases and interactions among analysts.

[11]More specifically, the fact that only operators specialized in commodity markets are being surveyed increase the proportion of "truly informed" agents in the survey population compared to a case in which cross-market analysts are being surveyed.

[12]As we take the cross-sectional average of investors' forecast as our proxy for market expectations, any non-coordinated strategic bias/error at the individual level is mitigated (see, e.g. Bernhardt and Kutsoati 1999, Hong et al. 2000, and Hong and Kubik 2003)

The survey allows to retrieve for each analyst the historical price forecasts and the related publication date. Analysts provides their expectations for spot prices in different days for fixed common maturities that correspond to calendar quarters, i.e. they provide discontinued fixed-calendar maturity quarterly expectations.[13] Such feature makes the use of the survey for operational purposes quite challenging as the quarterly analysts' forecasts submission are recorded daily and not evenly spaced in time.

To perform a sensible time-series comparison with the model-implied expectations, we need to transform analysts' responses in continued constant-horizon price forecasts. We aggregate responses at the monthly frequency to reduce the difference in the market information available between early and late submitters within a month. Then, we compute the forecasting horizon with respect to the end of the month of the last month in the quarter which is the object of the prediction. More specifically, at each point in time, we stack the forecasts with residual life that belongs to the following groups: 4 to 6; 7 to 9 and 10 to 12 months, then finally we approximate the aggregate expectations as the cross-sectional average prediction across analysts and time-horizons.

Short-term moving average effects are reduced by discarding the horizon between one and three months as the analysts take into account what has been the realized price over the first part of the quarter generating nowcasting dynamics which makes hard to disentangle the role of expectations versus current information in the dynamics of short-term risk premia.[14]

For the ease of exposition, we report the results for dollar value expectations at maturities $h = 2, 4$ quarters.[15] The sample period is from 12:2006 to 01:2016 for the survey, and is from 01:1995 to 01:2016 for the model-implied expectations. Figure 4 reports the results for $h = 2$. The red circles represent the monthly observed survey forecast, and the light-blue circles show the expectations obtained from the adaptive learning model. The shaded area underlying the

[13]Other surveys can be used to approximate the average investors' expectations, such as for instance the Energy & Metals Consensus Forecasts from Consensus Economics. However, Consensus forecasts do not count for agricultural and is lower/irregular frequency being collected every other month after April 2012 and on a quarterly basis before that date. Also, while Consensus forecasts are "contaminated" with forecasts from economists working in institutions that do not necessarily participate in commodity markets, Bloomberg's panel of respondents is uniquely composed of professional analysts' affiliated with banks and consultancy companies, which make the survey more suitable in approximating expectations by actual market participants.

[14]Also, contracts close to expiration are typically illiquid in commodity markets as futures traders do not want to take the risk of a physical delivering of the underlying.

[15]The empirical evidence for the intermediate horizon $h = 3$ are available upon request.

overlapping period between the survey and the model represents the difference between the two, i.e. a positive value means the model generates higher expected future spot prices than the survey and vice versa.

[Insert Figure 4 about here]

The survey forecasts and the adaptive expectations line up fairly well across the overlapping sample for WTI Crude Oil (top-left panel). This holds both during the dramatic rise and subsequent sharp fall in crude oil prices during the period 2008/2009, as well as during the market decline occurred since 2014. The "spread" between the model and the survey increases as high as 20$ across the great financial crisis, although is sensibly reduced over the remaining sample. The top-right panel shows the results for Copper. Similar to Oil, adaptive learning can mimic the drop in expected spot prices in the period 2008/2009, the subsequent rapid price recovery, as well as the downward trend from 2011 until the end of the sample. Over a short-term horizon, the model still generates higher expected prices compared to the survey for a fraction of the sample, although the gap is small in magnitude after 2010.

A comparison with observable expectations for Corn (bottom-left panel) is limited by the few observations available from the survey, which does not provide opinions from analysts in the period 2011-2013. The divergence around the great financial crisis is non-negligible as indicated by an 80 cents/bushel negative gap. However, over the last part of the sample adaptive learning closely replicates average survey forecasts. Results are stronger for Silver. The gap is fairly small, with the partial exception of a negative "spread" during the dramatic rise in spot prices occurred in the aftermath of the great financial crisis of 2008/2009. In a separate calculation, we show that the sample correlation between the model- and survey-implied risk premia across commodities and horizons is 0.81, on average. Figure 5 shows the results for a longer horizon,

[Insert Figure 5 about here]

Adaptive expectations line up fairly closely with survey average forecasts over a four-quarter horizon, although the similarity between the model and the survey partly deteriorates as indicated by a more persistent gap throughout the sample. As a whole, the model performance tends to deteriorate in the longer-term, where the correlation between the model- and the survey-implied risk premia decreases to an average value of 0.64.

In Appendix C we further test the null hypothesis that average survey forecasts are consistent with a recursive learning framework. In this respect, we test for internal consistency between the model outlined to generate expectations and the observable proxy represented by the survey. We find evidence in support of adaptivity in the expectations formation process across prediction horizons and commodity markets, meaning the elasticity of investors' expectations on future spot prices with respect to past forecasting errors is significant. However, notice that the evidence in favor of adaptive expectations does not rule out investors rationality (see Pesaran and Weale 2006 for more details).

4 Empirical Analysis

We cover four main commodity futures which represent the energy, agricultural, industrial and precious metals markets. We focus on these commodities as they are the most traded consumption commodities with the most complete sample of survey data. The necessity to compare the model-implied adaptive expectations with the survey of professional analysts limits the possibility to increase the cross-section of commodities. In this respect, the choice of the commodity to be included in the analysis is mostly dictated by the length of the corresponding survey and the number of professional analysts responding. Including other commodities would come at the cost of using averages of few respondents or time series with few observations.

4.1 Data

Data are obtained from different resources. Futures prices data on WTI Crude Oil are from the New York Mercantile Exchange (NYMEX), prices are in U.S. dollars per barrel. Futures on Silver and Copper are obtained from the Commodity Exchange (COMEX). Silver is quoted in U.S. dollars per troy ounce while Copper is quoted in U.S. cents/pound. We convert the price of Copper futures contracts to USD/tonne to match the measurement unit of the survey forecasts that instead refer to the London Metal Exchange (LME). Corn futures prices are from the Chicago Board of Trade

(CBOT) with price quotation in USD cents per bushel. As in Szymanowska et al. (2014) the spot price for each commodity is approximated by using the nearest contract to maturity, and the futures price is the price of the next to the nearest futures contract for a given maturity.

We define the futures price at time t with average quarterly time to maturity h as $F_t^{(h)}$, where the definition of the average time to maturity is consistent with the average forecasting horizon for the survey expectations. For example, the price of a future for delivery four quarters ahead is computed interpolating the prices of the contracts between 10 and 12 months ahead. The sample period is monthly 01:1993-01:2016.

In order to study the sources of time variation in commodity risk premia, we collect diverse determinants that are considered to capture alternative sources of risk and/or economic fundamentals. Fluctuations in the global supply-demand imbalance for each commodity are captured by using inventory stocks. We collect data on Copper and Crude Oil inventories from the London Metal Exchange (LME) and Energy Information Administration (EIA) respectively. Copper inventory levels are recorded daily from June 1974 and relate the previous day closing stock of commodities held in LME. Crude Oil inventories are recorded weekly by the EIA and published monthly since January 1945. Stocks levels are measured in thousands of barrels and exclude strategic petroleum reserves.[16] For Corn inventories, we use the U.S. ending stocks reported in thousands of metric tons. The time series is sampled at monthly frequency using the inventory level reported on the last business day of the month. Data are recorded from the United States Department of Agriculture (USDA) from January 1993. As far as Silver is concerned, we omit the inventory level variable as, similar to other precious metals, a considerable part of the existing reserves is privately held and therefore not reported in official statistics. In the regression specification we use the year-on-year growth rate of inventories as the levels are non-stationary and show the presence of a stochastic time trend.

Exchange rates is also a relevant risk factor as commodity trading takes place usually in U.S. Dollars, making FX a key factor for both producers and consumers that can directly affect profits and costs denominated in domestic currency. In order to account for the risk of appreciation and

[16]We include in the level of inventories those domestic and Customs-cleared foreign stocks held at, or in transit to, refineries and bulk terminals, and stocks in pipelines. Stocks include an adjustment of 10,630 thousand barrels (constant since 1983) to account for incomplete survey reporting of stocks held on producing leases.

depreciation in the U.S. Dollar, we include the growth rate of Federal Reserve U.S. trade weighted exchange rate index, normalized to be equal to one hundred in March 1973.

Furthermore, we include a measure of time series momentum among the risk factors in our analysis as it can be directly linked to asset demand by momentum traders as shown in Cutler et al. (1990), Moskowitz et al. (2012), and Kang et al. (2014). Momentum in commodity futures has been widely documented in the empirical finance literature, e.g. Erb and Harvey (2006), Miffre and Rallis (2007), Asness et al. (2013) and Szymanowska et al. (2014) among others. We construct time-series Momentum as the rolling return over the past 12 months skipping the most recent month on each commodity future. In addition, we include a Value factor which is assumed to be intimately interrelated to the dynamics of commodity risk premia, as it affects the propensity of market participants to trade in backwardation or in contango and can proxy the trading activity of speculators following mean-reversion type trading strategies. We follow Asness et al. (2013) and define Value as the average of the log spot price from 4.5 to 5.5 years ago, divided by the most recent spot price, which is essentially the negative of the spot rolling return over the last 60 months. In addition to time-series Value and Momentum, we also directly consider returns on the Standard and Poor's 500 and the MSCI Emerging Markets indexes as a proxy for financial risk. Beyond direct effects on financial flows, we incorporate stock indexes as they likely capture spillover effects to the real economy. As a measure of futures market uncertainty, we compute the Realized Volatility for a given maturity h as the sum of squared daily futures returns adjusted for roll-over and for delivery date $t + h$.

Finally, to capture market activity and risk sharing preferences in the economic mechanism that drives commodity risk premia we consider OI and HP (see e.g. Baker and Routledge 2011 and Singleton 2014). OI is measured as the total number of outstanding contracts that are held by market participants at the end of the month. An outstanding contract is when a seller and a buyer combine to create a single contract. For each seller of a futures there must be a buyer of that contract, therefore to determine the total OI for any given market we need to know the totals from one side or the other, buyers or sellers, not the sum of both. Increasing OI means that new cash is flowing into the marketplace while declining activity means that the market is liquidating, which can be interpreted as a signal of a price turning point. As for inventories, we use the year-on-year growth rate of OI as the levels are non-stationary.

Hedging pressure represents a measure of net positions of hedgers in commodity futures markets which is the result of risks that market participants do not want, or cannot trade because of market frictions, information asymmetries and limited risk capacity (see, e.g. Hong and Yogo 2012 and Kang et al. 2014). We compute the level of HP for different commodities as the net excess in short futures positions by commercial traders, i.e. short minus long positions, divided by the amount of outstanding contracts. The data on commercial traders futures positions are from the Commodity Futures Trading Commission (CFTC).

4.2 Dissecting Ex-Ante Risk Premia

The framework outlined in Section 3 allows to back out the dollar-valued time-varying ex-ante risk premia from adaptive expectations. In order to obtain the expected payoffs as a returns quantity, which is more suitable for our regression analysis, we took a log transformation for both futures prices and the model-implied expected future spot prices in Eq.(7). This allows to approximate risk premia in percentage returns, up to a negligible Schwarz inequality term.[17]

Panel A of Table 1 shows the in-sample descriptive statistics of the monthly risk premia (decimals). Unconditionally, the term structure of risk premia for Crude Oil and Copper is negatively sloped. Risk premia for these two commodities are negative and increasing over time in magnitude. This is consistent with the theory of Keynes (1930) and Hicks (1939), which posits that hedgers are net short and futures are set at a discount with respect to the future expected spot price. Conversely, for Corn and Silver average risk premia are positive and increasing as a function of time horizon. Hedging pressure theory states this is the result of hedgers predominantly being net-long with speculators willing to enter contracts with slightly negative payoff provided there are expectations of increasing future prices.

[Insert Table 1 about here]

[17]For the sake of completeness, we implement the empirical analysis in Section 4 by using Eq.(7) and rescale both futures prices and the model-implied expectations by current spot prices. The main results, available upon request from the authors, are in line with the log-transformation.

With the only exception in the short-term risk for Crude Oil and longer-term for Copper, the sample distribution of risk premia is far from Gaussian. Both Corn and Silver show a fairly large negative skewness and a substantial excess kurtosis. Departure from Normality is also mainly given to fatter tails in Oil and Copper. Overall, a Jarque-Bera test rejects the null hypothesis of Normality for nine out of twelve cases. Finally, the term structure of volatility for risk premia is positively sloped, i.e. the standard deviation of ex-ante risk premia increases with maturity. Panel B of Table 1 shows the in-sample cross-sectional correlations of risk premia for each expectations horizon. Cross-sectional correlations are inversely related to the investment horizon; for instance, the correlation of WTI Crude Oil with Copper is 0.355 at $h = 2$ and decreasing to 0.243 at $h = 4$. A similar path is found across commodities.

We now investigate the determinants of the ex-ante risk premia through a static regression. Table 2 shows the estimated standardized coefficients with the asymptotic t-statistics in parenthesis.[18] Few comments are in order; first, there is significant heterogeneity in the significance of each factor across commodities. While emerging markets are strongly significant for the sample variation of WTI and Copper risk premia, the same are not relevant for Corn and Silver. In this respect, Copper and Oil are directly affected by the demand from, e.g. China, while food and precious metals are much less dependent on spillovers effects from emerging markets. Similarly, realized volatility seems to significantly affect futures expected payoff only for Silver, which is consistent with the fact that precious metals are safe-haven assets during market turmoils.

[Insert Table 2 about here]

Second, surprisingly hedging pressure and inventories are not significant determinants of risk premia sample variation. This somewhat contradicts early work by De Roon et al. 2000, Basu and Miffre 2013, and Szymanowska et al. 2014. However, we show below that once the dynamics of risk premia is fully considered, HP turns out to be a key component. Similarly, except for futures on Copper at a two- and three-quarter horizon, inventories are not significantly related to the ex-ante

[18] The explanatory factors in the regression are pre-whitened, i.e. ortoghonalized to each other and standardized. Pre-whitening helps to reduce the spurious effect of cross-factor correlations, which can be arguably relevant in a linear model with many factors like ours, e.g. HP and OI or S&P500 and MXEF. We estimate the model by OLS with GMM corrected standard errors.

risk premia, after controlling for net supply-demand imbalances and spillover effects from emerging markets and currency fluctuations. Finally, sensitivity to past performances is significant and positive across commodities and horizons, with the only exception of short-term futures for Copper. This is consistent with Asness et al. (2013), and can be possibly rationalized by a "bandwagon" effect in market activity and trading behavior which increase the persistence of futures returns.

The regression results of Table 2 suggest that, unconditionally, risk sharing mechanism and market activity possibly explain the sample variation of the ex-ante risk premia. However, the fact that expected payoffs have their own dynamics could be the consequence of an heterogeneous exposure to different risk factors on a time scale. In this sense, the results of a static regression might be potentially incomplete, at best. For instance, the so-called financialization of commodity markets arguably increases the sensitivity of risk premia to market activity which is, by definition, contingent and time-varying and not necessarily linked to economic fundamentals. In the following, we use a dynamic regression modeling framework that explicitly allows for a time variation in the relationship between the risk premia $\hat{y}_{t+1}^{(h)}$ over the interval $[t, t+1)$ and the realizations of the explanatory factors observed at time t.

More specifically, we assume that the exposure of risk premia to each specific factor is a random walk (see, e.g. West and Harrison 1997, Kilian and Taylor 2003, and Ferreira and Santa-Clara 2011). Risk factors have been orthogonalized to avoid spurious effects due to cross-correlations in the explanatory variables. Methodologically, we opt for a Bayesian estimation framework, which allows to obtain robust finite-sample estimates that flexibly and explicitly accounts for different sources of uncertainty: uncertainty in the relative importance of predictors, uncertainty in the estimated coefficients and their degree of time-variation. Appendix B provide a detailed explanation of the regression design and model estimation strategy. One comment is in order; assuming regression betas evolve as a random walk implies that the elasticity of risk premia to a given factor drift to deterministic high or low values of $\hat{y}_t$, hence generating non-stationarity. However, an alternative more general AR(1) specification for the dynamics of the regression betas show that the state parameters are highly persistence with low conditional variance. In this respect, the random walk assumption represents an attractive approximation because of its parsimony, ease of computation and the smoothness it induces in the estimated sensitivities over time.[19]

For the ease of exposition we first investigate what is the actual amount of explanatory power that can be associated to each of these factors within our dynamic regression exercise, and then we show the time-varying betas for the sub-set of risk factors which show most of the significance. In particular, we first decompose the overall R^2 to measure the improvement resulting from including covariate k in a dynamic regression model that already contains the other covariates (see Genizi 1993 for more details). Given the regression covariates have been previously orthogonalized, this boils down to compute the ratio between the sum of explained residuals from the regressor k and the total sum of squares, as is done for a univariate regression with the single regressor k. Figure 6 shows the marginal contribution of each risk factor for the total R^2 of the regression.[20]

[Insert Figure 6 about here]

The top-left panel confirms that most of the explanatory power for the dynamics of WTI Oil risk premia comes from three key variables, namely MXEF, HP and TSMOM, especially for short maturities. Indeed, HP alone contributes to around 20% of the explained variation for $h = 2$, proportion than shrink to around 10% for $h = 3, 4$. As far as Copper is concerned, inventories contributed to a large fraction of the explained in-sample variation (around 15%) especially for longer-term maturities. Similar to WTI, we attribute most of the explanatory power to time-series momentum, particularly in the short-term (around 20% for $h = 2, 3$). Bottom-left panel shows the same decomposition for Corn. Most of the R^2 is attributed to open interests. Also, time-series momentum and USDTW carry a significant explanatory power, with the latter contributing to around 10% of the explained sample variation. Finally, bottom-right panel shows that most of the R^2 obtained by the dynamic regression model for Silver is due to market activity, past performances and uncertainty, as proxied by HP, momentum and realized volatility.

We now focus on those factors which shows most of the significance. Figure 7 shows the time-

[19]We share these findings with a large literature on returns predictability that assumes time variation in the predictive coefficients. Similar to our argument they find that assuming parameters are random walks in predicting excess returns we benefit from a substantial reduction of estimation error without effectively increasing the precision in the estimated dynamics in a finite sample.

[20]Notice that the percentages in the graph do not sum to one as we left aside the amount of sample variation explained by the intercept.

varying betas for each risk factor on WTI Crude Oil ex-ante risk premia. For the ease of exposition we report the results for $h = 2$ (blue line) and $h = 4$ (dark yellow line). We report both the posterior medians (solid marked line) and the 95% credibility intervals (dashed-dot lines). Results on the intermediate horizon $h = 3$ are available upon request.

[Insert Figure 7 about here]

The empirical evidence shows that the impact of emerging markets has become increasingly important in the aftermath of the great financial crisis of 2008/2009. A possible explanation is the presence of spillover effects due to the increasing weight of emerging economies in the global economic outlook.[21] Indeed, although the direct impact of shocks in stock valuations in emerging markets is relatively low due to moderate foreign investments, financial turbulence in this area is often perceived as a signal of a slowdown in global economic growth, and thus aggregate demand.

Betas on HP show that risk sharing/appetite preferences partly explain the dynamics of risk premia in the period that coincides with the dramatic rise in oil prices between 2001 to the end of 2005, and in the aftermath of the great financial crisis of 2008/2009. During this period the propensity to buy futures by consumers to lock in oil prices increased substantially. Pressure on the demand side of futures possibly decreased the premium required by speculators to take the short side of the contract. The period 2001-2005 is more difficult to rationalize as hedging pressure was widely fluctuating around zero during this period. A possible explanation relates to the scarce risk-bearing capacity of investors during a period characterized by overall higher uncertainty in the aftermath of 9/11 attacks and the following Iraq invasion of March 20th, 2003. In this respect, e.g. Acharya et al. 2013, Cheng et al. (2015), Etula 2013 and Hong and Yogo (2012) showed that when there are limits to the risk-bearing capacity of investors and/or constraints on the amount of capital different investor categories are willing to commit, large changes in market liquidity possibly affect prices both in the futures and spot markets and ultimately affect risk premia. As a whole, the strong relevance of HP for the dynamics of expected payoffs confirms the primary relevance of futures as a risk insurance market place, as postulated by Keynes (1930) and Hicks (1939).

[21] For instance, the IMF Economic Outlook 2016 states that growth in developing economies accounted for over 70 percent of global growth in 2016.

A substantial, positive, effect is also played by TSMOM, which can be generated by psychological biases of market participants and informational frictions that delay their learning about fundamentals (see, e.g. Cutler et al. 1990, Greenwood and Shleifer 2014, and Singleton 2014). More importantly, time series momentum aims at capturing the changes in trading activity of feedback traders. In fact, as shown by Cutler et al. (1990), the demand for futures contracts by feedback (momentum) traders depend on past market performances. By the same token, our results confirm the findings of Kang et al. (2014), that show the importance of speculators following momentum strategies in determining the market demand for liquidity, and ultimately risk premia. Indeed, as shown by Kang et al. (2014), momentum traders increase the demand for liquidity, which need to be absorbed by risk-averse market makers and hedgers who will require therefore appropriate risk compensation. Surprisingly, other economic fundamentals such as Inventories, Exchange rates, and Value do not play a sensible role in the dynamics of crude oil risk premia. Figure 8 shows the time-varying betas for each risk factor on Copper ex-ante risk premia.

[Insert Figure 8 about here]

Except for few differences, much of the results of Oil also holds for Copper, which is not surprising as industrial metals and energy commodities are commonly sensitive to fluctuations over the business cycle and share most of the risk factors exposures and similar storage costs (see, e.g. Bhardwaj et al. 2015). The impact of emerging markets is increasing in the aftermath of the great financial markets. As for crude oil, this is possibly due to the increasing impact of demand of Copper from Asian markets, and China in particular.

The positive effect of OI on risk premia is consistent with the idea that increasing market activity signals changes in economic conditions, which, in turn, increases the marginal propensity of hedgers to take a net long/short position, generating price pressure on futures. This result is in line with Hong and Yogo (2012) that showed how OI has a significant predictive power for realized risk premia in futures markets in the presence of hedging demand and limited risk capacity. Also, the significant betas $\beta_{OI,t}$ provide some indirect evidence on the financialization of commodity markets whereby commodity risk premia are no longer simply determined by their supply-demand but are also affected by aggregate investment behavior (see, e.g. Tang and Xiong 2012).

While the positive effect of TSMOM is similar to crude oil, the effect of inventories and realized volatility is much different. Indeed, in the longer-term, changes in inventories negatively affect risk premia. A possible explanation lies in the fact that inventories proxy supply-demand imbalances; a positive shock in stockpiles negatively correlates with prices, which in turn increases the risk premium required by speculators to take the long side of futures contracts. Figure 9 shows the time-varying betas for Corn.

[Insert Figure 9 about here]

Figure 6 shows that, unlike WTI Oil and Copper, risk premia on Corn are not affected by possible shocks from emerging markets. Similarly, except few nuances, betas on realized volatility, value and hedging pressure are not significant across the sample. Most of the explanatory power is limited to open interests, time-series momentum and USD TW index. Momentum in agriculturals is mostly generated by irregular production. Taking Corn as our example, consumer demand remains fairly stable throughout the year whilst production is seasonal and can vary hugely. For instance, a bad harvest in October/November in the U.S. (which represents around 40% of the global production) cannot be rebalanced until a good harvest occurs in the south hemisphere the next production cycle or in the U.S. the next year, increasing prices and possibly generating positive momentum as supply expectations are revised downward, and stockpiles decrease. The corresponding time-varying betas tell us that, except for the great financial crisis of 2008/2009, fluctuations in production make futures contracts more expensive on average. This is partly confirmed by the negative effect of changes in inventories, which becomes negative and significant towards the end of the sample.

Also, Figure 9 shows that risk premia on Corn turn out to be related to the exchange rate. Time-varying betas show a positive and significant effect of FX shocks mostly during the first decade of the 2000s, while for $h = 4$ major positive effects appear across 2011/2012. The positive effect of USD TW is somewhat expected as the U.S. represents on itself 40% of the global production for Corn. A strong dollar generally leads to lower exports for the U.S. as a consequence of lower demand given less competitive prices but also means that the production of Corn will become more profitable (see, e.g. Hamilton 2009). These effects combined makes overall more expensive to take the short side of a futures contract, therefore increasing the premium required by, for instance, speculators to sell contracts to hedgers. Another possible explanation relies on the increasing financialization of the agricultural commodity markets. As shown by Tang and Xiong (2012), after 2004, agricultural commodities included in financial indexes such as the Goldman Sachs Commodity Index (GSCI) and the Dow Jones (DJ)-AIG, became much more responsive to shocks to the U.S. dollar exchange rate. Finally, Figure 10 shows the time-varying betas for Silver.

[Insert Figure 10 about here]

Figure 6 shows that, similarly to Oil and Copper, betas on S&P500, Value and USD TW are not significant throughout the sample. On the other hand, β_{HP} tend to be negative for short-term contracts for the period 2003-2013. This period coincides with a massive increase in futures prices. The imbalance between short and long contracts was consistently positive during this period, i.e. hedgers were net short, although slightly decreasing. Given prices were constantly increasing, a further positive change in HP would make cheaper to take the long side of the contract, which means a lower premium is required to bear the risk of decreasing prices. Across the same period, β_{RVol} are negative and significant. The fact that the effect of uncertainty is opposite than Copper is no surprise. In fact, a closer look at expected price dynamics (see, e.g. Figures 5) shows that while uncertainty is associated with declining prices for Copper, the opposite holds for Silver.

4.2.1 How Reliable are Adaptive Expectations?

One may argue that the ex-ante risk premia extracted from equation (7) merely represent expectations errors which have nothing to do with investors' preferences and/or the actual expectation formation process. In this section, we address this concern by directly testing the consistency of our model output, i.e. expected spot prices and ex-ante risk premia, with the observable realized payoffs and future spot prices.

We first investigate whether $\hat{E}_t\left[S_{t+h}\right]$ from (6) can effectively approximate latent expectations $S_{t+h|t}$. More specifically, we compare the forecasting performance of the model-implied expectations against alternative specifications which are mostly used in the forecasting literature to predict future spot prices S_{t+h}. As a performance metric, we use the out-of-sample R^2 statistics, R^2_{OS}, suggested by Campbell and Thompson (2008) to compare our benchmark forecast with alternative predictions.

The R^2_{OS} is akin to the standard in-sample R^2 statistics and is computed as one minus the ratio of the Mean Squared Prediction Error (MSPE) obtained from the alternative model and the one obtained from our benchmark (see, e.g. Rapach et al. 2010). In this respect, the R^2_{OS} measures the improvement in forecasting future spot prices using adaptive expectations relative to the competing predictors. Thus, $R^2_{OS} > 0$ implies that adaptive learning is best performing according to the MSPE metric. Following Rapach et al. (2010), statistical significance for the R^2_{OS} statistic is based on the p-value for the Clark and West (2007) out-of-sample MSPE; the statistics corresponds to a one-side test of the null hypothesis that the competing specification for the expected future spot prices has equal forecasting performance that our benchmark adaptive expectations against the alternative that the competing model has a lower average square prediction error. We use the first ten years of data, i.e. 01:1995-12:2005 to train the model of adaptive expectations, such that the out-of-sample evaluation period on which R^2_{OS} is computed is 01:2006-01:2016. Table 3 reports the results; a number is highlighted in grey when the null hypothesis is rejected at least at a 5% significance level.

[Insert Table 3 about here]

The first raw shows the model performance with respect to $\hat{E}_t\left[S_{t+h}\right] = F_t^{(h)}$, i.e. using futures as proxy for expectations. Adaptive learning compares favorably in seven out of twelve cases, four of which are significant at 5% level. Except for Corn, simply using futures to predict future spot prices seem beneficial for Corn. This is in line with Alquist and Kilian (2010). A similar result is found by assuming expectations are restricted to be equal to current spot prices, i.e. $\hat{E}_t\left[S_{t+h}\right] = S_t$. However, for Corn there is a significant under-performance, although small in magnitude (from -0.8% at $h = 2$ to -2% at $h = 4$). Finally, we compare the forecasting ability of adaptive expectations against a baseline futures spread indicator, i.e. $\hat{E}_t\left[S_{t+h}\right] = S_t\left(1 + \ln\left(F_t^{(h)}/S_t\right)\right)$, (see Alquist and Kilian 2010). Our model of adaptive expectations compares favorably in eight out of twelve cases, i.e. $R^2_{OS} > 0$; the improvement is significant in five out of eight cases.

A second check should be made is to investigate the correlation between expected and realized payoffs. Expected payoffs are extracted from our model according to equation (7), and realized returns are computed as the excess rolling return in the generic contract for the same maturity of the corresponding model-based expectations. Figure 11-12 show the scatter plots of ex-ante vs realized risk premia for $h = 2$ and $h = 4$ maturities, respectively. The red line represents the regression line; betas and asymptotic t-statistics are reported within the graphs.

[Insert Figures 11-12 about here]

Few comments are in order; first, the scatter plots make clear that there is a significant positive correlation between the model-implied risk premia and the observable rolling returns, across maturities. The correlation is higher for Silver and lower for WTI Crude Oil. Second, as we would expect, the correlation between expected and realized payoffs becomes lower as the contracts maturity increases. This is possibly due to the fact that as the maturity of the contract increases, it is more likely that investors make mistakes in forecasting future spot prices, therefore increasing the gap between ex-ante and ex-post returns (see, eq. (2) and Section 2 for a full discussion).

5 Concluding Remarks

Our empirical analysis shows that investor expectations of future commodity spot prices can be approximated by an adaptive learning scheme in which expected future spot prices are revised in line with past prediction errors and changes in aggregate demand. We use this expectations formation mechanism to extract time-varying (ex-ante) risk premia from futures across different commodities and maturities.

By using a dynamic linear regression in which we accommodate uncertainty in the estimated coefficients and their degree of time-variation, we show that the dynamics of commodity risk premia is predominantly driven by market activity and the changing nature of market participants, as proxied by open interests, hedging pressure and time-series momentum. Further, we show that our model of adaptive expectations compares favorably to other commonly used specifications in forecasting future spot prices and generates expected payoffs which are consistently linked to the actual, observable, returns on same-horizon futures contracts.

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Appendix

A A Simple Model of Adaptive Expectations

We start from a simple rational expectations model which is closely related to the Muth (1961) market model with inventory speculation except demand shocks are predictable and not i.i.d. The market behavior is characterized by an infinite horizon, discrete time model with a market clearing condition that holds in each period, $t+1$;

$$C_{t+1} + I_{t+1} = Q_{t+1} + I_t, \tag{A.1}$$

where Q_{t+1} represents the output produced for a commodity in a period lasting as long as the production lag, C_{t+1} is the amount of commodity consumed in the same time period, and I_{t+1} the commodity inventories at the end of period $t+1$. The standard Muth (1961) market model posits there are three categories of economic agents active in the market for commodities; the buyers, the producers and the inventory holders. The latter can capture speculation effects. The utility of price-taking consumers is declining in the current market price S_{t+1} and affected by an aggregate persistent demand shock z_t. On the other hand, the utility of risk-averse producers is positively related to expected spot prices $E_t S_{t+1}$, while inventories decisions depend on the expected capital gain of holding a unit of commodity. As a result, aggregate demand, supply and holding functions are defined as

$$C_{t+1} = A - \delta S_{t+1} + z_{t+1}, \tag{A.2}$$

$$Q_{t+1} = \lambda E_t S_{t+1} + u_{t+1}, \tag{A.3}$$

$$I_{t+1} = \nu \left(E_t S_{t+1} - S_{t+1} \right), \tag{A.4}$$

with ν be a rescaled risk-aversion parameter. We extend the standard market model with inventory speculation assuming exogenous factors that affect aggregate demand are predictable and potentially persistent;

$$z_{t+1} = b z_t + e_{t+1}, \tag{A.5}$$

with e_{t+1} and u_{t+1} zero-mean i.i.d. disturbance terms. Storage costs are assumed to be zero to simplify the model. These equations and assumptions are the same of the original Muth (1961) model, except for the predictability of demand shocks. Substituting (A.2)-(A.5) in the equilibrium condition (A.1), the spot market equilibrium can be expressed in terms of prices, price expectations, demand shocks and disturbances;[22]

$$\begin{aligned} A - (\nu + \delta) S_{t+1} + b z_t + e_{t+1} &= \lambda E_t S_{t+1} + u_{t+1} - \nu S_t, \\ (\nu + \delta) S_{t+1} &= A + b z_t + e_{t+1} - \lambda E_t S_{t+1} - u_{t+1} + \nu S_t, \end{aligned} \tag{A.6}$$

which can be rewritten as a simple linear model as follows

$$S_{t+1} = \mu + \beta E_t S_{t+1} + \theta S_t + \omega z_t + \eta_{t+1}, \tag{A.7}$$

By taking expectations on both sides and substituting back in (A.7), we can obtain a unique reduced-form Rational Expectations Equilibrium (REE) as

$$S_{t+1} = \phi_0 + \phi_1 S_t + \phi_2 z_t + \eta_{t+1}, \tag{A.8}$$

with $\phi_0 = (1-\beta)^{-1} \mu$, $\phi_1 = (1-\beta)^{-1} \theta$, $\phi_2 = (1-\beta)^{-1} \omega$ and $\eta_{t+1} = e_{t+1} - u_{t+1}$. This solution is the same as the original Muth (1961)'s model except that future commodity spot prices now depends on aggregate demand. Notice that for a given level of commodity prices, Eq. (A.8) implies that a positive (negative) shock to aggregate demand

[22] We assume there is a period distance in the future where the forward expectations are equivalent, i.e. $E_t S_{t+1} \equiv E_{t+1} S_{t+2}$, (see, e.g. Beck 1993).

increases (decreases) future prices, while a positive (negative) shock in aggregate supply decreases (increases) prices.

A.1 Introducing a Futures Market

We now introduce a futures market upon the process of price formation and show that the functional form of the perceived law of motion under rational expectations is observationally equivalent to Eq.(A.8). By including a futures market we explicitly consider the effect of hedging in the decision-making process of a representative investor. We assume that suppliers, buyers and inventory holders now hedge their commodity positions by trading on futures. Following Turnovsky (1983), Kawai (1983), and Beck (1993), we start from the assumption that agents are making production, storage and hedging decisions simultaneously, depending on current futures and expectations of spot prices.[23] All agents are assumed to act as hedgers as well as speculators in the futures market. Assuming now futures prices and spot price expectations are linearly linked to each other, aggregate demand, supply and holding functions are defined as

$$
\begin{aligned}
C_{t+1} &= A - \delta F_t + z_{t+1}, && \text{(A.9)}\\
Q_{t+1} &= \lambda F_t + u_{t+1}, && \text{(A.10)}\\
I_{t+1} &= \xi\left(F_t - S_t\right), && \text{(A.11)}\\
-X_t^b &= \chi^b\left[F_t - E_t S_{t+1}\right] - \tilde{C}_{t+1}, && \text{(A.12)}\\
X_t^p &= \chi^p\left[F_t - E_t S_{t+1}\right] + \tilde{Q}_{t+1}, && \text{(A.13)}\\
X_t^i &= \chi^i\left[F_t - E_t S_{t+1}\right] + \tilde{I}_t, && \text{(A.14)}
\end{aligned}
$$

where ξ represents the inverse of storage cost per unit of commodity, and X_t^b, X_t^p and X_t^i represent the speculative positions, i.e. excess supply of futures contracts, by buyers, producers and inventory holders, respectively. Planned levels of consumption, production and inventories denoted as $\tilde{C}_{t+1}, \tilde{Q}_{t+1}$ and $\tilde{I}_{t+1}$, indicate that commodity positions are completely hedged in the futures market. The market clearing condition on the futures market states that the aggregate excess supply of futures contract should be zero, i.e.

$$X_t^p + X_t^i - X_t^b = 0,, \tag{A.15}$$

Substituting (A.9)-(A.11) in the market clearing condition (A.1), the spot market equilibrium can be expressed in terms of both futures and spot prices, demand shocks and disturbances, i.e.

$$A - \delta F_t + bz_t + e_{t+1} + \xi\left(F_{t+1} - S_{t+1}\right) = aF_t + u_{t+1} + \xi\left(F_t - S_t\right), \tag{A.16}$$

Similarly, by substituting (A.12)-(A.14) in (A.15) we obtain;

$$
\begin{aligned}
\chi^f\left[F_t - E_t S_{t+1}\right] + \tilde{Q}_{t+1} + \chi^i\left[F_t - E_t S_{t+1}\right] + \tilde{I}_t + \chi^b\left[F_t - E_t S_{t+1}\right] - \tilde{C}_{t+1} = 0,\\
\chi^f\left[F_t - E_t S_{t+1}\right] + \lambda F_t + \chi^i\left[F_t - E_t S_{t+1}\right] + \xi\left(F_t - S_t\right) + \chi^b\left[F_t - E_t S_{t+1}\right] - A + \delta F_t = 0,
\end{aligned}
$$

Where $\tilde{Q}_{t+1}, \tilde{I}_t$ and $\tilde{C}_{t+1}$ are defined as (A.9)-(A.11) without the error terms. Solving for F_t we have that

$$F_t = \overline{A} + \overline{\chi} E_t S_{t+1} + \overline{\xi} S_t, \tag{A.17}$$

[23] More specifically, we assume buyers are intermediate producers, which therefore as well willing to reduce risk hedging their positions participating in the futures contract.

with $\overline{A} = A/a$, $\overline{\chi} = \chi/a$ and $\overline{\xi} = \xi/a$, where $a = (\chi + \lambda + \xi - \delta)$ and $\chi = \chi^p + \chi^b + \chi^i$. Similarly, F_{t+1} can obtained as a function of $E_{t+1}S_{t+2}$ and S_{t+1}, and substitute these values into (A.16) to obtain;

$$\begin{aligned} \xi\left(\overline{\xi} - 1\right) S_{t+1} &= \delta\overline{A} + \overline{\chi}\,(a + \delta)\, E_t S_{t+1} + \overline{\chi}\,(\chi + \delta)\, S_t + b z_t + e_{t+1} - u_{t+1}, \\ S_{t+1} &= \mu + \beta E_t S_{t+1} + \theta S_t + \omega z_t + \eta_{t+1}, \end{aligned} \tag{A.18}$$

with $\mu = \delta\overline{A}/\xi\left(\overline{\xi} - 1\right)$, $\beta = \overline{\chi}\,(a + \delta)\,/\xi\left(\overline{\xi} - 1\right)$, $\theta = \overline{\chi}\,(\chi + \delta)\,/\xi\left(\overline{\xi} - 1\right)$, and $\omega = b/\xi\left(\overline{\xi} - 1\right)$, respectively. Equation (A.18) is analogous to (A.29) and can be solved in the same way. From (A.18), the solution procedure described above yields the same Perceived Law of Motion (PLM);

$$S_{t+1} = \phi_0 + \phi_1 S_t + \phi_2 z_t + \eta_{t+1}, \tag{A.19}$$

with $\phi_0 = (1 - \beta)^{-1}\mu$, $\phi_1 = (1 - \beta)^{-1}\theta$, $\phi_2 = (1 - \beta)^{-1}\omega$ and $\eta_{t+1} = e_{t+1} - u_{t+1}$. To summarize, we show that by introducing a futures market in which different type of investors hedge their positions in physical commodities, the reduced form PLM has the same functional form of the case without a futures market. In the following section we introduce recursive learning on the reduced-form parameters ϕ_0, ϕ_1 and ϕ_2 in Eq.(A.19).

A.2 Learning Dynamics

The key assumption to introduce learning is that the expectations of economic agents $E_t\left[S_{t+1}\right]$ are not necessarily rational as agents do not know the structural parameters. Expectations are instead formed on the basis of current observations and predictions of parameters which are updated over time. There are two key building blocks to explicit the agents' learning dynamics. First, agents beliefs are described by means of a dynamic model. We assume the PLM as the same functional form of the REE (A.19), where the true values $\phi = (\phi_0, \phi_1, \phi_2)$ are not known. Second, we need to describe how agents obtain estimates for the parameters of the PLM. We explicit agents' recursive estimates in terms of a Bayesian prior that describes how coefficients in the PLM drift at each time t;

$$\begin{aligned} S_{t+1} &= \phi_{t+1}' X_t + \eta_{t+1}, &\quad \text{with} \quad & \eta_{t+1} \sim N\left(0, \sigma^2\right), \\ \phi_{t+1} &= \phi_t + \epsilon_{t+1} &\quad \text{with} \quad & \epsilon_{t+1} \sim N\left(0, \Omega\right), \end{aligned} \tag{A.20}$$

with $\phi_t = (\phi_{0,t}, \phi_{1,t}, \phi_{2,t})$ and $X_t = (1, S_t, z_t)$. The shock η_{t+1} is uncorrelated with ϵ_{t+1}, and $\Omega << \sigma^2 I$. The innovation covariance matrix Σ governs the perceived volatility of increments to the parameters, and is a key component of the model (see Sargent and Williams 2005). Agents' recursive optimal estimate of ϕ_{t+1} conditional on information available up to time t. $\gamma_{t+1} = \hat{\phi}_{t+1|t}$ are provided by the Kalman filter recursion;

$$\begin{aligned} \gamma_{t+1} &= \gamma_t + K_t\left(S_{t+1} - \gamma_t' X_t\right), \\ R_{t+1} &= R_t - \frac{R_t X_t X_t' R_t}{X_t' R_t X_t + 1} + \sigma^{-2}\Omega, \end{aligned} \tag{A.21}$$

where $K_t = R_t X_t \left(X_t' R_t X_t + \sigma^2\right)^{-1}$ determines the degree of updating of agents' beliefs when faced when an unexpected commodity spot price $S_t - \gamma_t' X_t$, i.e. Kalman gain. The recursive learning dynamics (A.20) represents a generalization of a recursive learning with constant gain as specified in Evans and Honkapohja (2001), Sargent (2002), Cho et al. (2002), and Williams (2003), among others.

B Econometric Design

In the following we specify the dynamic regression model used to capture the time-varying linkages between the ex-ante risk premia and the corresponding explanatory variables. Denoting these by Z_t the set of economic predictors a dynamic regression can be specified as a state-space model;

$$y_t = Z_t'\theta_t + v_t, \qquad v_t \sim N(0, H), \tag{A.22}$$

$$\theta_t = \theta_{t-1} + \varepsilon_t, \qquad \varepsilon_t \sim N(0, W), \tag{A.23}$$

The vector θ_t consists of unobservable, time-varying, regression coefficients (see West and Harrison 1997 for more details on dynamic linear models). The observational H and state variances W are estimated using the whole sample of observations of risk premia and factors. As such, although the "betas" are time-varying, the structural variances are considered constant over time.[24]

The sequential model description in (A.22)-(A.23) requires that the defining quantities at time t be known at that time. Let D_0 contains the initial prior information about the elasticities and structural variances. We assume prior information about θ_0 is vague and centered around the initial hypothesis of no effect of risk factors on premia, i.e. $\theta_0|D_0 \sim N(c_0, C_0)$, with $c_0 = 0$ and $C_0 = 10,000$. Also, we assume that the impact of risk factors is highly uncertain and volatile, as captured by an Inverse-Wishart distribution with small degrees of freedom and large scale parameter, i.e. $W|D_0 \sim IW(a_0, A_0)$ with $a_0 = 3$ and $A_0 = 10,000$. As a result we assume that when no historical information on expected risk premia and factors is available, elasticities are mainly driven by idiosyncratic risk as proxied by an Inverse-Gamma distribution with uninformative hyper-parameters, i.e. $H|D_0 \sim IG(n_0/2, n_0N_0/2)$ with $n_0 = 0.001$ and $N_0 = 0.001$. Notice priors are constant for all maturities $h = 2, 3, 4$ quarters.

In the following we provide details of the Gibbs sampler we use for the estimation of the dynamic linear model (A.22)-(A.23). For the ease of exposition, we disregard the maturity super-script h. Let us denote $\mathbf{x}_{s:t} = (\mathbf{x}_s, \ldots, \mathbf{x}_t)$, $s \leq t$, the set of vectors $\mathbf{x}_u$. The collections of parameters is defined as $\Theta = (\theta_{1:T}, W, H)$, respectively, where $\theta_{1:T}$ represents the $(T \times N)$ matrix of state parameters. Let θ_0 represents the initial value of the dynamic sensitivity to the $k-$dimensional vector of regressors. The complete likelihood function can defined as

$$p(\mathbf{y}_{1:T}, \theta_{1:T}|\mathbf{Z}_{1:T}, W, H) = \prod_{t=2}^{T} p\left(\mathbf{y}_t|Z_t'\theta_t, H\right) p(\theta_t|\theta_{t-1}, W), \tag{A.24}$$

with $p(\mathbf{y}_t|Z_t'\theta_t, H) = N(Z_t'\theta_t, H)$ and $p(\theta_t|\theta_{t-1}, W) = N_k(\theta_{t-1}, W)$ two univariate and multivariate Gaussian distributions, respectively. Conditional on priors and the latent states $\theta_{1:T}$ the complete likelihood can be factorized as

$$\begin{aligned} p(\theta_{1:T}, W, H|\mathbf{y}_{1:T}, \mathbf{Z}_{1:T}) &\propto p(\mathbf{y}_{1:T}, \theta_{1:T}|\mathbf{Z}_{1:T}, W, H)\, p(\theta_0, W, H), \\ &= p(\mathbf{y}_{1:T}|\theta_{1:T}, \mathbf{Z}_{1:T}, H)\, p(\theta_{1:T}|W)\, p(\theta_0, W, H), \end{aligned}$$

The joint posterior distribution of the states and parameters is not tractable analytically such that the estimator for the parameters cannot be obtained in closed form. The latent variables $\theta_{1:T}$ are simulated alongside the model parameters H and W. At each iteration, the sampler sequentially cycles through the following steps:

1. Draw $\theta_{1:T}$ conditional on H, W and the data $\mathbf{y}_{1:T}, \mathbf{Z}_{1:T}$.
2. Draw W conditional on $\theta_{1:T}$.
3. Draw H conditional on $\mathbf{y}_{1:T}, \mathbf{Z}_{1:T}$, and $\theta_{1:T}$.

In what follows we provide details of each step of the Gibbs sampler.

B.1 Step 1. Sampling the Conditional Factor Sensitivities $\theta_{1:T}$

The full conditional posterior density for the time-varying factor loadings is computed using a Forward Filtering Backward Sampling (FFBS) approach as in Carter and Kohn (1994). The initial prior are sequentially updated via

[24] However, the framework could be easily extended by using an exponential weighted moving average recursion to obtain dynamic estimates for $H_{k,t}$ and $W_{k,t}$. We leave this for future research.

the Kalman filtering recursion. Conditionally on idiosyncratic risk H, state variance W, and assuming an initial distribution $\theta_0|y_0 \sim N(m_0, C_0)$, it is straightforward to show that the (see West and Harrison 1997 for more details)

$$\begin{aligned} \theta_t|Z_{1:t-1}, W &\sim N(a_t, R_t) && \textit{Propagation Density} \\ Y_t|Z_{1:t-1}, H &\sim N(f_t, Q_t) && \textit{Predictive Density} \\ \theta_t|Z_{1:t} &\sim N(m_t, C_t) && \textit{Filtering Density} \end{aligned}$$

with

$$\begin{aligned} a_t &= m_{t-1} & R_t &= C_{t-1} + W \\ f_t &= Z_t' a_t & Q_t &= Z_t R_t X_t' + H \\ m_t &= a_t + K_t e_t & C_t &= R_t - K_t Q_t K_t' \end{aligned} \tag{A.25}$$

and $K_t = R_t X_t Q_t^{-1}$ and $e_t = y_t - f_t$. Conditional thetas are drawn from the posterior distribution which is generated by backward recursion (see Frühwirth-Schnatter 1994, Carter and Kohn 1994, and West and Harrison 1997), i.e. $p(\theta_t|\mathbf{y}_{1:T}) = N_k\left(m_t^b, C_t^b\right)$, with

$$\begin{aligned} m_t^b &= (1 - B_t)\, m_t + B_t m_{t+1}^b, \\ C_t^b &= (1 - B_t)\, C_t + B_t^2 C_{t+1}^b, \qquad \text{with} \qquad B_t = \frac{C_t}{C_t + W}, \end{aligned}$$

B.2 Step 2. Sampling the State Variance Parameters W

Conditional on the risk exposures, the estimate of the state variance covariance matrix coincide with the update of an Inverse-Wishart distribution. Posterior estimates are obtained by updating the prior structure as

$$W|\theta_{1:T} \sim IW(a_1, A_1) \tag{A.26}$$

with

$$\begin{aligned} a_1 &= a_0 + T \\ A_1 &= A_0 + \hat{\boldsymbol{\varepsilon}}\hat{\boldsymbol{\varepsilon}}' \end{aligned}$$

where $\hat{\boldsymbol{\varepsilon}}' = (\hat{\varepsilon}_1, \ldots, \hat{\varepsilon}_T)$ and $\hat{\varepsilon}_t = \hat{\theta}_t - \hat{\theta}_{t-1}$ given $\hat{\theta}_t = m_t^b$.

B.3 Step 3. Sampling the Idiosyncratic Risk H

For the posterior estimates of the idiosyncratic risk we exploit the fact that the prior and the likelihood are conjugate. The updating scheme is easily derived as

$$H|\theta_{1:T}, \mathbf{Z}_{1:T}, \mathbf{y}_{1:T} \sim IG(\nu_1/2, \nu_1 N_1/2) \tag{A.27}$$

with

$$\begin{aligned} \nu_1 &= \nu_0 + T \\ \nu_1 N_1 &= \nu_0 N_0 + \hat{\boldsymbol{v}}\hat{\boldsymbol{v}}', \end{aligned}$$

where $\hat{\boldsymbol{v}}' = (\hat{v}_1, \ldots, \hat{v}_T)$ and $\hat{v}_t = y_t - Z_t'\hat{\theta}_{t-1}$ given $\hat{\theta}_{t-1} = m_{t-1}^b$

C Testing Extrapolative Expectations

At the outset of the paper we argue that our model of adaptive expectations closely track the average forecasts of professional analysts, which in turn represents an approximation of investors' expectations. In this section we test for the null hypothesis that average survey forecast is consistent with an adaptive learning framework. In its most general formulation, the model for adaptive expectations have a limited number of testable implications; the most important of which is the impact of past information on current forecasts (see, e.g. Frenkel and Froot 1987 and Pesaran and Weale 2006 for more details on testing rationality and adaptivity on survey forecasts).

We test for a general rule of updating by estimating the impact of current prices on expectations. Let $E_t\left[\Delta S_{t+h}\right]$ represents the investors' expectations at time t for a change in the future spot price from t to $t+h$. To test adaptivity we first estimate the following regression model;[25]

$$E_t\left[\Delta S_{t+h}\right] = \alpha + \beta \Delta S_t + e_t, \qquad \text{for} \qquad h = 2, 3, 4, \qquad \text{quarters}, \tag{A.28}$$

with $\Delta S_t = (S_t - S_{t-h})$ representing past changes in spot prices. We use past spot prices as, once become observable, they are assumed to summarize all the relevant current information which is readily available to professional analysts (see, e.g. Sockin and Xiong 2015). The regression equation (A.28) states that if a commodity has been recently depreciated, then it will be expected to depreciate in the near future as well. Strong rationality would imply the null hypothesis that there is no "learning" from past information, i.e. $H_0 : \beta = 0$. Panel A of Table C.1 shows the results.

[Insert Table C.1 about here]

Interestingly, the slope coefficients are all negative and strongly significant meaning that a recent depreciation of a commodity leads to an optimistic view on future spot prices, and vice versa. Such dynamics does not rule out the possibility of having positive autocorrelation in investors' expectations. Building on this result, we now test the further restriction that expectations are adaptive. Adaptive learning is the most prominent form of extrapolative expectations formation process (see, e.g. Nerlove 1958, Evans and Honkapohja 2001, Cho et al. 2002, Sargent 2002, Williams 2003, Sargent et al. 2004, Sargent and Williams 2005 and Malmendier and Nagel 2015, to cite a few). Under this model investors adjust their expectations in line with past prediction errors. In general, adaptive expectations need not be informationally efficient, and forecast errors can be serially correlated. We test the adaptive expectations hypothesis by regressing the expected price change on the lagged survey prediction error;

$$E_t\left[\Delta S_{t+h}\right] = \mu + \delta\left(E_{t-h} S_t - S_t\right) + \nu_t, \qquad \text{for} \qquad h = 2, 3, 4, \qquad \text{quarters}, \tag{A.29}$$

Panel B of Table C.1 shows the results. The slope coefficients are positive and statistically significant across forecasting horizons and commodity markets. This implies that investors, on average, place positive weight on previous prediction errors. To summarize, investors' expectations on future spot prices are not static; in fact, the elasticity of the expected future spot prices with respect to past forecasting errors is positive and significant. Notably, the support for a form of adaptivity in the expectations formation process does not depend on the prediction horizon and the specific commodity market.

[25] We estimate the model by OLS with GMM corrected standard errors to account for autocorrelation and heteroschedasticity in the residuals.

Table 1. Descriptive Statistics

This table reports the descriptive statistics for the risk premia for WTI Oil Crude, Copper, Corn and Silver. The ex-ante risk premia are obtained by subtracting from the futures prices the model-implied expected future spot prices for the same maturity, $h = 2, 3, 4$ quarters ahead. WTI Crude Oil prices are in U.S. Dollars per barrel, whereas Copper prices are transformed from USD Cents/Pound USD/Tonne to match the measurement unit used in the survey forecasts. Data on Crude Oil (WTI) are from the New York Mercantile Exchange (NYMEX) and Copper are obtained from the Commodity Exchange (COMEX). Data on Silver are obtained from the Commodity Exchange (COMEX), and data for Corn are from the Chicago Board of Trade (CBOT) with price quotation in USD cents per bushel. The sample period is 01:1995-01:2016, monthly.

Panel A: Descriptive Statistics

	WTI			Copper			Corn			Silver		
	$h=2$	$h=3$	$h=4$	$h=2$	$h=3$	$h=4$	$h=2$	$h=3$	$h=4$	$h=2$	$h=3$	$h=4$
Mean	-0.006	-0.012	-0.018	-0.013	-0.019	-0.024	0.020	0.023	0.027	0.019	0.022	0.027
Median	-0.015	-0.020	-0.029	-0.010	-0.012	-0.014	0.031	0.038	0.044	0.015	0.023	0.027
St. Dev.	0.100	0.111	0.123	0.090	0.097	0.101	0.119	0.128	0.137	0.086	0.087	0.096
Skewness	-0.021	0.008	0.022	0.068	-0.360	-0.250	-0.900	-0.785	-0.902	-0.455	-0.476	-0.610
Kurtosis	0.810	0.336	0.027	1.738	1.509	0.249	1.230	0.405	1.045	1.607	1.356	1.641
Jarque-Bera	7.286	1.252	0.029	33.68	30.99	3.47	52.66	29.11	48.19	37.83	30.40	46.33
p-value	0.026	0.535	0.985	0.000	0.000	0.176	0.000	0.000	0.000	0.000	0.000	0.000

Panel B: Correlations

Commodity	h=2				h=3				h=4			
WTI	1				1				1			
Copper	0.355	1			0.280	1			0.246	1		
Corn	0.149	0.139	1		0.139	0.129	1		0.116	0.114	1	
Silver	0.252	0.267	0.220	1	0.216	0.248	0.207	1	0.190	0.198	0.179	1

Table 2. Static Regression Analysis

This table shows the results of a static regression analysis. The set of predictors Z_t is pre-whitened, i.e. regressors are orthogonal to each other and have standard deviation equal to one, to improve the signal informativeness about the unconditional ex-ante risk premia. The ex-ante risk premia are obtained by subtracting from the futures prices the model-implied expected future spot prices for the same maturity, $h = 2, 3, 4$ quarters ahead. Futures prices data on Crude Oil (WTI) are from the New York Mercantile Exchange (NYMEX), prices are in U.S. dollars per barrel. Futures on Silver and Copper are obtained from the Commodity Exchange (COMEX). S&P500 and MXEF represent monthly returns for the Standard and Poor's 500 and the MSCI Emerging Markets indexes. Hedging pressure (HP) is defined as the net excess in short futures positions by commercial traders, i.e. short minus long positions, divided by the amount of outstanding contracts. Open Interest (OIN) is defined as the total number of outstanding contracts that are held by market participants at the end of the month. The data on commercial traders futures positions are from the Commodity Futures Trading Commission (CFTC). Inventories for Copper and Crude Oil are from the London Metal Exchange (LME) and Energy Information Administration (EIA) respectively. For Corn inventories, we use the U.S. ending stocks reported in thousands of metric tonnes. USD TW stands for the Federal Reserve U.S. trade-weighted exchange rate index, normalized to be equal to one hundred in March 1973. Realized volatility (RVol) is computed as the sum of squared daily returns adjusted for roll-over. For both open interests and inventories, we take the year-on-year growth as explanatory variable. A number is highlighted in grey when the null hypothesis is rejected at least at a 5% significance level. The sample period is 01:1995-01:2016, monthly.

Panel A: Regression Analysis

		S&P500	MXEF	HP	OI	TSM	Value	Inv	USDTW	RVol	R^2_{adj}
WTI	$h = 2$	-0.207	0.319	0.138	0.075	0.144	-0.006	0.214	-0.938	-0.116	0.306
		(-1.060)	(2.001)	(1.673)	(1.917)	(5.880)	(-0.483)	(1.516)	(-3.095)	(-0.768)	
	$h = 3$	-0.182	0.317	0.118	0.084	0.109	-0.004	0.253	-0.855	-0.056	0.376
		(-0.890)	(2.084)	(1.151)	(1.813)	(4.396)	(-0.256)	(1.605)	(-2.595)	(-0.305)	
	$h = 4$	-0.147	0.304	0.086	0.102	0.078	0.003	0.270	-0.754	-0.038	0.486
		(-0.736)	(2.132)	(0.774)	(1.890)	(2.676)	(0.136)	(1.717)	(-2.163)	(-0.185)	
Copper	$h = 2$	-0.021	0.412	0.037	0.060	0.050	0.004	-0.311	-1.021	-0.260	0.345
		(-0.155)	(3.871)	(1.224)	(2.229)	(1.910)	(0.492)	(-1.890)	(-2.153)	(-1.171)	
	$h = 3$	-0.118	0.453	0.036	0.091	0.160	0.010	-0.380	-0.971	-0.336	0.360
		(-0.692)	(3.924)	(1.029)	(2.678)	(2.340)	(1.160)	(-1.986)	(-1.659)	(-1.237)	
	$h = 4$	-0.013	0.395	0.000	0.133	0.287	0.036	-0.413	-0.087	-0.051	0.370
		(-0.085)	(3.257)	(0.001)	(3.002)	(3.078)	(3.815)	(-3.170)	(-1.881)	(-0.198)	
Corn	$h = 2$	-0.008	0.052	0.208	0.098	0.093	0.072	-0.076	-1.008	0.147	0.232
		(-0.035)	(0.348)	(1.137)	(2.403)	(1.994)	(1.686)	(-1.695)	(-1.479)	(0.574)	
	$h = 3$	-0.103	0.083	0.151	0.016	0.120	0.084	-0.084	-1.095	0.372	0.301
		(-0.473)	(0.589)	(1.405)	(2.155)	(2.278)	(1.830)	(-1.065)	(-1.007)	(1.209)	
	$h = 4$	-0.078	0.088	0.089	0.027	0.133	0.094	0.087	-0.962	0.350	0.332
		(-0.321)	(0.566)	(1.796)	(2.876)	(2.509)	(1.996)	(0.800)	(-1.744)	(1.033)	
Silver	$h = 2$	-0.312	0.380	0.105	0.037	0.058	0.021		-0.918	-0.633	0.290
		(-1.386)	(2.288)	(2.713)	(1.449)	(2.740)	(1.025)		(-2.955)	(-3.375)	
	$h = 3$	-0.287	0.417	0.118	0.007	0.044	0.002		-1.130	-0.230	0.256
		(-1.409)	(1.564)	(2.731)	(1.312)	(2.158)	(0.323)		(-3.341)	(-2.482)	
	$h = 4$	-0.233	0.398	0.155	0.007	0.041	-0.005		-1.112	-0.436	0.257
		(-1.173)	(1.811)	(2.721)	(1.273)	(1.980)	(-0.596)		(-3.296)	(-3.765)	

Table 3. Future Spot Prices Out-of-Sample Forecasting Comparison

This table reports the out-of-sample goodness-of-fit statistics R^2_{OS} computed as in Campbell and Thompson (2008). Statistical significance for the R^2_{OS} statistic is based on the p-value for the Clark and West (2007) out-of-sample Mean Squared Prediction Error (MSPE); the test statistics corresponds to a one-side test of the null hypothesis that the competing specification for the expected future spot prices has equal forecasting performance that our benchmark adaptive learning specification against the alternative that the competing model has a lower average square prediction error. A number is highlighted in grey when the null hypothesis is rejected at least at a 5% significance level. The sample period is 01:1995-01:2016, monthly.

Panel A: Out-of-sample R^2 statistics ($R^2_{OS}\%$)

Predictor	WTI			Copper			Corn			Silver		
	$h=2$	$h=3$	$h=4$	$h=2$	$h=3$	$h=4$	$h=2$	$h=3$	$h=4$	$h=2$	$h=3$	$h=4$
Futures	0.042	0.616	-0.111	-0.157	0.629	0.999	-0.076	-0.106	-0.595	0.070	0.281	0.042
Current Spot	-0.095	0.227	0.772	-0.266	0.410	0.616	-0.873	-1.195	-2.035	0.040	0.229	-0.043
Spread	0.059	0.669	1.332	-0.143	0.665	1.069	-0.027	-0.089	-0.632	0.069	0.279	0.038

Table C.1. Testing Extrapolative Expectations

This table shows the results of a test for extrapolative expectations on the cross-sectional average of individual Bloomberg survey price forecasts. The sample period for the survey is 12:2006-01:2016, aggregated monthly and collected for alternative commodities and time-horizons. We exclude from the analysis the survey for Corn as the survey has lots of missing data which would make the sample size subject to small-sample biases. Regressions are estimated by GMM correcting standard errors to account for autocorrelation and heteroskedasticity in the residuals. **Panel A:** shows the results for a the null hypothesis that expectations are extrapolative in its general form. **Panel B:** shows the results for the null hypothesis that expectations are revised in line with past prediction errors on future spot prices, i.e. adaptive expectations. Robust standard errors are in parenthesis. A number is highlighted in grey when the null hypothesis is rejected at least at a 5% significance level.

		Panel A: Extrapolative Expectations				**Panel B:** Adaptive Expectations			
Horizon (Quarters)	Commodity	α	β	adj R^2	DW	μ	δ	adj R^2	DW
$h=2$	Crude Oil (WTI)	-0.002	-0.668	0.466	0.567	0.003	0.244	0.558	1.261
		(0.017)	(0.095)			(0.009)	(0.038)		
	Copper	-0.019	-0.491	0.186	1.019	-0.004	0.226	0.349	1.441
		(0.018)	(0.193)			(0.013)	(0.056)		
	Silver	0.027	-0.695	0.513	1.123	0.020	0.290	0.396	1.240
		(0.012)	(0.069)			(0.009)	(0.042)		
$h=3$	Crude Oil (WTI)	0.020	-0.691	0.324	0.391	0.0216	0.269	0.669	1.199
		(0.023)	(0.127)			(0.009)	(0.031)		
	Copper	0.002	-0.497	0.166	0.876	0.006	0.197	0.334	1.140
		(0.019)	(0.199)			(0.015)	(0.031)		
	Silver	0.041	-0.723	0.487	0.932	0.036	0.173	0.201	1.013
		(0.013)	(0.079)			(0.013)	(0.038)		
$h=4$	Crude Oil (WTI)	0.041	-0.802	0.321	0.441	0.041	0.221	0.512	0.677
		(0.027)	(0.131)			(0.013)	(0.041)		
	Copper	0.009	-0.537	0.156	0.982	0.013	0.168	0.197	1.051
		(0.021)	(0.161)			(0.019)	(0.057)		
	Silver	0.059	-0.769	0.465	1.212	0.054	0.119	0.106	1.251
		(0.015)	(0.059)			(0.017)	(0.038)		

Figure 1. Expectations Errors for Future Spot Prices

This figure shows the unexpected changes in spot prices $E_t\left[S_{t+h}\right] - S_{t+h}$ for two different horizons, i.e. $h = 2, 4$. Expectations are proxied by the cross-sectional average of the individual Bloomberg's survey of professional analysts, i.e. **Panel A:** Shows the unexpected price changes for WTI Crude Oil (USD/Barrel). **Panel B:** Shows the unexpected price changes for Silver (USD/Ounce). Data on Crude Oil (WTI) are from the New York Mercantile Exchange (NYMEX) and on Silver are obtained from the Commodity Exchange (COMEX). The sample period for the Survey is 12:2006-01:2016, aggregated monthly.

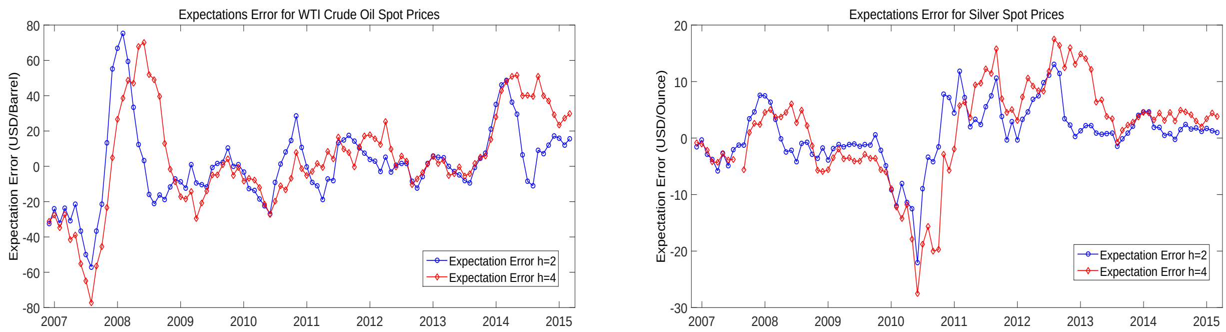
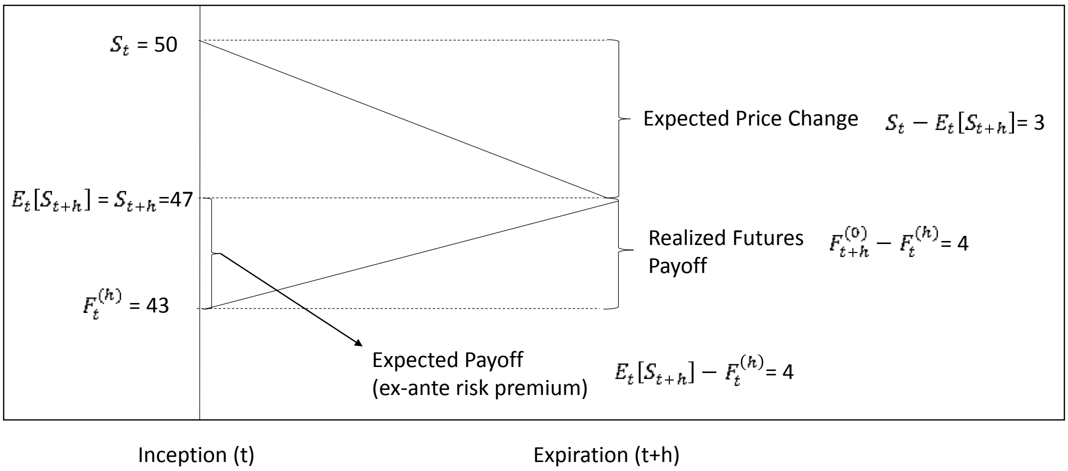


Figure 2. Ex-Ante vs Realized Risk Premia

This figure sketches the differences between the *expected* payoff, i.e. ex-ante risk premium, and the *realized* payoff of a futures position. **Panel A:** shows the payoff structure of a futures position keeping the contract until maturity under no unexpected changes in spot prices. In this case, the expected and the realized risk premia coincide. **Panel B:** shows the payoff structure of a futures position keeping the contract until maturity under a negative unexpected fluctuation in spot prices. In this case, the ex-ante and the realized risk premia diverge.



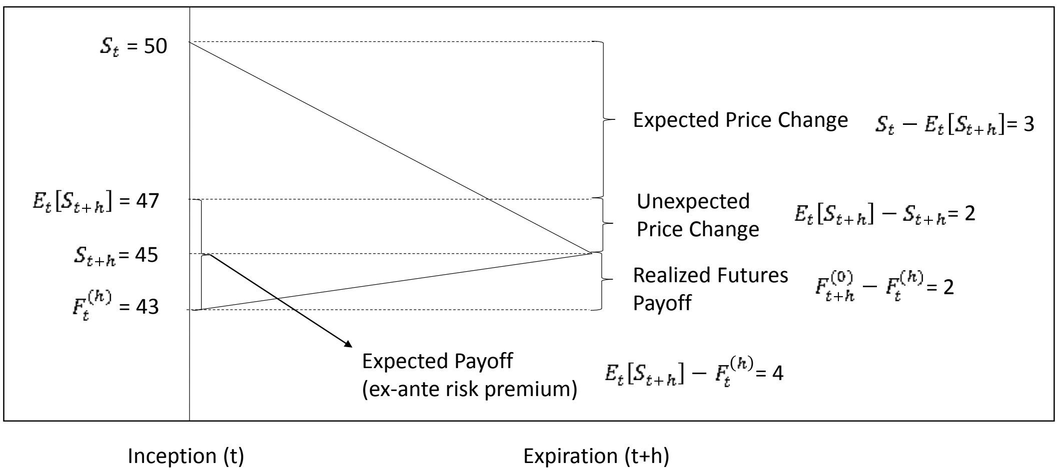


Figure 3. Spot Prices and Aggregate Demand

This figure shows the year-on-year growth rates for commodity spot prices (blue line) and the index of world industrial production (magenta line). Top panels compare the changes in world industrial production to the changes in WTI Crude Oil (top-left) and Copper (top-right) spot prices. Bottom panels compare the changes in world industrial production to the changes in Corn (top-left) and Silver (top-right) spot prices. Spot prices data on Crude Oil (WTI) are from the New York Mercantile Exchange (NYMEX), prices are in U.S. dollars per barrel. Spot prices on Silver and Copper are obtained from the Commodity Exchange (COMEX). Silver futures are quoted in U.S. dollars per troy ounce while Copper is quoted in U.S. cents/pound. The index of world industrial production published by the Netherlands Bureau of Economic and Policy Analysis, and contains aggregate information on industrial production from 81 countries worldwide, which account for about 97% of the global industrial production. The sample period is 01:1995-01:2016.

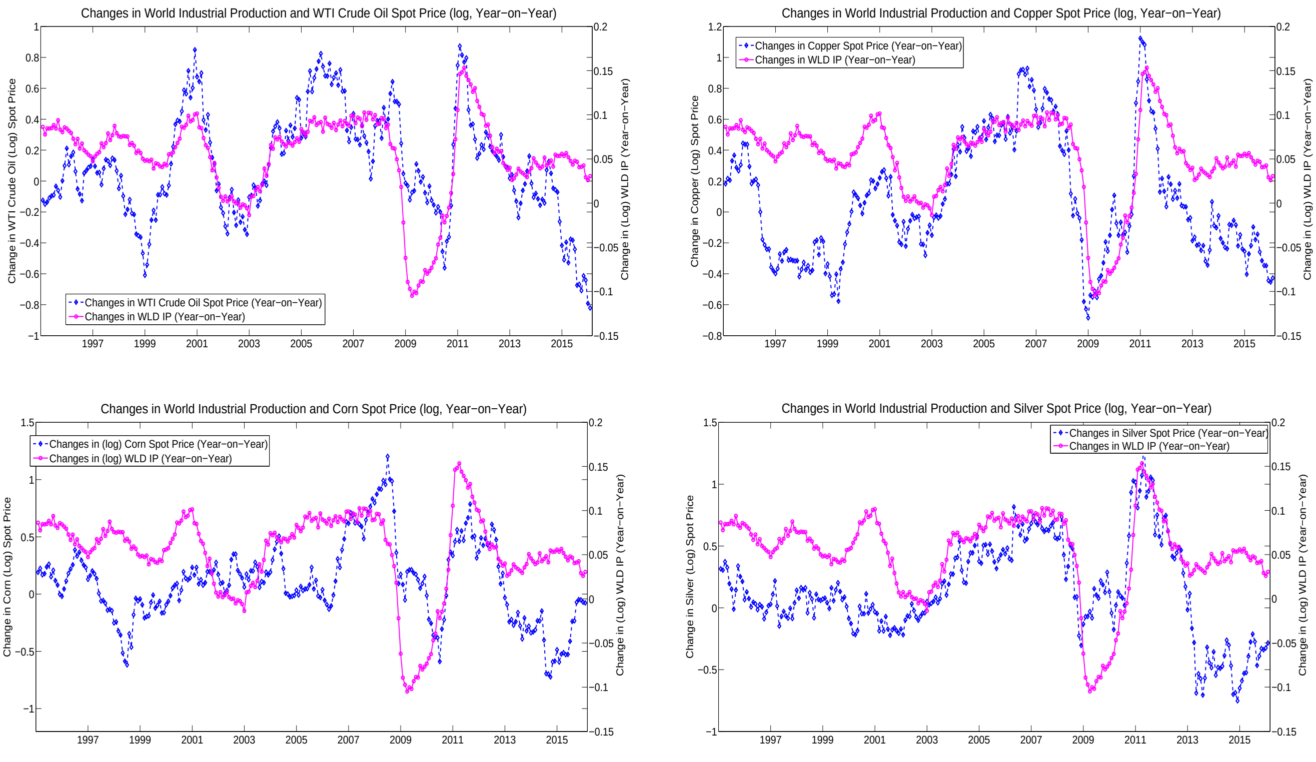


Figure 4. Model-Implied vs Survey Expectations ($h = 2$)

This figure compares the expected future spot prices implied by our model with the Survey Price Forecasts from Bloomberg's Commodity Price Forecasts Database for $h = 2$ quarters ahead. WTI Crude Oil prices are in U.S. Dollars per barrel, whereas Copper prices are transformed from USD Cents/Pound USD/Tonne to match the measurement unit used in the survey forecasts. Data on Crude Oil (WTI) are from the New York Mercantile Exchange (NYMEX) and Copper are obtained from the Commodity Exchange (COMEX). Data on Silver are obtained from the Commodity Exchange (COMEX), and data for Corn are from the Chicago Board of Trade (CBOT) with price quotation in USD cents per bushel. The shaded area at the bottom shows the difference between expectations from adaptive learning and the survey for the overlapping periods. The sample period for the Survey is 12:2006-01:2016, aggregated monthly. The sample period of the model-implied expected future spot price is 01:1995-01:2016.

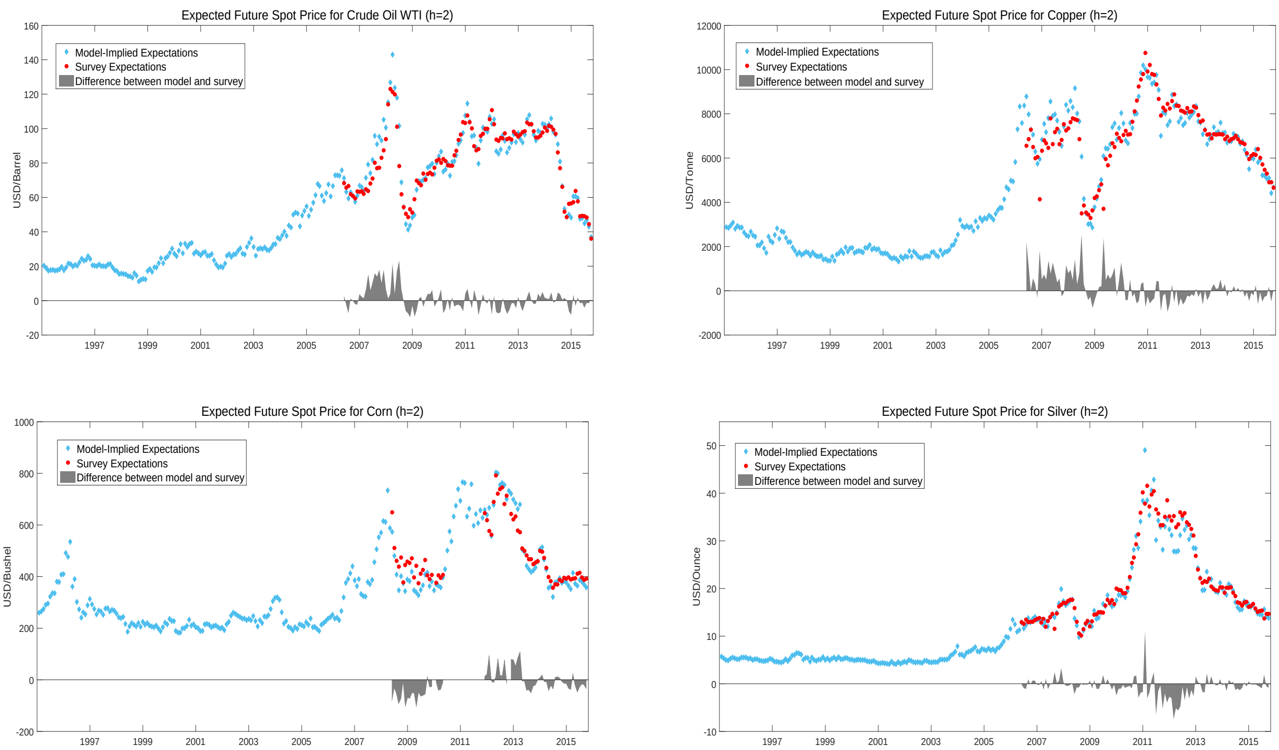


Figure 5. Model-Implied vs Survey Expectations ($h = 4$)

This figure compares the expected future spot prices implied by our model with the Survey Price Forecasts from Bloomberg's Commodity Price Forecasts Database for $h = 4$ quarters ahead. WTI Crude Oil prices are in U.S. Dollars per barrel, whereas Copper prices are transformed from USD Cents/Pound USD/Tonne to match the measurement unit used in the survey forecasts. Data on Crude Oil (WTI) are from the New York Mercantile Exchange (NYMEX) and Copper are obtained from the Commodity Exchange (COMEX). Data on Silver are obtained from the Commodity Exchange (COMEX), and data for Corn are from the Chicago Board of Trade (CBOT) with price quotation in USD cents per bushel. The shaded area at the bottom shows the difference between expectations from adaptive learning and the survey for the overlapping periods. The sample period for the Survey is 12:2006-01:2016, aggregated monthly. The sample period of the model-implied expected future spot price is 01:1995-01:2016.

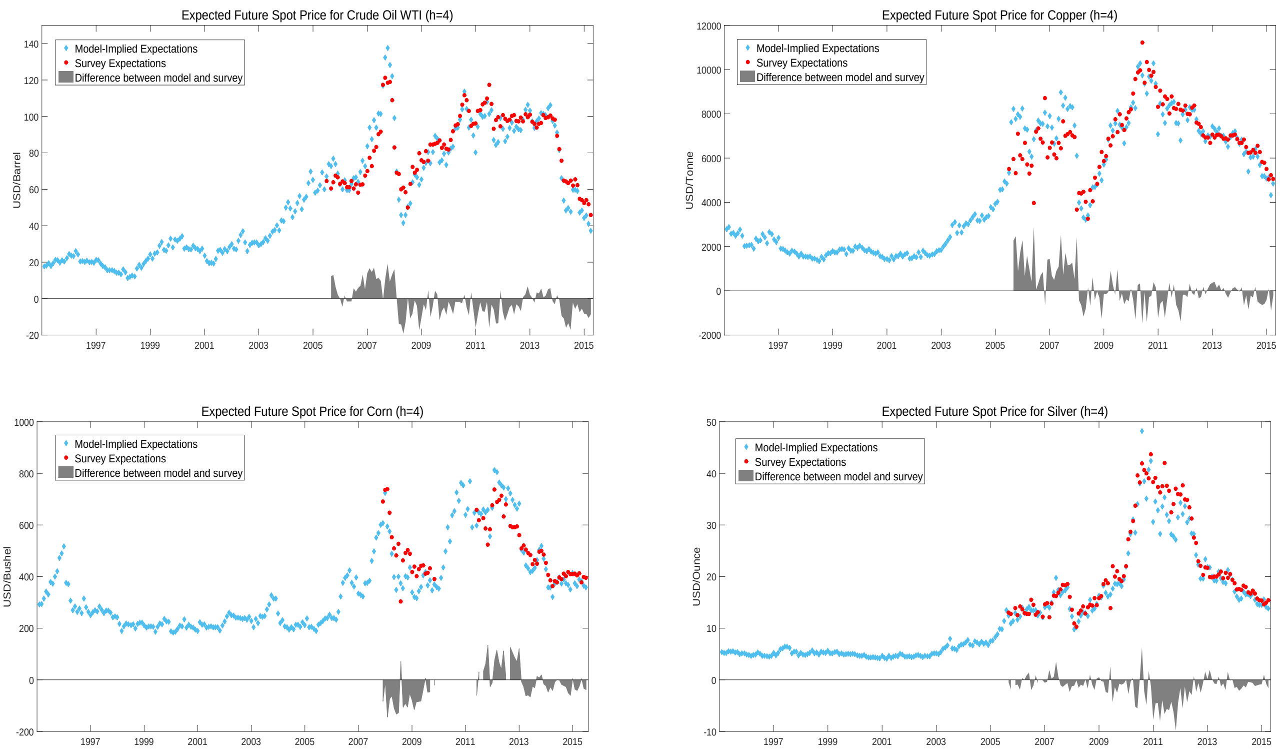


Figure 6. R^2 Decomposition

This figure shows a decomposition of the R^2 obtained from each explanatory variable in the dynamic regression of ex-ante risk premia on pre-whitened risk factors, across different commodities and expectations horizons. The ex-ante risk premia are extracted from the futures prices by using the model-implied expectations for $h = 2, 3, 4$ quarters ahead. Data are obtained from different resources. Futures prices data on Crude Oil (WTI) are from the New York Mercantile Exchange (NYMEX), prices are in U.S. dollars per barrel. Futures on Silver and Copper are obtained from the Commodity Exchange (COMEX). Silver is quoted in U.S. Dollars per troy ounce while Copper is quoted in U.S. cents/pound. We convert the price of Copper futures contracts to USD/tonne to match the measurement unit of the survey forecast, that instead refer to the London Metal Exchange (LME). Corn futures prices are from the Chicago Board of Trade (CBOT) with price quotation in USD cents per bushel. Details on the explanatory variables are outlined in Section 4. The sample period 1995:01-2016:01.

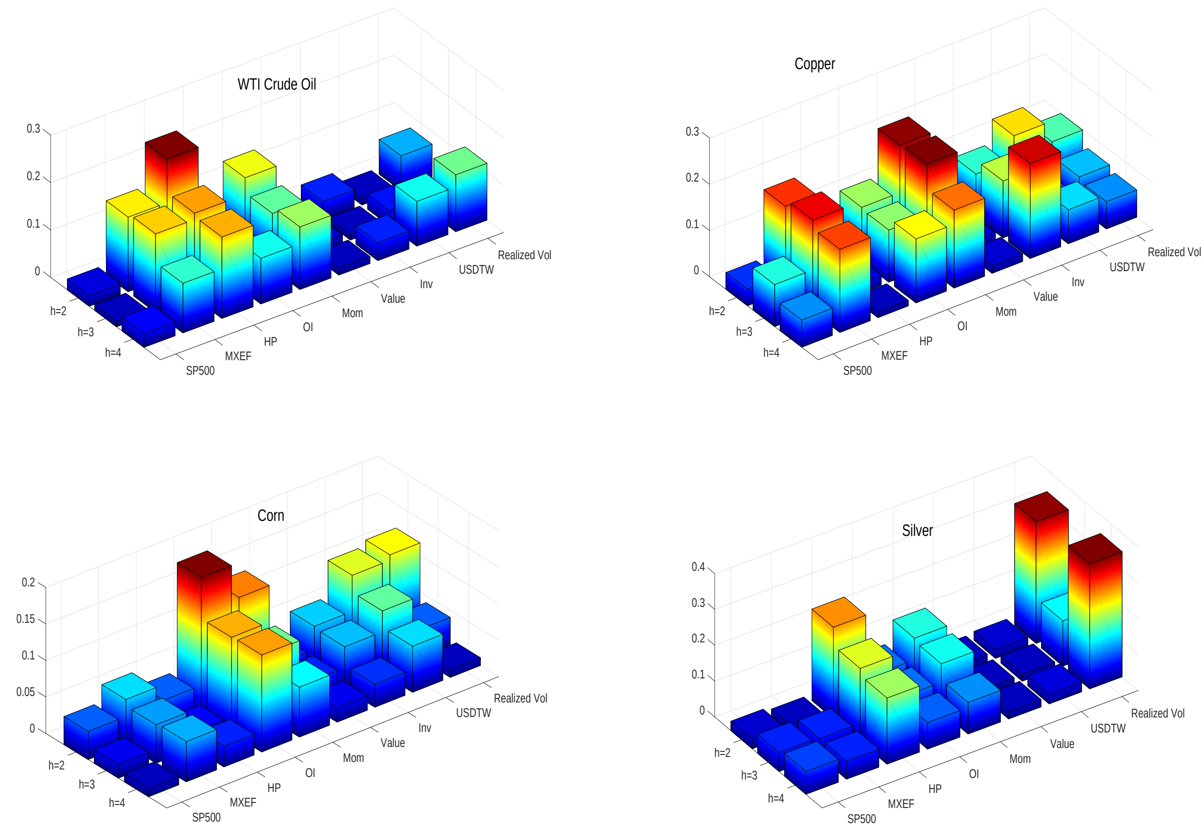


Figure 7. Time-Varying Betas for WTI Crude Oil

This figure shows the posterior median and credibility intervals for each explanatory variables on the ex-ante risk premia for futures on WTI Crude Oil. The solid blue (dark-yellow) line represents the estimated median beta for the two-quarter (four-quarter) ahead risk premia. The dashed-dot lines show the credibility intervals at the 5% significance level. The sample period 1995:01-2016:01.

Figure 8. Time-Varying Betas for Copper

This figure shows the posterior median and credibility intervals for each explanatory variables on the ex-ante risk premia for futures on Copper. The solid blue (dark-yellow) line represents the estimated median beta for the two-quarter (four-quarter) ahead risk premia. The dashed-dot lines show the credibility intervals at the 5% significance level. The sample period 1995:01-2016:01.

Figure 9. Time-Varying Betas for Corn

This figure shows the posterior median and credibility intervals for each explanatory variables on the ex-ante risk premia for futures on Corn. The solid blue (dark-yellow) line represents the estimated median beta for the two-quarter (four-quarter) ahead risk premia. The dashed-dot lines show the credibility intervals at the 5% significance level. The sample period 1995:01-2016:01.

Figure 10. Time-Varying Betas for Silver

This figure shows the posterior median and credibility intervals for each explanatory variables on the ex-ante risk premia for futures on Silver. The solid blue (dark-yellow) line represents the estimated median beta for the two-quarter (four-quarter) ahead risk premia. The dashed-dot line shows the credibility intervals at the 5% significance level. The sample period 1995:01-2016:01.

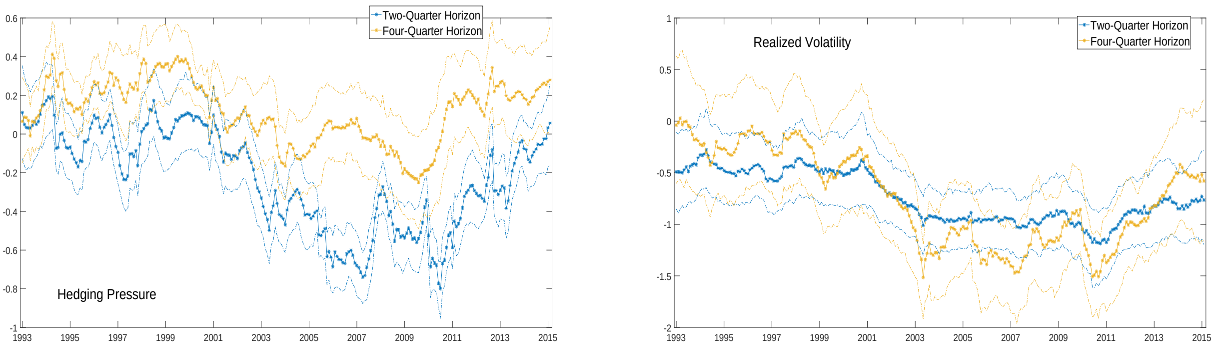


Figure 11. Ex-Ante vs Realized Risk Premia (Horizon $h = 2$)

This figure shows the scatter plot of ex-ante vs. realized risk premia. The ex-ante risk premia are extracted from the futures prices by using the model-implied expectations for $h = 2$ quarters ahead. Realized returns are computed as the excess rolling return in the generic contract for the same maturity of the corresponding model-implied expectations. Data are obtained from different resources. Futures prices data on Crude Oil (WTI) are from the New York Mercantile Exchange (NYMEX), prices are in U.S. dollars per barrel. Futures on Silver and Copper are obtained from the Commodity Exchange (COMEX). Silver is quoted in U.S. dollars per troy ounce while Copper is quoted in U.S. cents/pound. We convert the price of Copper futures contracts to USD/tonne to match the measurement unit of the survey forecasts, that instead refer to the London Metal Exchange (LME). Corn futures prices are from the Chicago Board of Trade (CBOT) with price quotation in USD cents per bushel. The red line represents the fitted value from a linear regression of the realized returns on the ex-ante risk premia. The sample period 1995:01-2016:01.

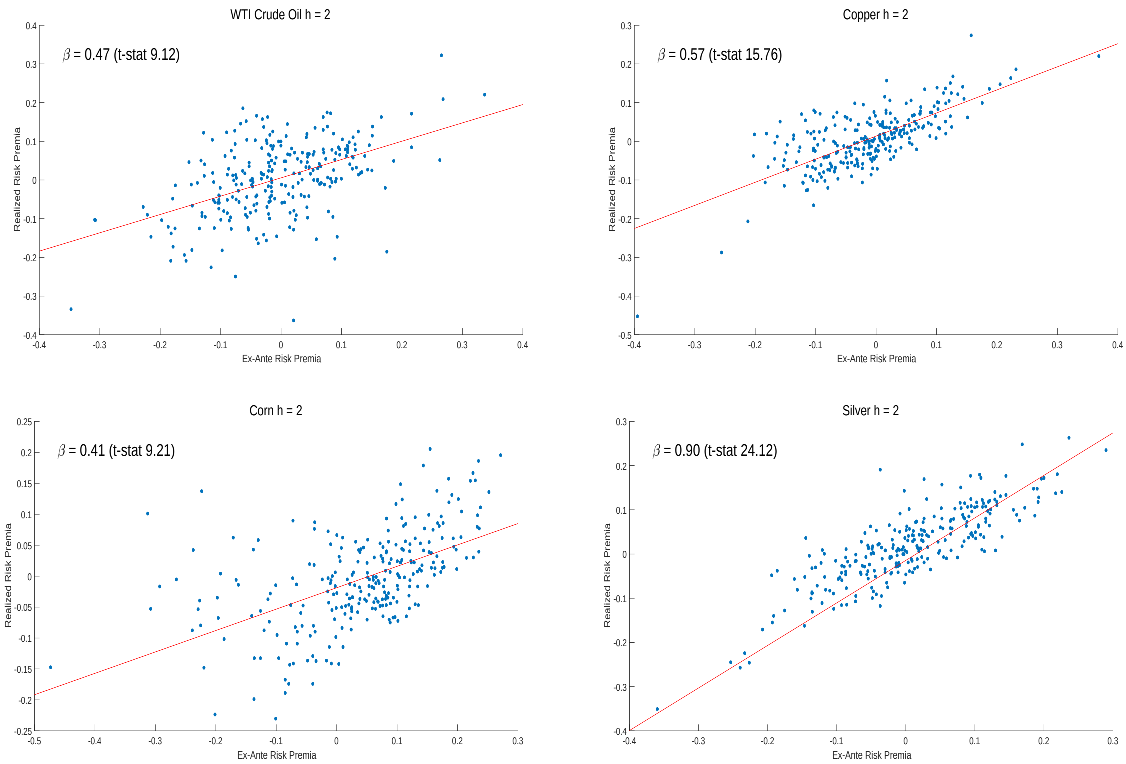


Figure 12. Ex-Ante vs Realized Risk Premia (Horizon $h = 4$)

This figure shows the scatter plot of ex-ante vs. realized risk premia. The ex-ante risk premia are extracted from the futures prices by using the model-implied expectations for $h = 4$ quarters ahead. Realized returns are computed as the excess rolling return in the generic contract for the same maturity of the corresponding model-implied expectations. Data are obtained from different resources. Futures prices data on Crude Oil (WTI) are from the New York Mercantile Exchange (NYMEX), prices are in U.S. dollars per barrel. Futures on Silver and Copper are obtained from the Commodity Exchange (COMEX). Silver is quoted in U.S. dollars per troy ounce while Copper is quoted in U.S. cents/pound. We convert the price of Copper futures contracts to USD/tonne to match the measurement unit of the survey forecasts, that instead refer to the London Metal Exchange (LME). Corn futures prices are from the Chicago Board of Trade (CBOT) with price quotation in USD cents per bushel. The red line represents the fitted value from a linear regression of the realized returns on the ex-ante risk premia. The sample period 1995:01-2016:01.