Commodity Financialization and Information Transmission

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Abstract

We study how commodity financialization affects information transmission and aggregation in a commodity futures market. The trading of financial traders injects both fundamental information and unrelated noise into the futures price. Thus, price informativeness in the futures market can either increase or decrease with commodity financialization. When the price-informativeness effect is negative, the futures price bias can increase with the population size of financial traders. Commodity financialization generally improves market liquidity in the futures market and strengthens the comovement between the futures market and the equity market. We find that operating profits and producer welfare move in opposite directions in response to commodity financialization, which provides important guidance for interpreting related empirical and policy studies.

Keywords: Commodity financialization, supply channel, price informativeness, futures price bias, liquidity, comovement, welfare **JEL Classifications**: D82, G14

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1 Introduction

Historically, futures markets were introduced for commodity producers (such as farmers) and demanders (such as manufacturers) to share later spot-price risks and control costs. Over the past decade, particularly after the year of 2004, commodity futures have become a popular asset to financial investors, such as hedge funds and commodity index traders. This process has been referred to as the "financialization of commodity markets" (Cheng and Xiong, 2014; Basak and Pavlova, 2016). Researchers and regulators are concerned about whether and how financialization has affected the functioning of commodity futures markets.

According to the 2011 Report of the G20 Study Group on Commodities (p. 29), "(t)he discussion centers around two related questions. First, does increased financial investment alter demand for and supply of commodity futures in a way that moves prices away from fundamentals and/or increase their volatility? And second, does financial investment in commodity futures affect spot prices?" A burgeoning empirical literature also links changes in futures price behavior to shifts in financial participation. For instance, Hamilton and Wu (2014) document that the risk premium in crude oil futures on average decreased since 2005, which is concurrent with the large inflow of institutional funds into commodity futures markets. Büyükşahin and Robe (2013, 2014) find that the equity-commodity correlation rises after 2004, which is largely driven by the trading of hedge funds that hold positions in both equity and commodity futures markets. (See the survey by Cheng and Xiong (2014) for more discussions.)

In this paper, we develop an asymmetric information model based on the classic work of Danthine (1978) and Grossman and Stiglitz (1980) to study commodity financialization. Our setting features one commodity good and two periods ($t = 0$ and 1). The commodity spot market opens at date 1, and there is a fundamental shock to commodity demand. The commodity supply is provided by commodity producers, who make their production decisions at date 0, after they see the equilibrium futures price. At date 0, the commodity futures market opens, commodity producers, financial traders, and noise traders trade futures contracts. Both commodity producers and financial traders have private information about the later commodity demand shock, and so they speculate on this information when trading futures. In addition, both types of traders trade futures for hedging purposes: Commodity producers hedge for their productions, while financial traders hedge for their positions in other assets such as stocks.

We first identify a supply channel through which the futures price affects the later spot price: A higher futures price induces commodity producers to supply more commodity, which in turn drives down the spot price through the spot market clearing mechanism. Thus, through affecting the futures price and hence commodity supply, financial investment in commodity futures can affect spot prices, which provides a positive answer to G20's second question. This supply channel also provides a natural setting for the "feedback effect" studied in the finance literature, which refers to the phenomenon that the price of a traded asset affects its cash flow (see Bond, Edmans, and Goldstein (2012) for a survey). In our setting, the traded asset is the futures contract whose cash flow is the later spot price. Thus, through the supply channel, the price of the futures contract naturally feeds back to its own cash flow. In Section 4.3.1, we show that this feedback effect helps to improve market liquidity because it makes demand functions more elastic to price, which means that changes in exogenous noise trading can be absorbed with a smaller price change.

We then use our analysis to speak to the implications of commodity financialization for market outcomes such as price informativeness, the future price bias, and producer welfare. We capture commodity financialization as an increase in the population size of financial traders active in the futures market. The results depend crucially on the trading behavior of financial traders. One key feature in our setting is that financial traders not only bring fundamental information, through their speculative trading, but also unrelated noise, through their hedging-motivated trading, into the futures price. As a result, adding more financial traders can either improve or harm price informativeness. Commodity financialization is beneficial to price informativeness if and only if the population size of financial traders is small.

The futures price biases refers to the deviations of the future price from the expected later spot price. In our setting, the futures market can either feature a normal backwardation (i.e., a downward bias in futures price) or a contango (i.e., an upward bias in future price). When the average commodity demand is relatively high, a normal backwardation ensues, and otherwise, a contango follows. Commodity financialization affects the futures price bias through two effects. First, adding more financial traders facilitates risk sharing, which tends to reduce the futures price bias. Second, as mentioned above, commodity financialization also affects price informativeness, which therefore affects the trading behavior of commodity producers. When commodity financialization harms price informativeness, the negative informational effect can be strong enough such that the futures price bias increases with the mass of financial traders. Thus, in response to the G20's first question, increased financial investment can indeed move the futures price away from fundamentals.

Commodity financialization helps to improve market liquidity and increase the comovement between the equity market and the commodity futures market. In particular, in our setting, the increase in equity-commodity comovement is driven by financial traders' hedging-motivated trades. This is consistent with the empirical channel documented by Büyükşahin and Robe (2013, 2014) who show link the increased correlation between commodities and stocks to the trading of hedge funds that are active in both equity and commodity futures markets.

Finally, we examine the real effect of commodity financialization, which offers important implications for related empirical analysis of commodity markets. Because welfare is not observable, empirical researchers often use operating profits as a proxy for producer welfare. Brogaard, Ringgenberg, and Sovich (2017) documents that commodity financialization negatively affects the profits of those companies that have significant economic exposure to index commodities. Our analysis suggests that when making normative statements, researchers should be careful in differentiating between operating profits and welfare. Specifically, we find that in our model, consistent with Brogaard, Ringgenberg, and Sovich (2017), price informativeness and operating profits move in the same direction in response to an increase in the mass of financial traders; however, producer welfare moves in the opposite direction. This provides important guidance for interpreting Brogaard, Ringgenberg, and Sovich's (2017) empirical findings: Although commodity producers earn lower profits following commodity financialization, it may be the case that only those producers who do not participate in futures market are harmed, while those producers who can trade futures may actually benefit from commodity financialization.

Related literature Our paper is broadly related to two strands of literature. The first is the literature on commodity financialization, which is largely empirical and documents the trading behavior of financial traders in futures markets and their pricing impact.[1] The theoretical research on the subject remains scarce. Basak and Pavlova (2016) construct dynamic equilibrium models to study how commodity financialization affects commodity futures prices, volatilities, and in particular, correlations among commodities and between equity and commodities. Fattouh and Mahadeva (2014) and Baker (2016) calibrate macro-finance models of commodities to quantify the effect of commodity financialization. Gorton, Hayashi, and Rouwenhorst (2012) and Ekeland, Lautier, and Villeneuve (2017) consider a combination of hedging pressure theory and storage theory to study commodity financialization. Knittel and Pindyck (2016) study a reduced-form setting of commodity financialization using a simple model of supply and demand in the cash and storage markets. Tang and Zhu (2016) model commodities as collateral for financing in a two-period economies with multiple countries and capital controls. While these existing models offer important insights, they all feature symmetric information, which is therefore not suitable for our goal of analyzing how financialization affects price discovery in futures markets.

Three existing theoretical studies also analyze the effects of informational frictions in the context of commodity financialization. Sockin and Xiong (2015) focus on information asymmetry in the spot market. They show that a high spot price may further spur the commodity demand through an informational channel, and in the presence of complementarity, this informational effect can be so strong that commodity demand can increase with the price. Goldstein, Li, and Yang (2014) argue that financial traders and commodity producers may respond to the same fundamental information in opposite directions, such that commodity financialization may have a negative informational effect. Leclercq and Praz (2014) consider how the entry of new speculators affects the average and volatility of spot prices. Relative to these three studies, in our setting, the futures price affects the spot price through affecting the production of commodity producers, and financial traders bring both information and noise into the futures price, which in turn affects the behavior of commodity producers.

The second strand of related literature is the classic and huge literature on futures markets

[1] See Irwin and Sanders (2011) and Cheng and Xiong (2014) for excellent surveys.

(see Section 1.1 of Acharya, Lochstoer and Ramadorai (2013) for a brief literature review on this literature). This literature has developed theories of "hedging pressure" (Keynes, 1930; Hicks, 1939; Hirshleifer, 1988, 1990) or "storage" (Kaldor, 1939; Working, 1949) to explain futures prices. Notably, the literature has also developed asymmetric information models on futures market (e.g., Grossman, 1977; Danthine, 1978; Bray, 1981; Stein, 1987). However, because commodity financialization is just a recent phenomenon, these models have focused on different research questions, for instance, on whether the futures market is viable (Grossman, 1977), on whether the futures price is fully revealing (Danthine, 1978; Bray, 1981), and on whether speculative trading can reduce welfare (Stein, 1987). Our paper is closest to Stein (1987) who shows that introducing a new speculative asset can harm welfare by generating price volatility due to a negative informational effect. However, the mechanism is different (see Footnote 4 for a technical discussion), and his analysis does not address questions specific to the debate on the financialization of commodities.

2 An Asymmetric Information Model of Commodity Financialization

The model has two periods: $t = 0$ and 1. The timeline of the economy is described by Figure 1. At date 0, the financial market opens, where a mass μ of financial traders—such as hedge funds or commodity index traders—trade futures contracts against commodity producers and noise traders. Here, we use parameter μ to capture financialization of commodities—i.e., the process of commodity financialization corresponds to an increase in μ. We normalize the mass of commodity producers as 1. Commodity producers make their investments on the commodity production at date 0, which in turn determines the commodity supply at the spot market that operates at date 1. In the following two subsections, we respectively describe the spot and futures markets.

2.1 The commodity spot market

There is only one commodity good in our setting. The spot market opens at date 1. As mentioned above, the supply of commodity will be determined by the production decisions of commodity producers, which we will discuss shortly in the next subsection. Following Hirshleifer (1988) and Goldstein, Li, and Yang (2014), we assume that the demand for the commodity good is implicitly derived from the preference of some (unmodeled) consumers and it is represented by a linear demand function:

$$y = \tilde{\theta} + \tilde{\delta} - \tilde{v}. \tag{1}$$

Here, $\tilde{v}$ is the commodity spot price, which will be endogenously determined in equilibrium, and $\tilde{\theta} + \tilde{\delta}$ represents an exogenous shock to consumers' commodity demand (which is the "fundamental" in our setting).

The demand shock is decomposed into two components, $\tilde{\theta}$ and $\tilde{\delta}$. Both components are normally distributed and mutually independent; that is, $\tilde{\theta} \sim N\left(\bar{\theta}, \tau_{\theta}^{-1}\right)$ and $\tilde{\delta} \sim N(0, \tau_{\delta}^{-1})$.[2] We have normalized the mean of $\tilde{\delta}$ as 0 since its mean can be absorbed by the mean of $\tilde{\theta}$. We assume that traders can learn information about $\tilde{\theta}$ but not about $\tilde{\delta}$. For example, $\tilde{\theta}$ can represent factors related to business cycles determining consumers' wealth level, on which there are many detailed macro data available that traders can purchase and analyze. In contrast, $\tilde{\delta}$ may represent noise affecting consumers' personal taste parameters, which are hard to predict given available data sources.

2.2 The commodity futures market

At date 0, the financial market opens. There are two tradable assets: a futures contract on the commodity and a risk-free asset. We normalize the net risk-free rate as zero. The payoff on the futures contract is the commodity spot price $\tilde{v}$ at date 1. Each unit of futures contract is traded at an endogenous price $\tilde{p}$. Commodity producers, financial traders, and noise traders participate in the financial market. Noise traders represent random transient demands in the futures market and they as a group demand $\tilde{\xi}$ units of the commodity futures,

[2] Throughout the paper, we use a tilde ($\tilde{}$) to signify a random variable, where a bar denotes its mean and τ denotes its precision (the inverse of variance). That is, for a random variable $\tilde{z}$, we have $\bar{z} \equiv E(\tilde{z})$ and $\tau_z = \frac{1}{Var(\tilde{z})}$.

where $\tilde{\xi} \sim N\left(\bar{\xi}, \tau_{\xi}^{-1}\right)$. We next describe in detail the behavior and information structure of commodity producers and financial traders.

2.2.1 Commodity producers

There is a continuum $[0,1]$ of commodity producers, indexed by i. Commodity producers are risk averse so that they have hedging motives in the futures market. Specifically, commodity producer i derives expected utility from his final wealth W_i at the end of date 1; he has a constant-absolute-risk-aversion (CARA) utility over wealth: $-e^{-\kappa W_i}$, where κ is the risk-aversion parameter. Commodity producers make decisions at date 0 and these decisions are twofold. First, they decide how many commodities to produce, which will in turn determine the commodity supply at the date-1 spot market. Second, they decide how many futures contracts to invest in the futures market to hedge their commodity production and to speculate on their private information.

Commodity producers are endowed with private information about the learnable component $\tilde{\theta}$ in the demand function of commodities. Specifically, commodity producer i receives a private signal $\tilde{s}_i$ which communicates information about $\tilde{\theta}$ in the following form:

$$\tilde{s}_i = \tilde{\theta} + \tilde{\varepsilon}_i, \tag{2}$$

where $\tilde{\varepsilon}_i \sim N\left(0, \tau_{\varepsilon}^{-1}\right)$ and $\left(\{\tilde{\varepsilon}_i\}_i, \tilde{\theta}, \tilde{\delta}\right)$ are mutually independent. The futures price $\tilde{p}$ is observable to all market participants and thus, commodity producer i's information set is $\{\tilde{s}_i, \tilde{p}\}$.

Commodity production incurs cost. When commodity producer i decides to produce x_i units of commodities, he needs to pay a production cost[3]

$$C\left(x_i\right) = c x_i + \frac{1}{2} x_i^2, \tag{3}$$

where c is a constant.

Thus, commodity producer i's problem is to choose commodity production x_i and futures

[3]The cost function $C\left(x_i\right)$ can be alternatively interpreted as an inventory cost. For instance, suppose that the date-0 wheat spot price is v_0 and carrying an inventory of x_i units of commodities incurs a cost of $c x_i + \frac{1}{2} x_i^2$. Then the total cost of storing x_i units of commodities is $C\left(x_i\right) = \left(c + v_0\right) x_i + \frac{1}{2} x_i^2$, which is essentially equation (3) with a renormalization of parameter c. However, this interpretation is made in a partial equilibrium setting as the date-0 spot price v_0 is exogenous. We can fully endogenize this spot price as well at the expense of introducing one extra source of uncertainty, because otherwise the prices of futures and current spot price will combine to fully reveal the shocks (see Grossman, 1977).

investment d_i (and investment in the risk-free asset) to maximize

$$E\left(-e^{-\kappa \tilde{W}_i}\middle|\, \tilde{s}_i, \tilde{p}\right) \tag{4}$$

subject to

$$\tilde{W}_i = \tilde{v}x_i - C(x_i) + (\tilde{v} - \tilde{p})\, d_i. \tag{5}$$

Here, $\tilde{v}x - C(x_i)$ is the profit from producing and selling x_i units of commodities: selling x_i units of commodities at a later spot price $\tilde{v}$ generates a revenue of $\tilde{v}x_i$, which, net of the production cost $C(x_i)$, gives rise to the profit. The term $(\tilde{v} - \tilde{p})\, d_i$ is the profit from trading d_i units of futures contracts. Specifically, at date 0, buying a futures contract promises to buy one unit of commodity at date 1 at a prespecified price $\tilde{p}$, and so from the perspective of date 0, this contract is an asset that costs $\tilde{p}$ and generates a payoff equal to the date-1 commodity spot price $\tilde{v}$. Thus, buying one futures contract leads to a trading gain/loss of $\tilde{v} - \tilde{p}$, which implies that $(\tilde{v} - \tilde{p})\, d_i$ is the profit from trading d_i units of futures contracts. In equation (5), we have normalized commodity producer i"s initial endowment as 0.

To better connect our setup to previous models, we have followed the literature (e.g., Danthine, 1978) and interpreted commodity producers as commodity suppliers. In effect, a more precise interpretation of commodity producers should be commercial hedgers, because as become clear later, their futures demand contains a hedging component (see equation (13)). In this sense, commodity producer can be either commodity providers or commodity demanders. Specifically, if $x_i < 0$, then in equation (5) the term $\tilde{v}x - C(x_i)$ can be interpreted as the utility from consuming $|x_i|$ units of commodities.

2.2.2 Financial traders

There is a mass $\mu \geq 0$ financial traders who derive utility only from their final wealth at the end of date 1. For simplicity, we assume that financial traders are identical in preference, investment opportunities, and information sets. They have a CARA utility with a risk aversion coefficient of $\gamma > 0$. We can show that in our setting, μ and γ affect the equilibrium only through the ratio $\frac{\mu}{\gamma}$, and thus the latter comparative static analysis in μ is equivalent to a comparative statics analysis in $\frac{1}{\gamma}$.

As commodity producers, financial traders trade futures both for speculation and for hedging motives. However, they hedge not for real production of commodities; instead, they hedge for their positions in other assets whose payoffs are correlated with the commodity market (and hence the payoffs on commodity futures). We borrow from Wang (1994), Easley, O'Hara, and Yang (2014), and Han, Tang, and Yang (2016) to model this hedging behavior of financial traders. Formally, we assume that at date 0, in addition to the risk-free asset and the futures contract, financial traders can invest in a private technology. This private technology may represent stock index in which financial traders typically invest. Another real-world example is commodity-linked notes (CLNs) that are traded over the counter and have payoffs linked to the price of commodity or commodity futures. As documented by Henderson, Pearson, and Wang (2015), the regular issuers of CLNs are big investment banks, who often invest in commodity futures to hedge their issuance of CLNs. More broadly, the private technology is introduced to capture the fact that in addition to accommodating commodity producers' hedging needs, financial traders trade futures also for their own reasons such as portfolio diversification and risk management, as emphasized by Cheng, Kirilenko, and Xiong (2015).

The net return on the private technology is $\tilde{\alpha} + \tilde{\eta}$, where $\tilde{\alpha} \sim N\left(0, \tau_\alpha^{-1}\right)$ and $\tilde{\eta} \sim N\left(0, \tau_\eta^{-1}\right)$. So, as the commodity demand shock, the net return on the private technology also has two components. Variable $\tilde{\alpha}$ represents the forecastable component and it is independent of all other random variables and privately observable to financial traders. The variable $\tilde{\eta}$ is the unforecastable component, and importantly, it is correlated with the unforecastable component $\tilde{\delta}$ in the commodity demand shock. Let $\rho \in (-1, 1)$ denote the correlation coefficient between $\tilde{\eta}$ and $\tilde{\delta}$. This correlation generates the hedging motives of financial traders in the futures market.

We assume that financial traders observe $\tilde{\theta}$.[4] This assumption is realistic to the extent that financial traders, such as hedge funds, generally have more sophisticated information

[4] Our result is robust to a general assumption that financial traders observe a noisy version of $\tilde{\theta}$, for instance, $\tilde{s}_F = \tilde{\theta} + \tilde{\varepsilon}_F$. This alternative assumption will introduce noise $\tilde{\varepsilon}_F$ into the price $\tilde{p}$, which complicates our analysis. Stein (1987) relies on such an assumption to generate a negative informational externality. However, under this alternative assumption, commodity financialization would always improve price informativeness in our setting, in the absence of the noise $\tilde{\alpha}$ generated from the hedging motive of financial traders. This is because both the private information of commodity producers and that of financial traders are about the same fundamental $\tilde{\theta}$. In contrast, in Stein's (1987) setting, financial traders and other traders have information about different variables, and financial traders' trading brings noise to the price, which impairs other traders' ability to make inferences based on current prices and their own information.

processing capacities than regular commodity producers. Of course, financial traders also observe the futures price $\tilde{p}$, and thus, financial traders' information set is $\{\tilde{\theta}, \tilde{\alpha}, \tilde{p}\}$. Their problem is to choose investment d_F in futures and investment z_F in the private technology (and investment in the risk-free asset) to maximize

$$E\left[-e^{-\gamma[(\tilde{v}-\tilde{p})d_F+(\tilde{\alpha}+\tilde{\eta})z_F]}\middle|\,\tilde{\theta}, \tilde{\alpha}, \tilde{p}\right]. \tag{6}$$

Here, $(\tilde{v}-\tilde{p})\,d_F$ captures the profit from trading futures and $(\tilde{\alpha}+\tilde{\eta})\,z_F$ captures the profit from investing in the private technology. Again, we have also normalized the initial endowment of financial traders to be zero.

3 Equilibrium Characterization

In our setting, $(\tilde{\theta}, \tilde{\delta}, \tilde{\xi}, \{\tilde{\varepsilon}_i\}_i, \tilde{\alpha}, \tilde{\eta})$ are underlying random variables that characterize the economy. They are all independent of each other, except that $\tilde{\delta}$ and $\tilde{\eta}$ are correlated with each other with the coefficient $\rho \in (-1,1)$. The tuple $\mathcal{E} = \left(\mu, \kappa, \gamma, c, \bar{\theta}, \bar{\xi}, \rho, \tau_\theta, \tau_\delta, \tau_\varepsilon, \tau_\xi, \tau_\alpha, \tau_\eta\right)$ defines an economy. Given an economy, an equilibrium consists of two subequilibria: the date-1 spot market equilibrium and the date-0 futures market equilibrium. At date 1, the commodity demand function clears the commodity supply provided by commodity producers at the prevailing spot price $\tilde{v}$. Because the commodity demand depends on the demand shock $\tilde{\theta}+\tilde{\delta}$ and the commodity supply depends on producers' private information $\{\tilde{s}_i\}$ and the futures price $\tilde{p}$, we expect that the spot price $\tilde{v}$ will be a function of $(\tilde{\theta}, \tilde{\delta}, \tilde{p})$. At date 0, we consider a noisy rational expectations equilibrium (NREE) in the futures market. Given that commodity producers have private information $\{\tilde{s}_i\}$, financial traders have private information $\{\tilde{\theta}, \tilde{\alpha}\}$, and noise trading is $\tilde{\xi}$, we expect that the futures price $\tilde{p}$ will depend on $(\tilde{\theta}, \tilde{\alpha}, \tilde{\xi})$. A formal definition of equilibrium is given as follows.

Definition 1 *An equilibrium consists of a spot price function,* $v(\tilde{\theta}, \tilde{\delta}, \tilde{p}) : \mathbb{R}^3 \to \mathbb{R}$*; a futures price function,* $p(\tilde{\theta}, \tilde{\alpha}, \tilde{\xi}) : \mathbb{R}^3 \to \mathbb{R}$*; a commodity production policy,* $x(\tilde{s}_i, \tilde{p}) : \mathbb{R}^2 \to \mathbb{R}$*; a trading strategy of commodity producers,* $d(\tilde{s}_i, \tilde{p}) : \mathbb{R}^2 \to \mathbb{R}$*; a trading strategy of financial traders,* $d_F(\tilde{\theta}, \tilde{\alpha}, \tilde{p}) : \mathbb{R}^3 \to \mathbb{R}$*; and a strategy of financial traders' investment on the private technology,* $z_F(\tilde{\theta}, \tilde{\alpha}, \tilde{p}) : \mathbb{R}^3 \to \mathbb{R}$*, such that:*

(a) At date 1, the spot market clears, i.e.,

$$\tilde{\theta}+\tilde{\delta}-v(\tilde{\theta},\tilde{\delta},\tilde{p})=\int_0^1 x\left(\tilde{s}_i,\tilde{p}\right)di; \tag{7}$$

(b) At date 0, given that $\tilde{v}$ is defined by $v(\tilde{\theta},\tilde{\delta},\tilde{p})$,

(i) $x\left(\tilde{s}_i,\tilde{p}\right)$ and $d\left(\tilde{s}_i,\tilde{p}\right)$ solve for commodity producers' problem given by (4) and (5);

(ii) $d_F(\tilde{\theta},\tilde{\alpha},\tilde{p})$ and $z_F(\tilde{\theta},\tilde{\alpha},\tilde{p})$ solve financial traders' problem (6); and

(iii) the futures market clears, i.e.,

$$\int_0^1 d\left(\tilde{s}_i,\tilde{p}\right)di+\mu d_F(\tilde{\theta},\tilde{\alpha},\tilde{p})+\tilde{\xi}=0. \tag{8}$$

We next construct an equilibrium in which the price functions $v(\tilde{\theta},\tilde{\delta},\tilde{p})$ and $p(\tilde{\theta},\tilde{\alpha},\tilde{\xi})$ are linear. As standard in the literature, we solve the equilibrium backward from date 1.

3.1 Spot market equilibrium

The commodity demand is given by equation (1). The commodity supply is determined by commodity producers' date-0 investment decisions. Solving commodity producers' problem (given by (4) and (5)) yields the following first-order conditions:

$$x_i+d_i = \frac{E\left(\tilde{v}\middle|\,\tilde{s}_i,\tilde{p}\right)-\tilde{p}}{\kappa Var\left(\tilde{v}\middle|\,\tilde{s}_i,\tilde{p}\right)}, \tag{9}$$

$$x_i = \tilde{p}-c. \tag{10}$$

The above expressions are similar to those in Danthine (1978). The intuition is as follows. Given both real investment x_i and financial investment d_i expose a commodity producer to the same risk source $\tilde{v}$, his overall exposure to this risk is given by the standard demand function of a CARA investor, as expressed in (9). Expression (10) says that after controlling the total risk given by (9), financial producers essentially treat $\tilde{p}$ as the commodity selling price when making real production decisions. Aggregating (10) across all commodity producers delivers the aggregate commodity supply at the spot market: $\int_0^1 x_i di=\tilde{p}-c$. By the market-clearing condition (7) and equations (1) and (10), we can solve the spot price $\tilde{v}$, which is given by the following lemma.

Lemma 1 (Spot prices) *The date-1 spot price $\tilde{v}$ is given by*

$$\tilde{v}=\tilde{\theta}+\tilde{\delta}+c-\tilde{p}. \tag{11}$$

This lemma formally establishes a supply channel through which the date-0 futures price $\tilde{p}$ affects the date-1 spot price $\tilde{v}$. Equation (11) therefore provides a positive answer to the following question raised in the 2011 G20 Report on Commodities: "(D)oes financial investment in commodity futures affect spot prices?" In our setting, financial traders' investments in futures will alter the behavior of $\tilde{p}$, which in turn changes the later spot price $\tilde{v}$ through equation (11). In other words, the futures market is not just a side show, and it has consequences for the real side. This phenomenon is labeled as the "feedback effect" in the finance literature;[5] that is, the price $\tilde{p}$ of a traded asset feeds back to its own cash flows $\tilde{v}$ (recall that for a futures contract, its cash flow is the later spot price).

3.2 Futures market equilibrium

We conjecture the following linear futures price function:

$$\tilde{p} = p_0 + p_\theta \tilde{\theta} + p_\alpha \tilde{\alpha} + p_\xi \tilde{\xi}, \tag{12}$$

where p_0, p_θ, p_α, and p_ξ are undetermined coefficients. We next compute the demand function of futures market participants and use the market-clearing condition to construct such a linear NREE price function.

By (9) and (10), commodity producer i's demand for the futures contract is

$$d(\tilde{s}_i, \tilde{p}) = \underbrace{\frac{E(\tilde{v}|\, \tilde{s}_i, \tilde{p}) - \tilde{p}}{\kappa Var(\tilde{v}|\, \tilde{s}_i, \tilde{p})}}_{\text{speculation}} - \underbrace{(\tilde{p} - c)}_{\text{hedging}}. \tag{13}$$

As mentioned before, a commodity producer trades futures for two reasons. First, he hedges his real commodity production of $x_i = \tilde{p} - c$. Second, because he also has private information $\tilde{s}_i$ on the later commodity demand, he also speculates on this private information.

By (12), the information contained in the futures price is equivalent to the signal $\tilde{s}_p$:

$$\tilde{s}_p \equiv \frac{\tilde{p} - p_0 - p_\xi \bar{\xi}}{p_\theta} = \tilde{\theta} + \pi_\alpha \tilde{\alpha} + \pi_\xi \left(\tilde{\xi} - \bar{\xi}\right), \text{ with } \pi_\alpha \equiv \frac{p_\alpha}{p_\theta} \text{ and } \pi_\xi \equiv \frac{p_\xi}{p_\theta}, \tag{14}$$

which is normally distributed with mean $\tilde{\theta}$ and precision τ_p, where

$$\tau_p = \left(\frac{\pi_\alpha^2}{\tau_\alpha} + \frac{\pi_\xi^2}{\tau_\xi}\right)^{-1}. \tag{15}$$

Variable τ_p measures how informative the futures price $\tilde{p}$ is about the later commodity demand ("fundamental"), and so we also refer to τ_p as "price informativeness."

Using the expression of $\tilde{v}$ in (11) and applying Bayes' rule to compute the conditional

[5] See Bond, Edmans, and Goldstein (2012) for a survey on the feedback effect literature.

moments in commodity producer i's demand function (13), we can compute

$$d\left(\tilde{s}_i, \tilde{p}\right) = \frac{\frac{\tau_\theta \bar{\theta} + \tau_\varepsilon \tilde{s}_i + \tau_p \tilde{s}_p}{\tau_\theta + \tau_\varepsilon + \tau_p} + c - 2\tilde{p}}{\kappa\left(\frac{1}{\tau_\theta + \tau_\varepsilon + \tau_p} + \frac{1}{\tau_\delta}\right)} - \left(\tilde{p} - c\right). \tag{16}$$

Solving financial traders' problem in (6), we can compute their futures demand as follows:

$$d_F(\tilde{\theta}, \tilde{\alpha}, \tilde{p}) = \underbrace{\frac{\tau_\delta\left(\tilde{\theta} + c - 2\tilde{p}\right)}{\gamma\left(1 - \rho^2\right)}}_{\text{speculation}} - \underbrace{\frac{\rho\sqrt{\tau_\delta \tau_\eta}}{\gamma\left(1 - \rho^2\right)}\tilde{\alpha}}_{\text{hedging}}. \tag{17}$$

As discussed in Section 2, financial traders invest in futures contracts also for two reasons. First, they speculate on their private information, in particular, on their superior information about commodity demand shock $\tilde{\theta}$. Second, they also have made informed investment on the private technology, whose payoff is correlated with the commodity market, and thus financial traders also trade futures to hedge their investment on the private technology.

Equation (17) reveals that the trading of financial traders injects both information $\tilde{\theta}$ (that is useful for predicting the later commodity demand) and "noise" $\tilde{\alpha}$ (that is orthogonal to the commodity demand shock) into the commodity futures market. Information is injected via financial traders' speculative trading, while noise is injected via their hedging-motivated trading. This observation has important implications for price informativeness, as we will explore in Section 4.

The equilibrium futures price function is derived as standard in the literature. That is, we insert demand functions (16) and (17) into the market-clearing condition (8) to solve the price in terms of $\tilde{\theta}$, $\tilde{\alpha}$, and $\tilde{\xi}$, and then compare with the conjectured price function in equation (12) to obtain a system defining the unknown p-coefficients. Solving this system yields the following proposition.

Proposition 1 (Futures market equilibrium) *For any given mass $\mu \geq 0$ of financial traders,* *there exists a unique linear NREE where the futures price $\tilde{p}$ is given by equation (12), where*

$$
\begin{aligned}
p_0 &= D^{-1}\left[\frac{\frac{\tau_\theta\bar{\theta}-\tau_p\pi_\xi\bar{\xi}}{\tau_\theta+\tau_\varepsilon+\tau_p}+c}{\kappa\left(\frac{1}{\tau_\theta+\tau_\varepsilon+\tau_p}+\frac{1}{\tau_\delta}\right)}+c+\frac{\mu\tau_\delta}{\gamma\left(1-\rho^2\right)}c\right],\\
p_\theta &= D^{-1}\left[\frac{\frac{\tau_\varepsilon+\tau_p}{\tau_\theta+\tau_\varepsilon+\tau_p}}{\kappa\left(\frac{1}{\tau_\theta+\tau_\varepsilon+\tau_p}+\frac{1}{\tau_\delta}\right)}+\frac{\mu\tau_\delta}{\gamma\left(1-\rho^2\right)}\right],\\
p_\alpha &= D^{-1}\left[\frac{\frac{\tau_p\pi_\alpha}{\tau_\theta+\tau_\varepsilon+\tau_p}}{\kappa\left(\frac{1}{\tau_\theta+\tau_\varepsilon+\tau_p}+\frac{1}{\tau_\delta}\right)}-\frac{\mu\rho\sqrt{\tau_\eta\tau_\delta}}{\gamma\left(1-\rho^2\right)}\right],\\
p_\xi &= D^{-1}\left[\frac{\frac{\tau_p\pi_\xi}{\tau_\theta+\tau_\varepsilon+\tau_p}}{\kappa\left(\frac{1}{\tau_\theta+\tau_\varepsilon+\tau_p}+\frac{1}{\tau_\delta}\right)}+1\right],
\end{aligned}
$$

where

$$
\begin{aligned}
D &= \frac{2}{\kappa\left(\frac{1}{\tau_\theta+\tau_\varepsilon+\tau_p}+\frac{1}{\tau_\delta}\right)}+1+\frac{2\mu\tau_\delta}{\gamma\left(1-\rho^2\right)},\\
\tau_p &= \left[\frac{\mu^2\rho^2\tau_\delta\tau_\eta}{\gamma^2\left(1-\rho^2\right)^2\tau_\alpha}+\frac{1}{\tau_\xi}\right]^{-1}\pi_\xi^{-2},\\
\pi_\alpha &= -\frac{\mu\rho\sqrt{\tau_\eta\tau_\delta}}{\gamma\left(1-\rho^2\right)}\pi_\xi,
\end{aligned}
$$

with $\pi_\xi \in \left(\left[\frac{\frac{\tau_\varepsilon}{\tau_\theta+\tau_\varepsilon}}{\kappa\left(\frac{1}{\tau_\theta+\tau_\varepsilon}+\frac{1}{\tau_\delta}\right)}+\frac{\mu\tau_\delta}{\gamma(1-\rho^2)}\right]^{-1}, \left[\frac{\mu\tau_\delta}{\gamma(1-\rho^2)}\right]^{-1}\right)$ *being determined by the unique root to the following equation:*

$$
\pi_\xi = \left[\frac{\frac{\tau_\varepsilon}{\tau_\theta+\tau_\varepsilon+\tau_p}}{\kappa\left(\frac{1}{\tau_\theta+\tau_\varepsilon+\tau_p}+\frac{1}{\tau_\delta}\right)}+\frac{\mu\tau_\delta}{\gamma\left(1-\rho^2\right)}\right]^{-1}.
$$

4 Commodity Financialization, Price Informativeness, and Asset Prices

4.1 Price informativeness

We measure price informativeness using τ_p, which concerns how much information the prevailing futures price $\tilde{p}$ conveys about the "fundamental," which refer to the commodity demand shock $\tilde{\theta}$ in our setting. As shown by the demand function (17) of financial traders, their speculative trading injects information $\tilde{\theta}$ into the price $\tilde{p}$, while their hedging-motivated trading injects noise $\tilde{\alpha}$ into the price $\tilde{p}$. So, in general, adding more financial traders has an ambiguous effect on price informativeness.

Proposition 2 (Price informativeness)

(a) When the population size of financial traders is sufficiently small, commodity financialization improves price informativeness. That is, $\frac{\partial \tau_p}{\partial \mu} > 0$ for sufficiently small μ.

(b) Suppose that the precision τ_ε of commodity producers' private signals is sufficiently high, then

$$\frac{\partial \tau_p}{\partial \mu} > 0 \Longleftrightarrow \mu < \frac{\kappa \gamma \tau_\alpha}{\tau_\delta \tau_\eta \tau_\xi} \left(\frac{1}{\rho^2} - 1 \right). \tag{18}$$

Proposition 2 suggests that increasing the population size μ of financial traders first improves price informativeness and then harms price informativeness. To understand this result, we examine in detail the demand functions of financial traders and commodity producers. In equation (17), we use

$$\begin{aligned} \beta_\theta &\equiv \frac{\partial d_F(\tilde{\theta}, \tilde{\alpha}, \tilde{p})}{\partial \tilde{\theta}} = \frac{\tau_\delta}{\gamma (1 - \rho^2)}, \\ \beta_\alpha &\equiv -\frac{\partial d_F(\tilde{\theta}, \tilde{\alpha}, \tilde{p})}{\partial \tilde{\alpha}} = \frac{\rho \sqrt{\tau_\delta \tau_\eta}}{\gamma (1 - \rho^2)}, \end{aligned}$$

to capture the sensitivities of financial traders' order flow to information $\tilde{\theta}$ and to noise $\tilde{\alpha}$. Similarly, we use ϕ_θ to measure the sensitivity of commodity producers' aggregate order flow to information $\tilde{\theta}$, i.e.,

$$\phi_\theta \equiv \frac{\partial \int_0^1 d(\tilde{s}_i, \tilde{p})\, di}{\partial \tilde{s}_i} = \frac{\frac{\tau_\varepsilon}{\tau_\theta + \tau_\varepsilon + \tau_p}}{\kappa \left(\frac{1}{\tau_\theta + \tau_\varepsilon + \tau_p} + \frac{1}{\tau_\delta} \right)},$$

where the second equality follows from equation (16).

Equipped with these notations and inserting the demand functions into the market-clearing condition (8), we have

$$\underbrace{\phi_\theta \tilde{\theta}}_{\text{info from com. prod.}} + \underbrace{\mu \beta_\theta \tilde{\theta}}_{\text{info from fin. trad.}} - \underbrace{\mu \beta_\alpha \tilde{\alpha}}_{\text{noise from fin. trad.}} + \underbrace{\tilde{\xi}}_{\text{exog. noise}} - L(\tilde{p}) = 0, \tag{19}$$

where $L(\tilde{p})$ is a known linear function that absorbs all the other terms unrelated to information or noise in the order flows of market participants. In the above market-clearing condition, the speculative trading of commodity producers and of financial traders injects information $\tilde{\theta}$ into the aggregate demand, the hedging-motivated trading of financial traders injects the endogenous noise $\tilde{\alpha}$ into the aggregate demand, while noise trading injects the exogenous noise $\tilde{\xi}$ into the aggregate demand.

In (19), moving $L(\tilde{p})$ to the right-hand side and dividing both sides by $(\phi_\theta + \mu\beta_\theta)$ yield the following signal:

$$\tilde{\theta} - \underbrace{\frac{\mu\beta_\alpha}{\phi_\theta + \mu\beta_\theta}\tilde{\alpha}}_{\text{added noise by fin. trad.}} + \underbrace{\frac{1}{\phi_\theta + \mu\beta_\theta}\tilde{\xi}}_{\text{exogenous noise trading}} = \frac{L(\tilde{p})}{\phi_\theta + \mu\beta_\theta} = \tilde{s}_p. \tag{20}$$

This signal gives the informational content in the aggregate order flow and in equilibrium, it must coincide with $\tilde{s}_p$ given by equation (14).

In equation (20), it is clear that increasing μ has two offsetting effects on the informativeness of $\tilde{s}_p$: First, it lowers the noise $\frac{1}{\phi_\theta+\mu\beta_\theta}\tilde{\xi}$ that is related to the exogenous noise trading; second, it increases the endogenous noise $\frac{\mu\beta_\alpha}{\phi_\theta+\mu\beta_\theta}\tilde{\alpha}$ brought by financial traders. When μ is small—for instance, when $\mu \approx 0$—the added noise $\frac{\mu\beta_\alpha}{\phi_\theta+\mu\beta_\theta}\tilde{\alpha}$ by financial traders is relatively small, and thus the main effect of increasing μ is to lower $\frac{1}{\phi_\theta+\mu\beta_\theta}\tilde{\xi}$. As a result, the price signal $\tilde{s}_p$ becomes more informative about $\tilde{\theta}$ when μ increases from a very small value. In contrast, as μ becomes very large, the added noise $\frac{\mu\beta_\alpha}{\phi_\theta+\mu\beta_\theta}\tilde{\alpha}$ eventually dominates the noise $\frac{1}{\phi_\theta+\mu\beta_\theta}\tilde{\xi}$, and the price signal $\tilde{s}_p$ becomes less informative about the fundamental $\tilde{\theta}$.

It is also useful to understand in detail the threshold value of μ in Part (b) of Proposition 2. A smaller threshold value implies that it is more likely for price informativeness to decrease with μ. First, when the correlation ρ between the private technology and the commodity demand is large, the threshold value of μ is small, because a large ρ implies that financial traders hedge more and so their trading brings more noise into the price. Second, for a similar reason, when $\tau_\delta\tau_\eta$ is large, there is little residual uncertainty in both the private technology and the futures payoff, and thus financial traders will also trade more aggressively and hedge more. Third, when $\frac{\tau_\alpha}{\tau_\xi}$ is small, the variance of the added noise by financial traders is large relative to the variance of the exogenous noise trading in the futures market, which means that the added noise is more effective in diluting information. Fourth, when the risk aversion κ of commodity producers is small, commodity producers trade aggressively and their trading already injects a lot of information into the price. In this case, adding financial traders is more likely to adversely affect the aggregation of commodity traders' information. Finally, lowering risk aversion γ of financial traders is equivalent to scaling up the total order flow of financial traders, and thus the threshold value of μ decreases with γ as well.

Figure 2 provides a graphical illustration for the effect of μ on price informativeness τ_p. In this example, we set the parameter values as follows: $\tau_\theta = \tau_\delta = \tau_\varepsilon = \tau_\xi = \tau_\alpha = \tau_\eta = 1, \gamma = \kappa = 0.1$, and $\rho = 0.5$. The pattern is robust to the choice of parameter values. Indeed, we see that price informativeness τ_p first increases and then decreases with the mass μ of financial traders. This suggests that commodity financialization is beneficial to price informativeness if and only if the population size of new financial traders in the futures market is moderate.

4.2 Futures price biases

The literature has long been interested in "futures price bias," that is, the deviation of the futures price from the expectation of the later spot price, $E(\tilde{v} - \tilde{p})$. A downward bias in the futures price is termed "normal backwardation," while an upward bias in the futures price is termed "contango." A major branch of literature on futures pricing has attributed bias to hedging pressures of commodity producers (e.g., Keynes, 1930; Hicks, 1939; Hirshleifer 1988, 1990). Hamilton and Wu (2014) document that the futures price bias in crude oil futures on average decreased since 2005. Regulators are also very concerned about how commodity financialization affects the average futures price. For instance, the 2011 G20 Report on Commodities asked: "(D)oes increased financial investment alter demand for and supply of commodity futures in a way that moves prices away from fundamentals and/or increase their volatility?"

In our setting, we can compute the futures price bias $E(\tilde{v} - \tilde{p})$ as follows:

$$E(\tilde{v} - \tilde{p}) = \frac{\frac{\bar{\theta} - c}{2} - \bar{\xi}}{\underbrace{\frac{(\tau_\theta + \tau_\varepsilon + \tau_p)\tau_\delta}{\kappa(\tau_\theta + \tau_\varepsilon + \tau_p + \tau_\delta)}}_{\text{learning by commodity producers}} + \underbrace{\frac{\mu\tau_\delta}{\gamma(1-\rho^2)}}_{\text{risk sharing}} + \frac{1}{2}}. \tag{21}$$

Thus, there can be either a downward bias or an upward bias in futures prices, depending on the sign of $\frac{\bar{\theta} - c}{2} - \bar{\xi}$: $E(\tilde{v} - \tilde{p}) > 0$ if and only if $\frac{\bar{\theta} - c}{2} > \bar{\xi}$. Intuitively, when the average commodity demand shock $\bar{\theta}$ is high relative to the production cost parameter c, commodity producers tend to produce more commodities and thus they will short more futures to hedge their commodity production. If their shorting pressure overwhelms the average demand $\bar{\xi}$ from noise traders, then on average, the futures price is depressed relative to its fundamental value, which leads to a downward bias in futures price (normal backwardation). By contrast, when $\frac{\bar{\theta}-c}{2}$ is small relative to $\bar{\xi}$, the futures price is biased upward, leading to a contango. Fama and French (1987) used 21 commodities to test the futures risk premium hypothesis, and indeed, they found that some markets feature "normal backwardation," while others feature "contango." According to our theory, this difference can be explained by the relative sizes of the hedging pressure $\frac{\bar{\theta}-c}{2}$ from commodity producers average and the average noise demand $\bar{\xi}$ in futures market.

In equation (21), increasing the population size μ of financial traders affect the futures price bias $|E(\tilde{v}-\tilde{p})|$ in two ways. First, the newly added financial traders directly share more risk that is loaded off from the hedging needs of commodity producers. This tends to reduce the futures price bias. Second, increasing μ also affects price informativeness τ_p, which in turn changes the risk perceived by commodity producers through affecting their learning from prices. As shown in Proposition 2, τ_p can either increase or decrease with μ. When τ_p increases with μ, the learning effect works in the same direction as the risk-sharing effect, and thus the futures price bias $|E(\tilde{v}-\tilde{p})|$ decreases with μ. When τ_p decreases with μ, the learning effect works against the risk-sharing effect, which can generate a non-monotonic relation between $|E(\tilde{v}-\tilde{p})|$ and μ.

Proposition 3 (Futures price bias)

(a) T*here is a downward bias (i.e., normal backwardation) in the futures price relative to the expected value of the later spot price if and only if* $\frac{\bar{\theta}-c}{2} > \bar{\xi}$. *That is,* $E(\tilde{v}-\tilde{p}) > 0$ *if and only if* $\frac{\bar{\theta}-c}{2} > \bar{\xi}$.

(b) When price informativeness τ_p *increases with the mass* μ *of financial traders, commodity financialization reduces the futures price bias; that is, if* $\frac{\partial \tau_p}{\partial \mu} > 0$, *then* $\frac{\partial |E(\tilde{v}-\tilde{p})|}{\partial \mu} < 0$. *In contrast, if* $\frac{\partial \tau_p}{\partial \mu} < 0$, *then it is possible that* $\frac{\partial |E(\tilde{v}-\tilde{p})|}{\partial \mu} > 0$.

Corollary 1 *When the population size of financial traders is small, commodity financialization reduces the futures price bias. That is,* $\frac{\partial |E(\tilde{v}-\tilde{p})|}{\partial \mu} < 0$ *for sufficiently small* μ.

Figure 3 plots price informativeness τ_p and the futures price bias $|E(\tilde{v}-\tilde{p})|$ against the mass μ of financial traders. In the top panels, the parameters are the same as in Figure 2, that is, $\tau_\theta = \tau_\delta = \tau_\varepsilon = \tau_\xi = \tau_\alpha = \tau_\eta = 1, \gamma = \kappa = 0.1$, and $\rho = 0.5$. We have also set $\bar{\theta} = 2, c = 1$, and $\bar{\xi} = 0$, so that $E(\tilde{v}-\tilde{p}) > 0$ by Part (a) of Proposition 3. As we discussed in the previous subsection, price informativeness τ_p first increases and then decreases with μ in Panel a1. In Panel a2, the futures price bias $E(\tilde{v}-\tilde{p})$ monotonically decreases with μ, because the risk-sharing effect always dominates the learning effect in determining the overall effect of increasing μ on the futures price bias.

In the bottom two panels of Figure 3, we have increased the values of τ_δ, τ_η, and τ_ξ from 1 to 5. This change strengthens the negative effect on τ_p, because according to Part (b) of Proposition 2, the μ-threshold decreases with $\tau_\delta \tau_\eta \tau_\xi$. This can be seen from a left shift of the peak in Panel b1. In addition, we also increase the risk aversion γ of financial traders from 0.1 to 0.5 while still keeping the risk aversion κ of commodity producers at 0.1, so that commodity producers play a larger role in determining $E(\tilde{v}-\tilde{p})$ in equation (21). Both changes in parameters can make it more likely for the learning effect to dominate the risk-sharing effect, so that the futures price bias can increase with μ. Specifically, in Panel b1, we see that $E(\tilde{v}-\tilde{p})$ first decreases with μ (as predicted by Corollary 1), then increases with μ (because the learning effect dominates), and finally decreases with μ again (because the risk-sharing effect will eventually dominate, i.e., $E(\tilde{v}-\tilde{p}) \to 0$ as $\mu \to \infty$ in (21)).

4.3 Market liquidity

Market liquidity refers to a market's ability to facilitate the purchase or sale of an asset without drastically affecting the asset's price. The literature has used the coefficient p_ξ on exogenous noise trading $\tilde{\xi}$ in price function (12) to inversely measure market liquidity: A smaller p_ξ means that noise trading $\tilde{\xi}$ has a smaller price impact, and thus the market is deeper and more liquid. This measure of market liquidity is often referred to as Kyle's (1985) lambda. Using Proposition 1, we can compute

$$Market\ liquidity \equiv \frac{1}{p_\xi} = \frac{\overbrace{\frac{2\tau_\delta(\tau_p+\tau_\theta+\tau_\varepsilon)}{\kappa(\tau_p+\tau_\theta+\tau_\delta+\tau_\varepsilon)}}^{\text{market making by com. prod.}} + \overbrace{\frac{2\mu\tau_\delta}{\gamma(1-\rho^2)}}^{\text{market making by fin. traders}} + 1}{\underbrace{\frac{\tau_\delta\tau_p\pi_\xi}{\kappa(\tau_\delta+\tau_\theta+\tau_\varepsilon+\tau_p)}}_{\text{adverse selection of com. prod.}} + 1}. \tag{22}$$

Increasing the population size μ of financial traders has three effects on market liquidity $\frac{1}{p_\xi}$. The first effect is a direct positive effect: By submitting demand schedules, financial traders are effectively making the market to noise traders and thus, the more financial traders are in the market, the smaller is the price change induced by a change in the exogenous noise trading.

The other two effects are driven by the trading behavior of commodity producers that is influenced by μ via the price-informativeness channel. To fix ideas, let us assume that price informativeness τ_p increases with μ, which is true when μ is small (see Proposition 2). First, commodity producers now can learn more information from the price. This in turn makes commodity producers face less uncertainty and trade more aggressively against noise traders, enhancing their market making capacity. As a result, changes in noise trading are absorbed with a smaller price change. Second, when price becomes more informative, commodity producers also face a more severe adverse-selection problem. This is because commodity producers cannot disentangle information-driven trades from noise-driven trades. Thus, when the price contains more information, commodity producers make more inference from the price change induced by noise trading, which worsens market liquidity.

The overall liquidity effect of increasing μ is determined by the interactions among the above three effects. Proposition 4 provides a sufficient condition under which liquidity $\frac{1}{p_\xi}$ increases with μ. The condition is satisfied when $\tau_\delta \tau_\xi$ is sufficiently small, and/or when $(\tau_\theta + \tau_\varepsilon)$ or κ is sufficiently large. In addition, we use Figure 4 to plot $\frac{1}{p_\xi}$ against μ under the same parameter configuration as in Figure 2. In this example, market liquidity $\frac{1}{p_\xi}$ increases with the mass μ of financial traders.

Proposition 4 (Market liquidity) *If* $2\kappa^2 (\tau_\theta + \tau_\delta + \tau_\varepsilon)^2 > \tau_\delta \tau_\xi [\kappa (\tau_\theta + \tau_\delta + \tau_\varepsilon) + 2\tau_\delta (\tau_\theta + \tau_\varepsilon)]$, *then* $\frac{\partial p_\xi^{-1}}{\partial \mu} > 0$ *for sufficiently small* μ.

4.3.1 Why is the futures market so liquid?

Futures markets are typically very liquid. One typical argument is that futures contracts are standardized contracts traded on organized exchanges. Our analysis also highlights the other two possible forces that make a futures market particularly liquid: (1) the feedback effect of the futures price $\tilde{p}$ on the later spot price $\tilde{v}$, as captured by equation (11); and (2) the hedging needs of commodity producers, as captured by the second term in the right-hand side of equation (13). Both forces improve the market-making capacity of financial traders and commodity producers by making their demand functions more elastic, which therefore implies that any changes in exogenous noise trading can be absorbed with a smaller price change, leading to a higher market liquidity.

To illustrate this point, let us consider a hypothetical benchmark setting that shuts down the feedback effect and the hedging needs of commodity producers. Specifically, first, instead of using equation (11) to endogenously determine the spot price $\tilde{v}$, let us assume that the spot price is given exogenously as $\tilde{v} = \tilde{\theta}+\tilde{\delta}+c-E(\tilde{p})$, i.e., we replace $\tilde{p}$ with its unconditional average in (11). Second, for commodity producers' demand (13), we only keep its speculative component, i.e., $d(\tilde{s}_i, \tilde{p}) = \frac{E(\tilde{v}|\tilde{s}_i,\tilde{p})-\tilde{p}}{\kappa Var(\tilde{v}|\tilde{s}_i,\tilde{p})}$. In this benchmark economy, we can compute market liquidity as follows:

$$Market\ liquidity_{benchmark} = \frac{\frac{\tau_\delta(\tau_p+\tau_\theta+\tau_\varepsilon)}{\kappa(\tau_p+\tau_\theta+\tau_\delta+\tau_\varepsilon)} + \frac{\mu\tau_\delta}{\gamma(1-\rho^2)}}{\frac{\tau_\delta\tau_p\pi_\xi}{\kappa(\tau_\delta+\tau_\theta+\tau_\varepsilon+\tau_p)} + 1}. \tag{23}$$

Comparing equation (22) with equation (23), we find that market liquidity in our setting is higher than that in the benchmark setting for two reasons. First, recall that commodity producers treat the futures price $\tilde{p}$ as the effective selling price of commodities and thus, when $\tilde{p}$ is high, they produce more commodities, which in turn implies that they will short more futures. As a result, the demand of commodity producers is further reduced in addition to the position adjustment in the benchmark setting. This makes commodity producers' demand function more sensitive to price. Formally, this effect is captured by the term $-\tilde{p}$ in the hedging component of demand function (13). This translates to the extra term "+1" in the numerator of $\frac{1}{p_\xi}$ in equation (22), which leads to an extra liquidity in our setting.

Second, the feedback effect causes both financial traders and commodity producers to respond more to price changes in our setting. This is reflected by the multiple "2" in the two terms, $2 \times \frac{\tau_\delta(\tau_p+\tau_\theta+\tau_\varepsilon)}{\kappa(\tau_p+\tau_\theta+\tau_\delta+\tau_\varepsilon)}$ and $2 \times \frac{\mu\tau_\delta}{\gamma(1-\rho^2)}$, in the numerator of $\frac{1}{p_\xi}$ in equation (22). This comes from the fact that the sensitivity to price $\tilde{p}$ in the speculative components of demand functions in (16) and (17) is -2, instead of -1, which is the price sensitivity in the benchmark setting. Intuitively, when the futures price $\tilde{p}$ increases by an amount dp, both financial traders and commodity producers reduce their speculative positions in the futures contract for two reasons. First, as in a standard NREE model, a higher asset price leads to lower demand (Note that this has taken into account the fact that traders make inference about $\tilde{\theta}$ from the price). Second, specific to our setting, both types of traders expect that the later spot price $\tilde{v}$ will drop by the amount of dp via equation (11). Since $\tilde{v}$ is the payoff on the futures contract, both types of traders will reduce further their futures positions. Thus, the demand functions are double more elastic and hence the market is more liquid in our setting than in the benchmark setting without a feedback effect.

4.4 Commodity-equity market comovement

There exists empirical evidence documenting that commodity financialization increases the comovement between the commodity futures market and the equity market. Gorton and Rouwenhorst (2006) demonstrated that before 2004, commodity returns had negligible correlations with equity returns. Tang and Xiong (2012) documented that the correlation between the Goldman Sachs Commodity Index (GSCI) and the S&P 500 stock returns rose after 2004, and was especially high in 2008, which is concurrent with the financialization of commodities. Büyükşahin and Robe (2013, 2014) further link the increased correlation between commodities and stocks to the trading of hedge funds, especially those funds that are active in both equity and commodity futures markets. (See the survey by Cheng and Xiong (2014) for more discussions.)

Our model can shed light on this commodity-equity comovement driven by commodity financialization. Specifically, we can interpret financial traders' extra investment opportunity in our setting as stocks in reality, and consistent with Büyükşahin and Robe (2013, 2014), financial traders can represent hedge funds who hold positions in both equity and commodity futures markets. The return on stocks is simply $\tilde{\alpha}+\tilde{\eta}$, while the return on futures is $\tilde{v}-\tilde{p}$. We can capture the commodity-equity comovement by $Cov\left(\tilde{v}-\tilde{p},\tilde{\alpha}+\tilde{\eta}\right)$, and examine how $Cov\left(\tilde{v}-\tilde{p},\tilde{\alpha}+\tilde{\eta}\right)$ changes with the mass μ of financial traders.

Proposition 5 (Commodity-equity market comovement)
(a) The covariance between stock returns $\tilde{\alpha}+\tilde{\eta}$ *and futures returns* $\tilde{v}-\tilde{p}$ *is positive if and only if the correlation* ρ *between the unforecastable component* $\tilde{\eta}$ *in stock returns and the unforecastable component* $\tilde{\delta}$ *in commodity demand is positive. That is,* $Cov\left(\tilde{v}-\tilde{p},\tilde{\alpha}+\tilde{\eta}\right)>0\Longleftrightarrow Cov(\tilde{\delta},\tilde{\eta})>0$.
(b) When the population size μ *of financial traders is sufficiently small, commodity finan-*

cialization strengthens commodity-equity market comovement. *That is,* $\frac{\partial|Cov(\tilde{v}-\tilde{p},\tilde{\alpha}+\tilde{\eta})|}{\partial\mu} > 0$ *for sufficiently small* μ.

Figure 5 plots the correlation $Corr(\tilde{v}-\tilde{p},\tilde{\alpha}+\tilde{\eta})$ between futures returns $\tilde{v}-\tilde{p}$ and stock returns $\tilde{\alpha}+\tilde{\eta}$ for the same parameter configuration as in Figure 2. In particular, $\rho = 0.5 > 0$, and thus, consistent with Part (a) of Proposition 5, we observe that $Corr(\tilde{v}-\tilde{p},\tilde{\alpha}+\tilde{\eta}) > 0$. In addition, we see that $Corr(\tilde{v}-\tilde{p},\tilde{\alpha}+\tilde{\eta})$ increases with the mass μ of financial traders. Intuitively, the hedging-motivated trades of financial traders injects the forecastable component $\tilde{\alpha}$ in stock returns into the futures price $\tilde{p}$, which leads to the extra comovement between futures returns $\tilde{v}-\tilde{p}$ and stock returns $\tilde{\alpha}+\tilde{\eta}$. Thus, in our setting, it is financial traders, active in both the equity and commodity futures markets, that connect further these two markets, which is consistent with the empirical channel documented by Büyükşahin and Robe (2013, 2014). This view also complements Basak and Pavlova (2016) who obtain the increase in equity-commodity comovement through benchmarking institutional investors to a commodity index that serves as a new common factor on which all assets load positively.

5 Real Effect of Commodity Financialization

Recent empirical literature, e.g., Brogaard, Ringgenberg, and Sovich (2017), documents that commodity financialization affects the production decisions and profits of those companies that have significant economic exposure to index commodities. In this section, we use our framework to explore the effect of commodity financialization on commodity producers, which helps to understand and interpret the recent empirical findings.

The realized operating profits of a representative commodity producer i are $\tilde{v}x_i - C(x_i)$. Using the equilibrium production decision and price functions, equation (10), (11) and (12), we can compute the average operating profits as follows:

$$\begin{aligned}\text{Operating profits} &\equiv E[\tilde{v}x_i - C(x_i)] \\ &= \left[\bar{\theta} - E(\tilde{p})\right][E(\tilde{p}) - c] - \frac{[E(\tilde{p}) - c]^2}{2} \\ &\quad + \frac{p_\theta}{\tau_\theta} - \frac{3}{2}Var(\tilde{p}).\end{aligned}$$

In our setup, commodity producers do not maximize expected profits. Instead, they maximize expected utility, as given by equations (4) and (5). In particular, we can rewrite commodity producer i's terminal wealth in (5) as

$$\begin{aligned}\tilde{W}_i &= \tilde{v}x_i - C(x_i) + (\tilde{v}-\tilde{p})\,d_i \\ &= \underbrace{(\tilde{v}-\tilde{p})(x_i+d_i)}_{\text{risk exposure}} + \underbrace{[\tilde{p}x_i - C(x_i)]}_{\text{riskless production}}.\end{aligned}$$

This allows us to decompose the producer's maximization problem (4) as follows:

$$\begin{aligned}&\max_{x_i,d_i}\left[E\left(\tilde{W}_i\middle|\,\tilde{s}_i,\tilde{p}\right) - \frac{\kappa}{2}Var\left(\tilde{W}_i\middle|\,\tilde{s}_i,\tilde{p}\right)\right] \\ \Longleftrightarrow\ &\max_{x_i+d_i}\left[E\left(\tilde{v}-\tilde{p}|\,\tilde{s}_i,\tilde{p}\right)(x_i+d_i) - \frac{\kappa}{2}(x_i+d_i)^2\,Var\left(\tilde{v}-\tilde{p}|\,\tilde{s}_i,\tilde{p}\right)\right] \\ &+\max_{x_i}\left[\tilde{p}x_i - C(x_i)\right].\end{aligned}$$

Thus, as we discussed in Section 3.1, in terms of commodity production, producer i treats the futures price $\tilde{p}$ as the effective selling price, and then adjusts the position d_i in futures contracts to reach the optimal exposure to the risk $\tilde{v}$. In particular, when making production decisions, commodity producer i does not face uncertainty since he sees $\tilde{p}$. He still actively learns from futures prices $\tilde{p}$, but this learning does not affect his production decision on x_i.

The optimal decisions are respectively given by equations (9) and (10) in Section 3. Inserting these optimal decisions yields the indirect utility of commodity producers as follows:

$$E\left(\tilde{W}_i\middle|\,\tilde{s}_i,\tilde{p}\right) - \frac{\kappa}{2}Var\left(\tilde{W}_i\middle|\,\tilde{s}_i,\tilde{p}\right) = \underbrace{\frac{\left[E\left(\tilde{v}-\tilde{p}|\,\tilde{s}_i,\tilde{p}\right)\right]^2}{2\kappa Var\left(\tilde{v}|\,\tilde{s}_i,\tilde{p}\right)}}_{\text{trading gains}} + \underbrace{\frac{(\tilde{p}-c)^2}{2}}_{\text{effective profits}}. \tag{24}$$

The first term reflects producer i's benefits from trading futures contracts, after he observes information $\{\tilde{s}_i,\tilde{p}\}$ and trades futures. The second term is the *effective profits* from producing commodities that can be sold at the effective price $\tilde{p}$.

We then compute the ex ante certainty equivalent to capture the welfare of commodity producers,

$$CE_i \equiv -\frac{1}{\kappa}\log\left[E\left(e^{-\kappa\left[E\left(\tilde{W}_i\middle|\tilde{s}_i,\tilde{p}\right)-\frac{\kappa}{2}Var\left(\tilde{W}_i\middle|\tilde{s}_i,\tilde{p}\right)\right]}\right)\right],$$

whose exact expression is given in Appendix C. To better understand CE_i, we can also simply take expectation on the indirect utility in (24) to average out uncertainty driven by $\{\tilde{s}_i,\tilde{p}\}$, and thus roughly decompose CE_i into the following two terms:

$$\begin{aligned}\text{Trading gains} &\equiv E\left[\frac{\left[E\left(\tilde{v}-\tilde{p}|\,\tilde{s}_i,\tilde{p}\right)\right]^2}{2\kappa Var\left(\tilde{v}|\,\tilde{s}_i,\tilde{p}\right)}\right], \\ \text{Effective profits} &\equiv E\left[\frac{(\tilde{p}-c)^2}{2}\right].\end{aligned}$$

This decomposition is "rough" because commodity producers are expected utility maximizers. Mathematically, it changes commodity producers' ex ante objective $-E\left(e^{-\gamma \tilde{W}_i}\right)$ to $-E\left[\log E\left(e^{-\gamma \tilde{W}_i}\middle| \tilde{s}_i, \tilde{p}\right)\right]$, which is also commonly used in recent models on acquisition of information or attention (e.g., Kacperczyk, Van Nieuwerburgh, and Veldkamp, 2016).

Due to the complexity of the expressions for profits and welfare variables, we use Figure 6 to conduct a numerical analysis. Here, we plot price informativeness τ_p, operating profits $E\left[\tilde{v}x_i - C\left(x_i\right)\right]$, trading gains $E\left[\frac{[E(\tilde{v}-\tilde{p}|\tilde{s}_i,\tilde{p})]^2}{2\kappa Var(\tilde{v}|\tilde{s}_i,\tilde{p})}\right]$, effective profits $E\left[\tilde{p}x_i - C\left(x_i\right)\right] = E\left[\frac{(\tilde{p}-c)^2}{2}\right]$, and producer welfare CE_i, against the mass μ of financial traders. The parameter values are the same as those in Figure 2. The patterns of most variables, except effective profits $E\left[\tilde{p}x_i - C\left(x_i\right)\right]$, are robust to parameter choices. For some other parameter configurations, such as those in Panel b of Figure 3, effective profits $E\left[\tilde{p}x_i - C\left(x_i\right)\right]$ can exhibit a nonmontone pattern (i.e., effective profits first increase, then decrease, and finally increase again with μ). But the patterns of price informativeness τ_p, operating profits $E\left[\tilde{v}x_i - C\left(x_i\right)\right]$, and producer welfare CE_i remain unchanged.

Panel a of Figure 6 simply reproduces Figure 2, that is, price informativeness first increases and then decreases with μ. Panel b of Figure 6 shows that operating profits exhibit a similar pattern. This is consistent with Brogaard, Ringgenberg, and Sovich (2017), who find that after 2004 following commodity financialization, the information efficiency of futures index prices decreases and at the same time, those firms using index commodities earn lower profits. However, the lower profits do not necessarily translate into a lower welfare of producers. In contrast, Panel c3 of Figure 6 shows that the pattern of producer welfare is opposite of operating profits; that is, CE_i is U-shaped in μ, while operating profits are hump-shaped. This result is driven by the behavior of trading gains $E\left[\frac{[E(\tilde{v}-\tilde{p}|\tilde{s}_i,\tilde{p})]^2}{2\kappa Var(\tilde{v}|\tilde{s}_i,\tilde{p})}\right]$ and effective profits $E\left[\tilde{p}x_i - C\left(x_i\right)\right]$ in Panels c1 and c2, respectively.

In Panel c1, trading gains are also U-shaped in μ. This is driven by the hump shape of price informativeness τ_p. Intuitively, as futures price becomes more informative, commodity producers will have fewer opportunities to explore their information advantage and so trading gains will deteriorate. This intuition shares similiarity to the well-known Hirshleifer effect (1971), which says that more public information harms investor welfare through destroying trading opportunities (see Goldstein and Yang (2017) for more discussions on this kind of negative welfare effect due to trading destructions). In Panel c2, effective profits $E\left[\tilde{p}x_i - C\left(x_i\right)\right]$ increase with μ. The intuition is as follows. Since commodity producers make production after seeing futures price $\tilde{p}$, they can adjust production to accommodate variations in $\tilde{p}$ (and thus effective profits are convex in $\tilde{p}$). More financial traders can make the futures price $\tilde{p}$ more volatile, which therefore benefits commodity producers.

Taken together, both price informativeness and operating profits first increase and then decrease with commodity financialization, while producer welfare first decreases and then increases with commodity financialization. Because welfare is not observable, empirical researchers often use operating profits as a proxy for welfare. Our analysis suggests that these two measures are aligned only if commodity producers do not take positions in futures markets. In contrast, if commodity producers also trade futures, researchers should carefully differentiate between operating profits and welfare when making normative statements. For instance, in Brogaard, Ringgenberg, and Sovich (2017), both price efficiency and operating profits deteriorate after 2004, which may suggest that commodity financialization actually benefits rather than harms those commodity producers who actively use futures to hedge their positions.

6 Conclusion

In the past decade, there is a large inflow of financial investors to commodity futures markets, which is labelled as the financialization of commodities. In this paper, we develop a model to study how commodity financialization affects information transmission in the futures market. One key insight is that the trading of financial traders not only brings fundamental information but also unrelated noise into the futures price. As a result, adding more financial traders can either improve or decrease price informativeness. This information effect in turn affects the decisions of other existing traders in the market, such as commodity producers. When the information effect is positive, commodity financialization reduces the futures price bias. However, when the information effect is negative, commodity financialization can aggravate the futures price bias, because the negative information effect can overwhelm the positive risk-sharing effect. In general, commodity financialization can improve market liquidity in the futures market and increase the comovement between the commodity futures market and the equity market. Finally, our analysis shows that operating profits and producer welfare often move in opposite directions in response to commodity financialization, which provides important guidance for interpreting empirical and policy studies on the real effect of commodity financialization.

Appendix A: Lemmas

In this Appendix, we provide two lemmas that will be used for future proofs.

Lemma A1 *(a)*

$$\frac{\partial \pi_\xi}{\partial \mu} = -\frac{\pi_\xi^2 \left[\left(\frac{\tau_\delta}{\tau_\theta + \tau_\varepsilon + \tau_p + \tau_\delta} \right)^2 \frac{2\mu\rho^2\tau_\eta\tau_\varepsilon}{\gamma^2\kappa(1-\rho^2)^2\tau_\alpha} \tau_p^2 \pi_\xi^2 + \frac{\tau_\delta}{\gamma(1-\rho^2)} \right]}{1 + \frac{2\tau_\delta\tau_\varepsilon}{\kappa(\tau_\theta+\tau_\varepsilon+\tau_p+\tau_\delta)^2}\tau_p\pi_\xi} < 0, \tag{A1}$$

$$\frac{\partial \tau_p}{\partial \mu} = -\frac{2\mu\tau_\delta\tau_\eta\rho^2}{\gamma^2\left(1-\rho^2\right)^2\tau_\alpha}\tau_p^2\pi_\xi^2 - 2\tau_p\pi_\xi^{-1}\frac{\partial \pi_\xi}{\partial \mu}; \tag{A2}$$

(b)

$$\frac{\partial \tau_p}{\partial \mu} > 0 \Longleftrightarrow \frac{\mu}{\left[\frac{\mu^2\rho^2\tau_\eta\tau_\delta}{\gamma^2(1-\rho^2)^2\tau_\alpha} + \frac{1}{\tau_\xi} \right]\pi_\xi} < \frac{\gamma\tau_\alpha\left(1-\rho^2\right)}{\rho^2\tau_\eta}. \tag{A3}$$

Proof. We apply the implicit function theorem to equations (B2) and (B4) to compute equations (A1) and (A2). We obtain condition (A3) as follows. Inserting (A1) into (A2), we can show that

$$\frac{\partial \tau_p}{\partial \mu} > 0 \Longleftrightarrow \mu\tau_p\pi_\xi < \frac{\gamma\tau_\alpha\left(1-\rho^2\right)}{\rho^2\tau_\eta}.$$

Then we use equation (B4) to express τ_p in terms of π_ξ on the left-hand-side (LHS) in the above condition. ■

Lemma A2 *As $\tau_\varepsilon \to \infty$, we have*

$$\pi_\xi \to \left[\frac{\tau_\delta}{\kappa} + \frac{\mu\tau_\delta}{\gamma\left(1-\rho^2\right)} \right]^{-1} \text{ and } \tau_p \to \left[\frac{\mu^2\rho^2\tau_\delta\tau_\eta}{\gamma^2\left(1-\rho^2\right)^2\tau_\alpha} + \frac{1}{\tau_\xi} \right]^{-1} \left[\frac{\tau_\delta}{\kappa} + \frac{\mu\tau_\delta}{\gamma\left(1-\rho^2\right)} \right]^2.$$

Proof. These expressions are obtained directly from equations (B2) and (B4). ■

Appendix B: Proofs

Proof of Proposition 1

We plug the demand functions (16) and (17) into the market-clearing condition (8) to write the equilibrium price $\tilde{p}$ as functions of $(\tilde{\theta}, \tilde{\alpha}, \tilde{\xi})$. This gives the expressions of the p-coefficients in Proposition 1.

By the expressions of the p-coefficients, we have

$$\pi_\alpha \equiv \frac{p_\alpha}{p_\theta} = -\frac{\frac{\mu\rho\sqrt{\tau_\eta\tau_\delta}}{\gamma(1-\rho^2)}}{\frac{\frac{\tau_\varepsilon}{\tau_\theta+\tau_\varepsilon+\tau_p}}{\kappa\left(\frac{1}{\tau_\theta+\tau_\varepsilon+\tau_p}+\frac{1}{\tau_\delta}\right)} + \frac{\mu\tau_\delta}{\gamma(1-\rho^2)}}, \tag{B1}$$

$$\pi_\xi \equiv \frac{p_\xi}{p_\theta} = \frac{1}{\frac{\frac{\tau_\varepsilon}{\tau_\theta+\tau_\varepsilon+\tau_p}}{\kappa\left(\frac{1}{\tau_\theta+\tau_\varepsilon+\tau_p}+\frac{1}{\tau_\delta}\right)} + \frac{\mu\tau_\delta}{\gamma(1-\rho^2)}}. \tag{B2}$$

Using these two equations, we can express π_α in terms of π_ξ as in Proposition 1:

$$\pi_\alpha = -\frac{\mu\rho\sqrt{\tau_\eta\tau_\delta}}{\gamma\left(1-\rho^2\right)}\pi_\xi. \tag{B3}$$

Combining (15) and (B3), we have

$$\tau_p = \left[\frac{\mu^2\rho^2\tau_\delta\tau_\eta}{\gamma^2\left(1-\rho^2\right)^2\tau_\alpha} + \frac{1}{\tau_\xi}\right]^{-1}\pi_\xi^{-2}. \tag{B4}$$

Inserting equation (B4) into equation (B2) generates an equation that is defined in terms of a single unknown π_ξ. Now we prove that there exists a unique solution of π_ξ.

First, by the intermediate value theorem, there exists a solution of π_ξ. To see this, note that when $\pi_\xi = 0$, we have $\tau_p = \infty$ by (B4), and so the right-hand-side (RHS) of (B2) is $\frac{\gamma\left(1-\rho^2\right)}{\mu\tau_\delta} > 0$. When $\pi_\xi = \infty$, we have $\tau_p = 0$ by (B4) and the RHS of (B2) is $\frac{1}{\frac{\frac{\tau_\varepsilon}{\tau_\theta+\tau_\varepsilon}}{\kappa\left(\frac{1}{\tau_\theta+\tau_\varepsilon}+\frac{1}{\tau_\delta}\right)}+\frac{\mu\tau_\delta}{\gamma(1-\rho^2)}} < \infty$.

Second, note that the RHS of (B2) is increasing in τ_p. By equation (B4), τ_p is decreasing in π_ξ. Thus, the RHS of (B2) is decreasing in π_ξ. As a result, the solution of π_ξ is unique.

Finally, since the RHS of (B2) is increasing in τ_p, we set $\tau_p = 0$ and $\tau_p = \infty$ to generate the lower and upper bounds for the equilibrium value of π_ξ in Proposition 1.

Proof of Proposition 2

Part (a): We prove Part (a) by checking the sign of $\frac{\partial\tau_p}{\partial\mu}$ at $\mu = 0$. By (A1), we know that π_ξ decreases with μ. Thus, as $\mu \to 0$, π_ξ does not go to zero. As a result, condition (A3) is always satisfied at $\mu = 0$. That is, $\left.\frac{\partial \tau_p}{\partial \mu}\right|_{\mu=0} > 0$.

Part (b): Suppose $\tau_\varepsilon \to \infty$. Inserting the expression of π_ξ in Lemma A2 into condition (A3) , we can show that

$$\frac{\mu}{\left[\frac{\mu^2\rho^2\tau_\eta\tau_\delta}{\gamma^2(1-\rho^2)^2\tau_\alpha} + \frac{1}{\tau_\xi}\right]\pi_\xi} < \frac{\gamma\tau_\alpha\left(1-\rho^2\right)}{\rho^2\tau_\eta} \Longleftrightarrow \mu < \frac{\kappa\gamma\tau_\alpha\left(1-\rho^2\right)}{\tau_\delta\tau_\xi\tau_\eta\rho^2}.$$

Proof of Proposition 3

Part (a): By equation (21), it is straightforward that $E\left(\tilde{v}-\tilde{p}\right) > 0 \Longleftrightarrow \frac{\bar{\theta}-c}{2} > \bar{\xi}$. Thus, the key is to compute equation (21). By demand functions (13) and (17) and the market-clearing condition (8), we can show that

$$\left[\frac{1}{\kappa Var\left(\tilde{v}|\,\tilde{s}_i,\tilde{p}\right)} + \frac{\mu\tau_\delta}{\gamma\left(1-\rho^2\right)}\right]E\left(\tilde{v}-\tilde{p}\right) = E\left(\tilde{p}-c\right) - \bar{\xi}. \tag{B5}$$

Then, we use the expression of $\tilde{v}$ in (11) to obtain

$$E\left(\tilde{p}-c\right) = \frac{\bar{\theta}-c}{2} - \frac{1}{2}E\left(\tilde{v}-\tilde{p}\right). \tag{B6}$$

From equations (B5) and (B6), we can compute equation (21).

Part (b): If $\frac{\partial \tau_p}{\partial \mu} > 0$, then $\frac{\partial}{\partial \mu}\frac{(\tau_\theta+\tau_\varepsilon+\tau_p)\tau_\delta}{\kappa(\tau_\theta+\tau_\varepsilon+\tau_p+\tau_\delta)} = \frac{\tau_\delta^2}{\kappa(\tau_\theta+\tau_\varepsilon+\tau_p+\tau_\delta)^2}\frac{\partial \tau_p}{\partial \mu} > 0$. Clearly, $\frac{\partial}{\partial \mu}\frac{\mu\tau_\delta}{\gamma(1-\rho^2)} = \frac{\tau_\delta}{\gamma(1-\rho^2)} > 0$. Thus, by equation (21), we have $\frac{\partial|E(\tilde{v}-\tilde{p})|}{\partial \mu} < 0$. If $\frac{\partial \tau_p}{\partial \mu} < 0$, Figure 3 constructs an example to show that $\left|E\left(\tilde{v}-\tilde{p}\right)\right|$ first increases and then decreases with μ.

Proof of Corollary 1

The proof follows directly from combining Part (a) of Proposition 2 and Part (b) of Proposition 3.

Proof of Proposition 4

Proposition 1, when $\mu = 0$, we have

$$\tau_p = \tau_\xi\pi_\xi^{-2}, \tag{B7}$$

where π_ξ is determined by the unique positive root of the following cubic:

$$\tau_\delta\tau_\varepsilon\pi_\xi^3 - \kappa\left(\tau_\theta+\tau_\delta+\tau_\varepsilon\right)\pi_\xi^2 - \kappa\tau_\xi = 0.$$

By Part (a) of Lemma A1, we have

$$\left.\frac{\partial \pi_\xi}{\partial \mu}\right|_{\mu=0} = -\frac{\pi_\xi^2 \frac{\tau_\delta}{\gamma(1-\rho^2)}}{1+\frac{2\tau_\delta \tau_\varepsilon}{\kappa(\tau_\theta+\tau_\varepsilon+\tau_p+\tau_\delta)^2}\tau_p \pi_\xi}, \tag{B8}$$

$$\left.\frac{\partial \tau_p}{\partial \mu}\right|_{\mu=0} = -2\tau_\xi \pi_\xi^{-3}\frac{\partial \pi_\xi}{\partial \mu}. \tag{B9}$$

Taking derivative of equation (22) with respect to μ and using equations (B7)–(B9), we can compute

$$\frac{\partial}{\partial \mu}\frac{1}{p_\xi} = \frac{\tau_\delta}{\gamma\left(1-\rho^2\right)} \frac{1}{\left(\frac{\tau_\delta}{\kappa}\frac{\tau_p \pi_\xi}{\tau_\delta+\tau_\theta+\tau_\varepsilon+\tau_p}+1\right)^2}\frac{Q_1}{Q_2},$$

where

$$Q_1 \equiv \kappa\left[\tau_\xi+\pi_\xi^2\left(\tau_\theta+\tau_\delta+\tau_\varepsilon\right)\right]\left[\kappa\left(\tau_\xi+\tau_\theta\pi_\xi^2+\tau_\delta\pi_\xi^2+\tau_\varepsilon\pi_\xi^2\right)^2+2\tau_\delta\tau_\xi\tau_\varepsilon\pi_\xi^3\right]>0,$$

$$\begin{aligned}
Q_2 \equiv\ & \left(\tau_\theta+\tau_\delta+\tau_\varepsilon\right)\left\{2\kappa^2\left(\tau_\theta+\tau_\delta+\tau_\varepsilon\right)^2-\tau_\delta\tau_\xi\left[\kappa\left(\tau_\theta+\tau_\delta+\tau_\varepsilon\right)+2\tau_\delta\left(\tau_\theta+\tau_\varepsilon\right)\right]\right\}\pi_\xi^6 \\
& +2\kappa\tau_\delta\tau_\xi\left(\tau_\theta+\tau_\delta+\tau_\varepsilon\right)\left(\tau_\theta+3\tau_\delta+3\tau_\varepsilon\right)\pi_\xi^5 \\
& +2\tau_\xi\left(3\kappa^2\left(\tau_\theta+\tau_\delta+\tau_\varepsilon\right)^2+\tau_\delta^3\tau_\xi+2\tau_\delta^2\tau_\xi\tau_\varepsilon\right)\pi_\xi^4 \\
& +4\kappa\tau_\delta\tau_\xi^2\left(\tau_\theta+2\tau_\delta+2\tau_\varepsilon\right)\pi_\xi^3 \\
& +\tau_\xi^2\left(6\kappa^2\tau_\theta+6\kappa^2\tau_\delta+6\kappa^2\tau_\varepsilon+2\tau_\delta^2\tau_\xi+\kappa\tau_\delta\tau_\xi\right)\pi_\xi^2 \\
& +2\kappa\tau_\delta\tau_\xi^3\pi_\xi+2\kappa^2\tau_\xi^3.
\end{aligned}$$

Note that in the expression of Q_2, only the coefficient on π_ξ^6 has an undetermined sign. If $2\kappa^2\left(\tau_\theta+\tau_\delta+\tau_\varepsilon\right)^2>\tau_\delta\tau_\xi\left[\kappa\left(\tau_\theta+\tau_\delta+\tau_\varepsilon\right)+2\tau_\delta\left(\tau_\theta+\tau_\varepsilon\right)\right]$, then this coefficient is positive, and thus $Q_2>0$. As a result, $\frac{\partial}{\partial\mu}\frac{1}{p_\xi}>0$.

Proof of Proposition 5

Part (a): By equations (11) and (12), we have

$$\begin{aligned}
& Cov\left(\tilde{\alpha}+\tilde{\eta},\tilde{v}-\tilde{p}\right) = Cov\left(\tilde{\alpha}+\tilde{\eta},\left(1-2p_\theta\right)\tilde{\theta}+\tilde{\delta}-2p_\alpha\tilde{\alpha}-2p_\xi\tilde{\xi}\right) \\
= \ & Cov\left(\tilde{\alpha},-2p_\alpha\tilde{\alpha}\right)+Cov\left(\tilde{\eta},\tilde{\delta}\right) \\
= \ & -2p_\alpha\frac{1}{\tau_\alpha}+\frac{\rho}{\sqrt{\tau_\eta\tau_\delta}}.
\end{aligned}$$

By Proposition 1, we have

$$\begin{aligned} p_\alpha &= D^{-1}\left[\frac{-\frac{\mu\rho\sqrt{\tau_\eta\tau_\delta}}{\gamma(1-\rho^2)}\pi\xi\frac{\tau_p}{\tau_\theta+\tau_\varepsilon+\tau_p}}{\kappa\left(\frac{1}{\tau_\theta+\tau_\varepsilon+\tau_p}+\frac{1}{\tau_\delta}\right)}-\frac{\mu\rho\sqrt{\tau_\eta\tau_\delta}}{\gamma\left(1-\rho^2\right)}\right], \\ &= -\frac{\mu\rho\sqrt{\tau_\eta\tau_\delta}}{\gamma\left(1-\rho^2\right)}D^{-1}\left[\frac{\pi\xi\frac{\tau_p}{\tau_\theta+\tau_\varepsilon+\tau_p}}{\kappa\left(\frac{1}{\tau_\theta+\tau_\varepsilon+\tau_p}+\frac{1}{\tau_\delta}\right)}+1\right]. \end{aligned}$$

Thus,

$$Cov\left(\tilde{\alpha}+\tilde{\eta},\tilde{v}-\tilde{p}\right)=\frac{\rho}{\sqrt{\tau_\eta\tau_\delta}}\left(2\frac{\mu\tau_\eta\tau_\delta}{\gamma\left(1-\rho^2\right)}D^{-1}\left[\frac{\pi\xi\frac{\tau_p}{\tau_\theta+\tau_\varepsilon+\tau_p}}{\kappa\left(\frac{1}{\tau_\theta+\tau_\varepsilon+\tau_p}+\frac{1}{\tau_\delta}\right)}+1\right]\frac{1}{\tau_\alpha}+1\right),$$

which implies that $Cov\left(\tilde{\alpha}+\tilde{\eta},\tilde{v}-\tilde{p}\right)>0$ if and only if $\rho>0$.

Part (b): Without loss of generality, let us assume $\rho>0$. When $\mu=0$, we have $Cov\left(\tilde{\alpha}+\tilde{\eta},\tilde{v}-\tilde{p}\right)=\frac{\rho}{\sqrt{\tau_\eta\tau_\delta}}$. When $\mu>0$, we have $Cov\left(\tilde{\alpha}+\tilde{\eta},\tilde{v}-\tilde{p}\right)>\frac{\rho}{\sqrt{\tau_\eta\tau_\delta}}$. Thus, it must be the case that $Cov\left(\tilde{\alpha}+\tilde{\eta},\tilde{v}-\tilde{p}\right)$ is increasing in μ at $\mu=0$.

Appendix C: Welfare Expressions of Commodity Producers in Section 5

The ex ante utility of commodity producer i is:

$$\begin{aligned} E\left[-e^{-\kappa\tilde{W}_i}|\tilde{s}_i,\tilde{p}\right] &= E\left(e^{-\kappa\left[E\left(\tilde{W}_i\middle|\tilde{s}_i,\tilde{p}\right)-\frac{\kappa}{2}Var\left(\tilde{W}_i\middle|\tilde{s}_i,\tilde{p}\right)\right]}\right) \\ = \quad E&\left[-\exp\left(-\left[\frac{\left[E\left(\tilde{v}-\tilde{p}|\,\tilde{s}_i,\tilde{p}\right)\right]^2}{2Var\left(\tilde{v}|\,\tilde{s}_i,\tilde{p}\right)}+\kappa\frac{\left(\tilde{p}-c\right)^2}{2}\right]\right)\right]. \end{aligned}$$

To compute the above expectation, we use the following formula. If $\tilde{X}\sim N\left(\bar{X},\sigma_X^2\right),\tilde{Y}\sim N\left(\bar{Y},\sigma_Y^2\right)$ and $Cov\left(\tilde{X},\tilde{Y}\right)=\sigma_{XY}$, then

$$E\left[e^{-\tilde{X}^2-\tilde{Y}^2}\right]=\left|I+2\Sigma\right|^{-1/2}\exp\left[2\begin{bmatrix}\bar{X}\\ \bar{Y}\end{bmatrix}'\left(I+2\Sigma\right)^{-1}\Sigma\begin{bmatrix}\bar{X}\\ \bar{Y}\end{bmatrix}-\bar{X}^2-\bar{Y}^2\right]$$

where I is 2 by 2 identity matrix and Σ is the variance matrix of $\left(\tilde{X},\tilde{Y}\right)'$.

Define:

$$\tilde{X}\equiv\frac{E\left(\tilde{v}-\tilde{p}|\,\tilde{s}_i,\tilde{p}\right)}{\sqrt{2Var\left(\tilde{v}|\,\tilde{s}_i,\tilde{p}\right)}},\tilde{Y}\equiv\sqrt{\frac{\kappa}{2}}\left(\tilde{p}-c\right).$$

We have

$$\bar{X}=\frac{E\left(\tilde{v}-\tilde{p}\right)}{\sqrt{2Var\left(\tilde{v}|\,\tilde{s}_i,\tilde{p}\right)}},\bar{Y}=\sqrt{\frac{\kappa}{2}}\left(E\left(\tilde{p}\right)-c\right),$$

$$\sigma_X^2 = \frac{\frac{\tau_\varepsilon}{\tau_\theta+\tau_\varepsilon+\tau_p}\left(\frac{1}{\tau_\theta}+\frac{1}{\tau_\varepsilon}\right)+\left(\frac{\tau_p\frac{1}{p_\theta}}{\tau_\theta+\tau_\varepsilon+\tau_p}-2\right)^2 Var\left(\tilde{p}\right)+2\frac{\tau_\varepsilon}{\tau_\theta+\tau_\varepsilon+\tau_p}\left(\frac{\tau_p\frac{1}{p_\theta}}{\tau_\theta+\tau_\varepsilon+\tau_p}-2\right)p_\theta\frac{1}{\tau_\theta}}{2Var\left(\tilde{v}\right|\tilde{s}_j,\tilde{p})},$$

$$\sigma_Y^2 = \frac{\kappa}{2}Var\left(\tilde{p}\right),$$

$$\sigma_{XY} = \frac{1}{\sqrt{2Var\left(\tilde{v}\right|\tilde{s}_i,\tilde{p})}}\sqrt{\frac{\kappa}{2}}\left[\frac{\tau_\varepsilon}{\tau_\theta+\tau_\varepsilon+\tau_p}p_\theta\frac{1}{\tau_\theta}+\left(\frac{\tau_p\frac{1}{p_\theta}}{\tau_\theta+\tau_\varepsilon+\tau_p}-2\right)Var\left(\tilde{p}\right)\right].$$

Thus,

$$CE_i = \frac{1}{2\kappa}\log\left|I+2\Sigma\right| - \frac{1}{\kappa}\left[2\begin{bmatrix}\bar{X}\\ \bar{Y}\end{bmatrix}'\left(I+2\Sigma\right)^{-1}\Sigma\begin{bmatrix}\bar{X}\\ \bar{Y}\end{bmatrix}-\bar{X}^2-\bar{Y}^2\right],$$

$$\begin{aligned}\text{Trading gains} &= \frac{\bar{X}^2+\sigma_X^2}{\kappa},\\ \text{Effective profits} &= \frac{\bar{Y}^2+\sigma_Y^2}{\kappa}.\end{aligned}$$

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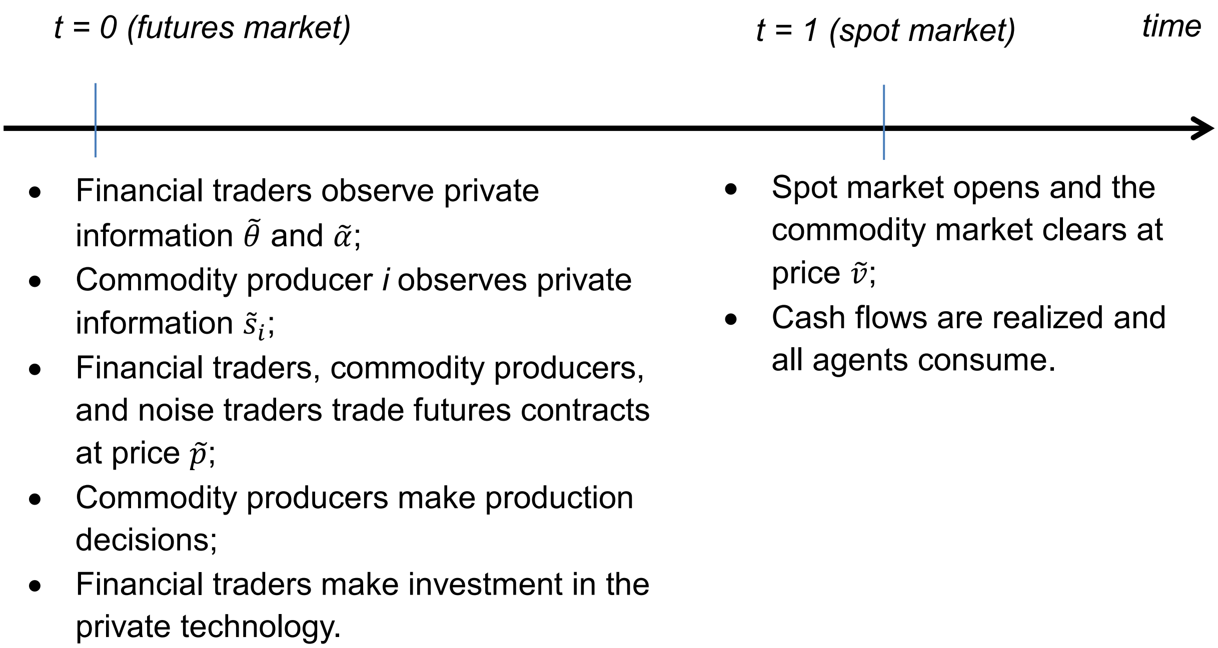
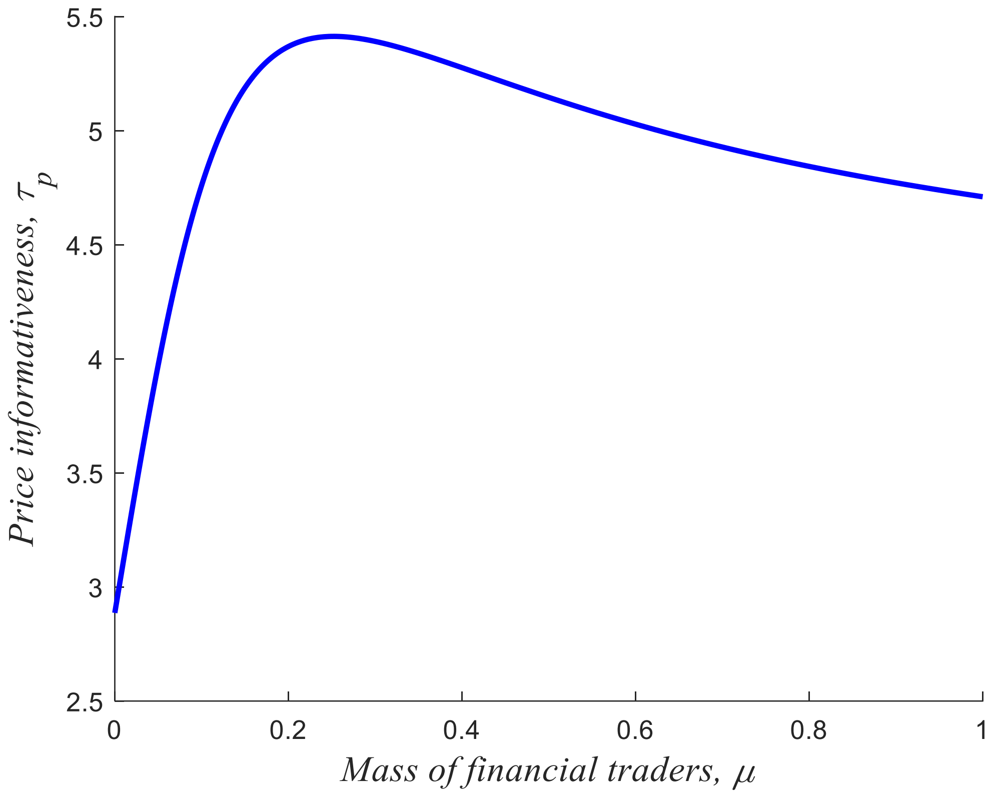
Figure 1: Timeline



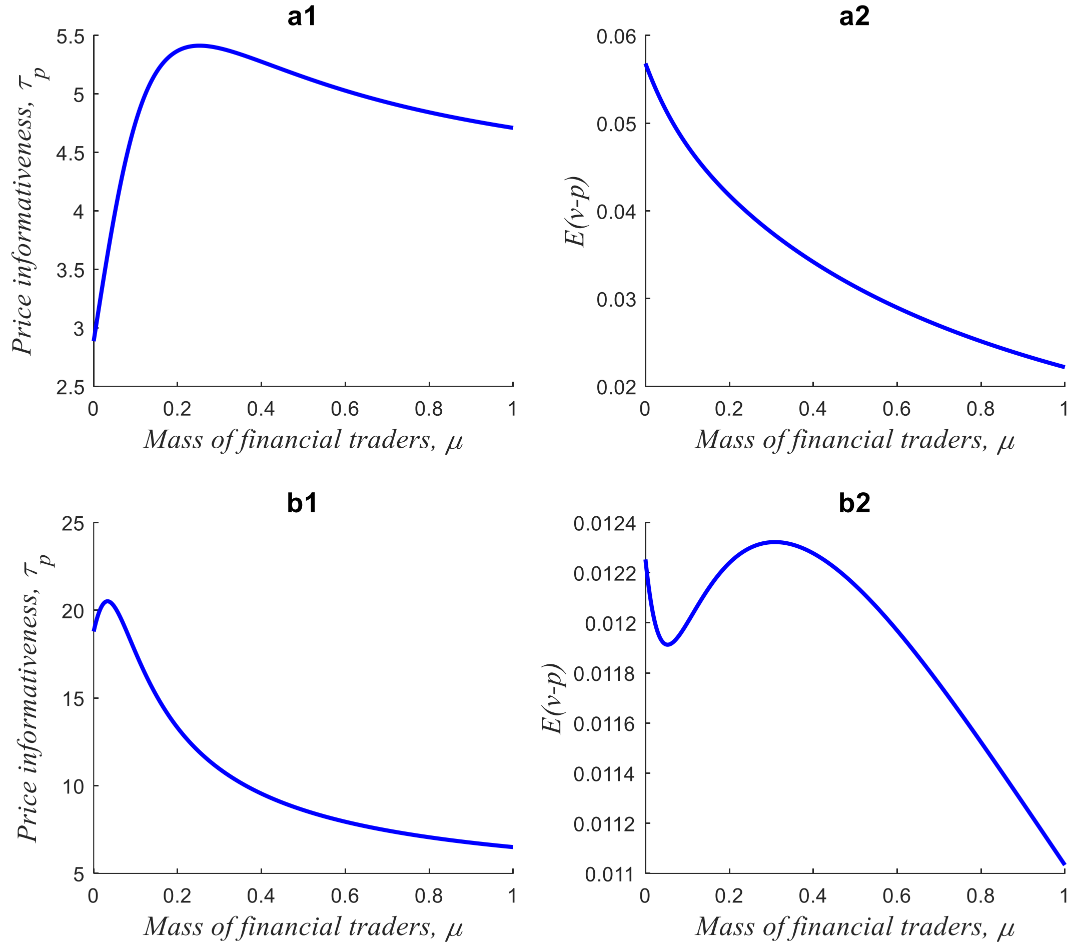
Note: This figure plots the order of events in the economy.

Figure 2: Price Informativeness



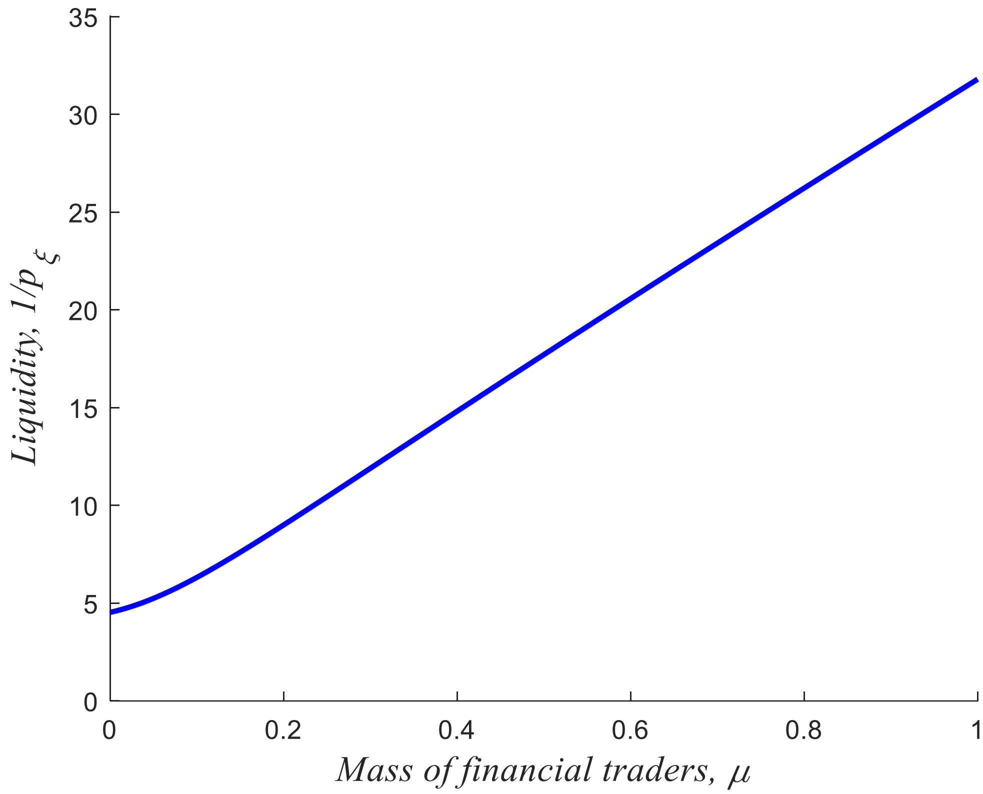
Note: This figure plots price informativeness τ_p against the population size μ of financial traders. The other parameters are: $\tau_\theta = \tau_\delta = \tau_\varepsilon = \tau_\xi = \tau_\alpha = \tau_\eta = 1$, $\gamma = \kappa = 0.1$, and $\rho = 0.5$.

Figure 3: Futures Price Biases



Note: This figure plots price informativeness τ_p and futures price biases $E(\tilde{v} - \tilde{p})$ against the population size μ of financial traders. In Panels a1 and a2, we set $\tau_\theta = \tau_\delta = \tau_\varepsilon = \tau_\xi = \tau_\alpha = \tau_\eta = 1$, $\gamma = \kappa = 0.1$, $\bar{\theta} = 2$, $c = 1$, and $\rho = 0.5$. In Panels b1 and b2, we set $\tau_\theta = \tau_\varepsilon = \tau_\alpha = 1$, $\tau_\delta = \tau_\eta = \tau_\xi = 5$, $\gamma = 0.5$, $\kappa = 0.1$, $\bar{\theta} = 2$, $c = 1$, $\bar{\xi} = 0$, and $\rho = 0.5$.

Figure 4: Market Liquidity



Note: This figure plots market liquidity $1/p_{\xi}$ against the population size μ of financial traders. The other parameters are: $\tau_{\theta} = \tau_{\delta} = \tau_{\varepsilon} = \tau_{\xi} = \tau_{\alpha} = \tau_{\eta} = 1$, $\gamma = \kappa = 0.1$, and $\rho = 0.5$.

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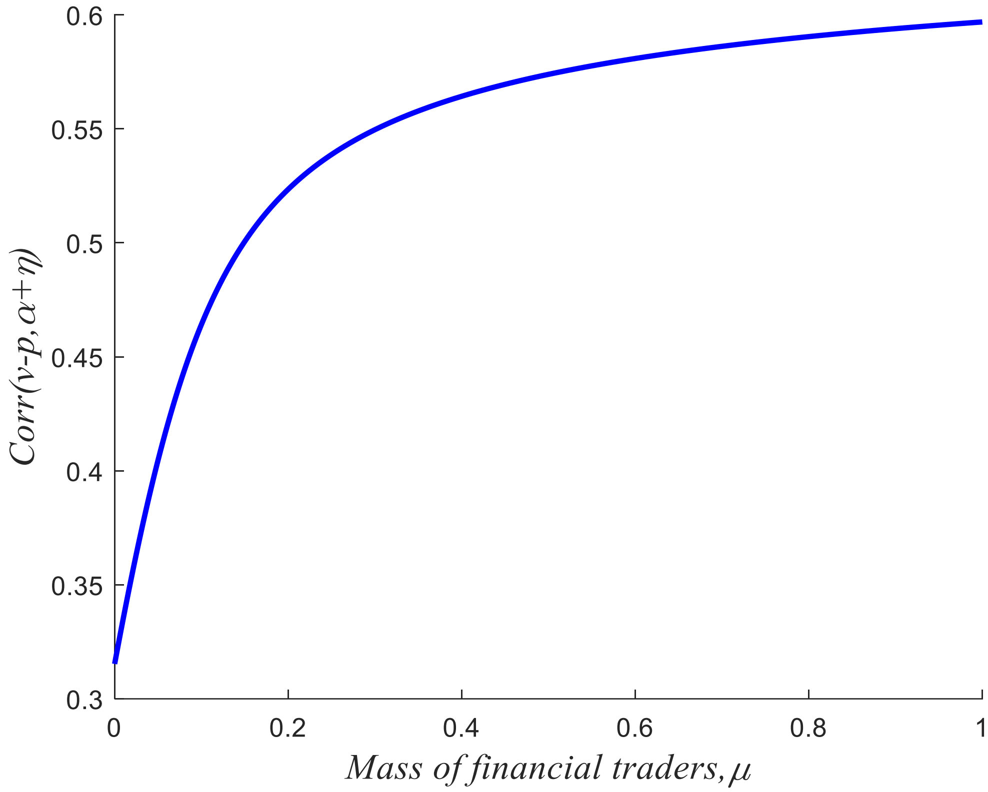
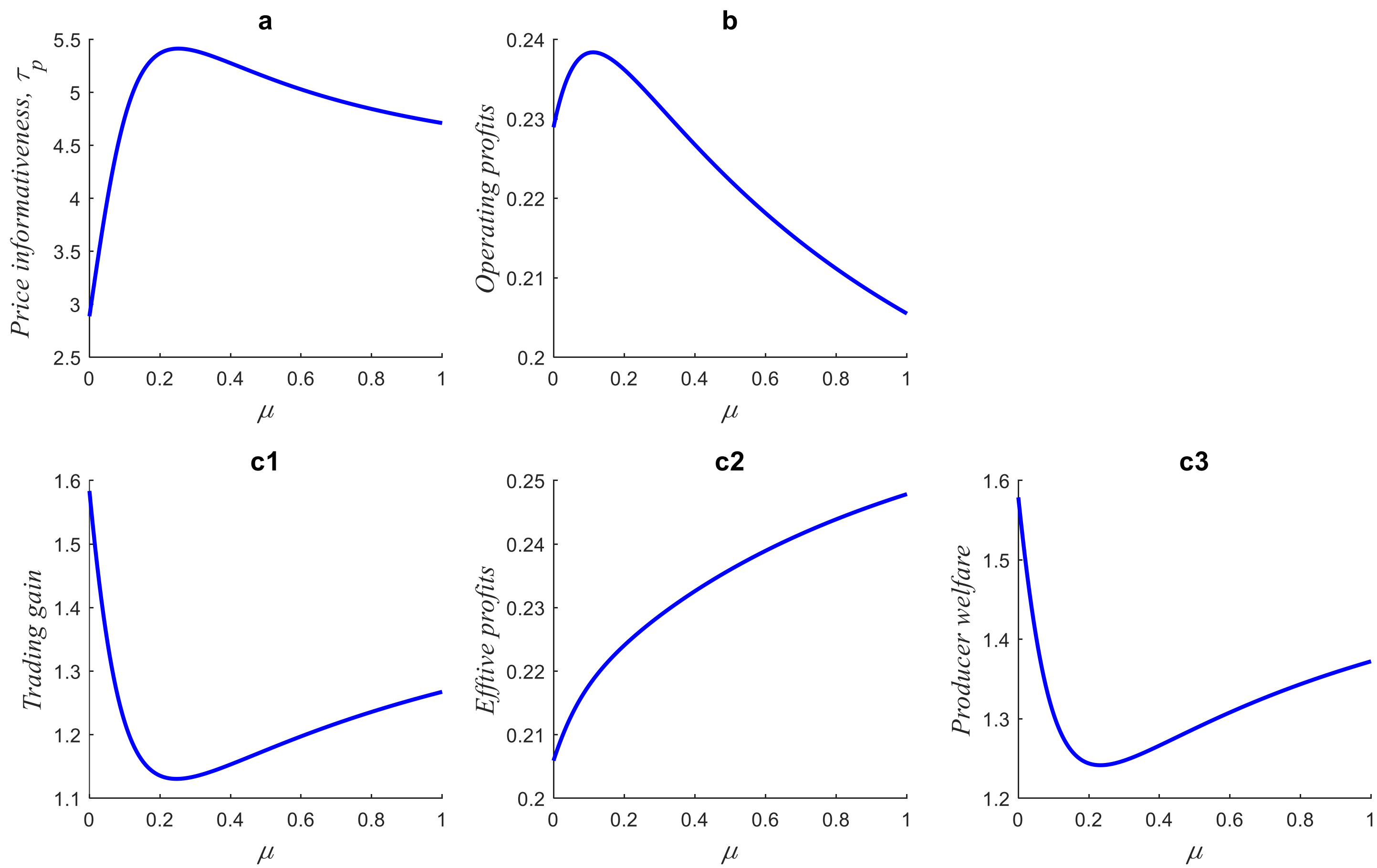
Figure 5: Comovement between Futures Market and Stock Market



Note: This figure plots the correlation coefficient $Corr(\tilde{v} - \tilde{p}, \tilde{\alpha} + \tilde{\eta})$ between commodity futures returns and stock market returns against the population size μ of financial traders. The other parameters are: $\tau_\theta = \tau_\delta = \tau_\varepsilon = \tau_\xi = \tau_\alpha = \tau_\eta = 1$, $\gamma = \kappa = 0.1$, and $\rho = 0.5$.

Figure 6: Real Effect of Commodity Financialization



Note: This figure plots price informativeness τ_p, operating profits $E[\tilde{v}x_i - C(x_i)]$, trading gain $E\left[\frac{[E(\tilde{v}-\tilde{p}|\tilde{s}_i,\tilde{p})]^2}{2\kappa Var(\tilde{v}|\tilde{s}_i,\tilde{p})}\right]$, effective profits $E[\tilde{p}x_i - C(x_i)]$, and commodity producers' ex ante welfare against the population size μ of financial traders. The other parameters are: $\tau_\theta = \tau_\delta = \tau_\varepsilon = \tau_\xi = \tau_\alpha = \tau_\eta = 1, \gamma = \kappa = 0.1$, and $\rho = 0.5$.