A Theory of Capital Structure, Price Impact, and Long-Run Stock Returns under Heterogeneous Beliefs

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Abstract

We analyze a firm's financing decision in an environment of heterogeneous beliefs and short sales constraints. We study a setting in which the insiders of a firm, owning a certain fraction of its equity, choose between equity, debt, or convertible debt to raise additional financing to implement a positive net present value project. The insiders' objective is to maximize their long-run wealth conditional on their own beliefs about their firm's future prospects. Market participants, each of whom have limited wealth, have heterogeneous beliefs about the firm's long-run value. We analyze two different economic settings: one in which there are no issue costs or costs of financial distress, and another in which these two costs are significant. We show that, in the absence of these two costs, the average belief of outsiders ("optimism") and the dispersion in outsider beliefs are the crucial determinants of the firm's security choice. When outsider beliefs are highly optimistic relative to firm insiders and the dispersion in outsider beliefs is high, the firm issues equity alone; when outsider beliefs are less optimistic (and less dispersed), the firm issues a combination of equity and debt. Neither straight debt alone nor convertible debt alone is optimal in this setting. Once the two costs are significant, we show that the firm issues equity when outsider beliefs are optimistic and highly dispersed; they issue debt when outsider beliefs are quite pessimistic and least dispersed; and convertible debt when outsider beliefs are between the above two extremes. Our analysis generates a pecking order of external financing under heterogeneous beliefs. In addition to our predictions for security choice, our model has several testable predictions for the price impact of equity, debt, and convertible debt issues and for the firm's long-run stock returns following the issuance of these securities.

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1 Introduction

Starting with Miller (1977), a number of authors have theoretically examined the stock price implications of heterogeneous beliefs and short sale constraints on stock valuations. Miller (1977) argued that when investors have heterogeneous beliefs about the future prospects of a firm, its stock price will reflect the valuation that optimists attach to it, because the pessimists will simply sit out the market (if they are constrained from short-selling). A number of subsequent authors have formalized Miller's (1977) intuition and brought out one of the most interesting cross-sectional implications of Miller's logic: the greater the divergence in the valuations of the optimists and the pessimists, the higher the current price of a stock in equilibrium, and hence lower the subsequent returns (see, e.g., Harrison and Kreps (1978), Morris (1996), Duffie, Gârleanu, and Pedersen (2002), and Chen, Hong, and Stein (2002) for contributions to this literature, and Scheinkman and Xiong (2004) for a review).[1] The above proposition that greater heterogeneity in investor beliefs about the future prospects of a firm leads to poorer subsequent long run stock returns has also been subject to some empirical testing (see, e.g., Diether, Malloy, and Scherbina (2002) or Chen, Hong, and Stein (2002)).

However, while the implications of heterogeneous beliefs among investors for long-run stock returns have been examined in some detail, the corporate finance implications of such beliefs have not been adequately studied. Several interesting questions arise in this context. For example, does heterogeneity in investor beliefs about the future prospects of a firm affect its security choice when raising external financing? Does increased investor optimism about a firm's future prospects result in its being more likely to issue equity over debt, or a combination of the two? Under what situations is it optimal to issue convertible debt? Does heterogeneity in investor beliefs affect the price-impact (i.e., the abnormal

[1] Several authors have argued that prior beliefs should be viewed as primitives in the economic environment (Kreps (1990)) and that it may be appropriate for economists to allow for differences in prior beliefs to understand economic phenomena (Morris (1996)). Kurz (1994) provides the foundations for heterogeneous but rational priors. Morris (1995) provides a detailed discussion of the role of the common prior assumption in economic theory.

return to equity upon a new security issue on the date the security is actually issued) of a firm's equity, debt, or convertible debt issue? What explains the fact that, while the long-term stock returns of both equity and debt issuers are negative, the long-term return of equity issuers is significantly more negative than that of debt issuers? Finally, how does the level of outside investor optimism about a firm's future prospects and the dispersion in outsider's beliefs affect the long-term stock returns to issuers of equity, debt, and convertible debt?

The objective of this paper is to answer many of the above questions theoretically in a heterogeneous beliefs framework. We analyze a firm's financing decision in an environment of heterogeneous beliefs and short sales constraints. The setting we study is one in which insiders of a firm, owning a certain fraction of equity in the firm, choose between equity, debt, or convertible debt to raise external financing to implement a positive net present value project. Market participants, each of whom have limited wealth to invest in the firm, have heterogeneous beliefs about the long-term value of the firm, which differs from that of the insiders. We can think of the average outsider belief as the level of "optimism" among outsiders, and the spread among investor beliefs as the "dispersion" in outside investors' beliefs. The objective of firm insiders is to make their choice of security (or combination of securities) to issue such that they maximize the long-term wealth of the current shareholders.

The paper consists of four parts. In the first part of the paper (section 2), we characterize the solution to the above described problem of the firm in a setting where there are no other market imperfections (i.e., no security issue costs or costs of financial distress) than heterogeneity in outside investors' beliefs. We show that, in the above setting, insiders of the firm will issue equity if and only if they expect the beliefs of the marginal outside investor to whom they will sell equity to be above their own beliefs about their firm's prospects, since they will otherwise have to sell equity that is "undervalued" with respect to their own valuation. Thus, if the marginal outside investor's belief (if the firm were to

raise external financing by issuing equity alone) is below their own belief, firm insiders will choose to issue a combination of equity and debt instead, taking advantage of the fact that the valuation of debt is relatively insensitive to outsider beliefs. The above implies that, the more optimistic or the more dispersed outsider beliefs are about the firm (or both), the more likely the firm is to issue equity rather than debt (or to use a larger fraction of equity to raise the required investment amount). Further, the larger the amount of external financing raised, the more likely the marginal investor's beliefs are to be below the insider beliefs so that the firm is more likely to issue debt.

In the second part of the paper (section 3), we introduce issue costs (e.g., investment banking fees) and costs of financial distress into our earlier setting. We then compare situations under which the firm issues combinations of securities (e.g., a combination of debt and equity) to those under which it will issue equity alone or debt alone. We show that, while the firm may issue equity alone in preference to a combination of debt and equity even when issue costs are low (this occurs when outsiders are significantly more optimistic than firm insiders), issuing a combination of equity and debt always dominates issuing debt alone if issue costs are low. Finally, we also compare situations under which the firm issues a combination of debt and equity to those under which it issues convertible debt.

In the third part of the paper (section 4), we study the price impact of equity, debt, and convertible debt issues, and study how the dispersion in investor beliefs affects the price impact of an equity issue. By price impact, we mean the abnormal return to the firm's equity from the price prevailing before the external security issue (not the announcement date) to the price prevailing after the issue. Since the market already is aware that a security issue has been announced, one would expect a price impact of zero in the absence of heterogeneity in investor beliefs. We demonstrate that, in the presence of heterogeneous beliefs among outside investors, the price impact of an equity issue will be negative, while that of debt and convertible debt issues will be zero. Further, we show that the price impact of

an equity issue will be more negative if the dispersion in outsider beliefs is greater.

In the fourth part of the paper (section 5), we characterize the long term stock returns of firms following equity, debt, and convertible debt issues. First, our analysis implies that the long-term stock returns after an equity issue will be negative. Second, it implies that the long-term stock returns after a debt issue will be greater (more positive or less negative) on average than those after an equity issue. Finally, our analysis predicts that the long-term stock returns after an equity issue will be more negative as the optimism level of outsider beliefs and the dispersion in beliefs is greater. We discuss several other testable implications of the model in section 6.

The implications of our model have motivated a recent empirical study by Chemmanur, Nandy, and Yan (2008). They test some of the above implications of our model using measures of investor optimism developed by Baker and Wurgler (2003), and the two standard proxies for heterogeneity in investor beliefs used in the literature, namely, the dispersion in analyst forecasts and abnormal share turnover. Their results can be summarized as follows. First, they find that the probability of a firm issuing equity rather than debt is increasing in both the level of optimism of outside investors and the dispersion in outsider beliefs. Second, they find, consistent with our predictions, that the price impact on a firm's equity is negative for an equity issue and zero for a debt issue. These results are robust to controlling for the fact that the choice of security to issue (debt versus equity) is itself determined by the average level of outsider beliefs (optimism) and the dispersion in these beliefs. Finally, they find that, while the long-run stock returns to both debt and equity issuers are negative, the stock returns to equity issuers is significantly more negative than that to debt issuers. Further, the more optimistic outside investors are at the time of an equity issue and more dispersed their beliefs, the more negative the long-term (one and two year) stock returns are to the firm after equity issuance.

The empirical results of Chemmanur, Nandy, and Yan (2008) indicate that outside investor optimism

and the dispersion in outside investor beliefs are indeed important determinants of the external financing choices made by a firm. Further, it is difficult to justify the existence of phenomena such as the negative price impact of an equity issue using other imperfections such as asymmetric information: since no new information arrives in the market on the day of an equity issue beyond that released on the day of the announcement of the issue (i.e., prior to the actual issue), the price impact should be zero in the absence of heterogeneous beliefs. Their results on the long-term returns to equity and debt issuers are also difficult to rationalize in the absence of heterogeneous beliefs among outside investors.

Our paper is related to several strands in the literature. Allen and Gale (1999) examine how heterogeneous priors affect new firm financing. Garmaise (2001) examines the implications of heterogeneous beliefs for security design. Dittmar and Thakor (2007) study how difference of opinion between managers and investors as a group, affect a firm's choice between debt and equity.[2]In contrast to their paper, our primary focus is on how heterogeneity in beliefs among investors affects the firm's choice of security to issue. Harris and Raviv (1993) use differences of opinion to explain empirical regularities about the relation between stock price and volume. Apart from the papers discussed earlier, a number of other papers have theoretically discussed asset trading and the asset pricing implications of heterogeneous beliefs: see, e.g., Mayshaw (1982), Jarrow (1980), Varian (1989), Biais and Bossaerts (1998), Brav and Heaton (2002), Harris and Raviv (1993), Kandell and Pearson (1995), Kyle and Lin (2003), Cao and Ou-Yang (2004), and Viswanathan (2000). The empirical literature on the relationship between the tightness of short sale constraints on a stock and subsequent stock returns is also related to this paper: see, e.g., Nagel (2005) and Asquith, Pathak, and Ritter (2005). Finally, our paper is broadly related to the large theoretical and empirical literature on corporate capital structure driven by considerations other than heterogeneity in beliefs (see Harris and Raviv (1991) for a review).

[2]Boot, Gopalan, and Thakor (2006) study an entrepreneur's choice between private and public financing in a similar setting of disagreement between insiders and outside investors as a group.

The rest of the paper is organized as follows. Section 2 outlines the basic model, without issue costs or costs of financial distress. Section 3 introduces issue costs and costs of financial distress into the model. Section 4 discusses the implications of our model with regard to the pecking order of security issuance of a firm under heterogeneous beliefs. Section 5 analyzes the price impact following equity, debt, and convertible debt issues, and section 6 analyzes the long-run returns following security issues. Section 7 highlights some of the testable implications of the model, and section 8 concludes. All proofs are confined to the appendix.

2 The Basic Model

There are three dates in the model: time 0, 1, and 2. At time 0, insiders of a firm own a fraction α of the firm's equity. The remaining $1-\alpha$ is held by a group of outside shareholders. The total number of shares in the firm is normalized to 1, so that α can be thought of as either the fraction of shares or the number of shares held by insiders. At time 1, the firm needs to raise an amount of I from outside investors to fund the firm's project. At time 2, the cash flows from the firm's project are realized and become common knowledge to all market participants, which can be either X^H or X^L, where $X^H > X^L > 0$.[3]

There is a continuum of investors in the market, with an aggregate wealth amount of $W > 0$. Each investor has the same amount of wealth. Market participants have heterogeneous beliefs about the future (time 2) cash flows of the firm. Firm insiders believe that with probability θ^f, the cash flow will be X^H, and with probability $1-\theta^f$, the cash flow will be X^L. We assume that $\theta^f X^H + (1-\theta^f)X^L > I$ so that firm insiders believe that the project has positive net present value. Outside investors' beliefs about the value of the firm are uniformly distributed over the interval $[\theta^m - d, \theta^m + d]$. We can think θ^m as the "average" or "mean" belief of outsiders, and d as the dispersion in outsiders' beliefs (we will

[3]Note that the values of X^H and X^L are conditional on the project being financed, and they already include the value of the new project.

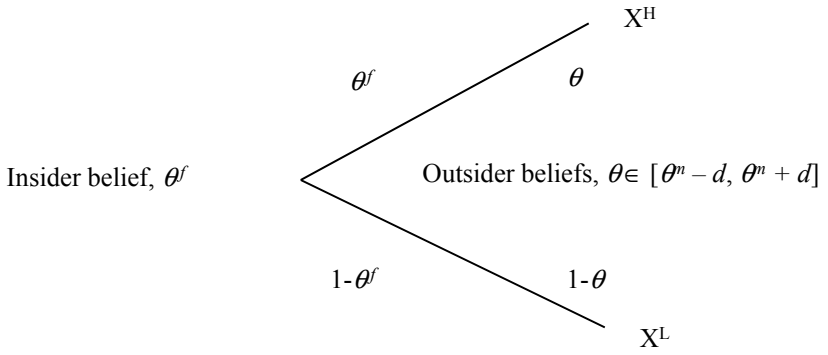

Figure 1: Beliefs of insiders and outsiders about the cash flow of the firm

sometimes refer to θ^m as the level of "optimism" among potential outside investors). We use θ to index an agent whose belief is θ. Agent θ believes that with probability θ the firm's time-2 cash flow will be X^H, and with probability $1-\theta$, the cash flow will be X^L. Clearly, outsiders who already hold the firm's stock at time 0 will be the most optimistic outside investors, and their beliefs are greater than $\theta^m + d$. We assume that the outside shareholders holding the outstanding stock in the firm have already exhausted their wealth so that they cannot buy any further securities issued by the firm at time 1. The beliefs about the cash flows of the firm are illustrated in Figure 1.

The menu of securities available to the firm consists of common equity, straight debt, and convertible debt. In the basic model (section 2), we assume that the firm does not incur any frictional cost of issuing securities (i.e., no issue or underwriting costs). Further, in the basic model we also assume that the firm does not incur any deadweight cost of financial distress even if it is in default on its promised payment on debt issued (either straight debt or convertible debt). We will introduce both the above costs in our full-fledged model (starting with section 3). Throughout the paper, we assume that all investors are subject to a short-sale constraint; i.e., no short selling in the firm's security is allowed in the economy. We also assume that the amount of total wealth available to all investors is relatively large compared to the amount of money the firm wants to raise, so that $W > 2I$.

The objective of firm insiders is to choose the optimal security to issue such that they maximize the expected time-2 payoff of current shareholders, based on firm insiders' belief, θ^f.[4] There is a risk-free asset in the economy, the net return on which is normalized to 0. All agents are risk-neutral. Thus, firm insiders choose the optimal security, S, to maximize the following objective function

$$\max_{S} \quad E_1[CF_2^{equity}|S, \theta^f] \tag{1}$$

where $E_1[CF_2^{equity}|S, \theta^f]$ is the time-1 expected value (according to firm insiders' belief) of the time-2 cash flows to the current equity holders of the firm, conditional on issuing security S, where S can be either equity, straight debt, or convertible debt.

2.1 The Case where the Firm issues Equity alone

We first analyze the case where the firm issues only equity to outside investors in order to raise the required amount of investment I at time 1. The issuing firm maximizes its expected payoff based on insiders' own belief about the firm's future cash flow at time 2. The following lemma characterizes the main properties of this equity issue.

Lemma 1 *When the issuing firm chooses to issue common stock alone to raise the amount of investment I, it has to issue a total of*

$$E_1 = \frac{I}{\widehat{\theta}X^H + (1-\widehat{\theta})X^L - I} \tag{2}$$

shares of new stock to outside investors at the price $PE_1^{Equity} = \widehat{\theta}X^H + (1-\widehat{\theta})X^L - I$, where the marginal investor in the firm's equity has the belief $\widehat{\theta} = \theta^m + d - \frac{2dI}{W}$ about the firm's cash flow at time 2. The equity price PE_1^{Equity} is decreasing in the amount of investment I.

Under heterogeneous beliefs and short-sale constraints, the firm will offer equity only to the most optimistic investors in the market. How much these investors are willing to pay for the firm's equity

[4]Since firm insiders hold a fraction α of the firm's shares, maximizing the value of current shareholders is equivalent to maximizing the value of shares held by firm insiders.

depends on two factors. The first factor is the average belief of investors in the market: the higher this average belief, the higher the beliefs of the group of outside investors who buy equity in the firm. The other factor is the dispersion in outside investors' beliefs. Holding the average belief constant, a higher dispersion in outside investors' beliefs means that more outside investors will be very optimistic about the future prospects of the firm so that the firm can sell its equity at a higher price to its new investors. The price at which the firm sells shares to outsiders depends on the belief of the marginal outside investor in the firm's equity, denoted by $\widehat{\theta}$. This marginal investor is determined by starting with the most optimistic outside investor willing to invest in the firm (whose belief is given by $\theta^m + d$) and working down the ladder of outside investors' beliefs until the entire amount I required for investment in the firm is raised by selling equity. Clearly, the higher the amount of money the firm needs to raise from outsiders, the lower down the ladder the firm needs to go in terms of outsiders' beliefs, and therefore the less optimistic the marginal investor holding the firm's equity subsequent to the equity issue will be. Hence, the less optimistic the marginal investor holding the firm's equity, the lower the price of the firm's equity will be.

2.2 The Case where the Firm issues Straight Debt alone

We now assume that the firm issues straight debt alone to raise the required investment amount of I. We assume that the face value of each unit of straight debt is 1. The price of the firm's straight debt is determined by the belief of the marginal investor in the firm's debt. The following lemma characterizes the main properties of this debt issue.

Lemma 2 *When the issuing firm chooses to issue straight debt alone to raise the required amount of investment I:*

i) If $I > X^L$, the firm issues risky straight debt. The price of each unit of debt is given by:

$$PD_1 = \frac{\widehat{\theta} I}{I - (1 - \widehat{\theta}) X^L}. \tag{3}$$

The firm needs to issue a total of

$$F = \frac{I - (1 - \widehat{\theta})X^L}{\widehat{\theta}} \tag{4}$$

units of straight debt to raise the amount I, where the marginal investor in the firm's debt has the belief $\widehat{\theta} = \theta^m + d - \frac{2dI}{W}$ *about the firm's cash flow at time 2.*

ii) If $I \leq X^L$*, the firm issues risk-free straight debt. The price* PD_1 *of each unit of debt is 1, and the firm needs to issue a total of* $F = I$ *units of straight debt to raise the required amount of financing* I*.*

When the firm issues straight debt alone in order to raise the required amount of new financing I, it raises these funds from the same group of investors as in the above case where it issues equity alone. In other words, similar to an equity issue, the firm starts with the outside investor who is most optimistic about the firm's future cash flows and works down the ladder of outsiders' beliefs until the entire amount I is raised by selling straight debt. Therefore, Lemma 2 shows that the marginal investor in the firm's debt is the same as the marginal investor in its equity if the firm were to issue equity only as in Lemma 1. We therefore denote the belief of this marginal debt investor also by $\widehat{\theta}$ which is equal to $\theta^m + d - \frac{2dI}{W}$.[5]

The price at which each unit of straight debt is sold, denoted by PD_1, is the price at which the marginal investor breaks even, given his belief $\widehat{\theta}$. The firm issues F units of straight debt such that it is able to raise the entire investment amount I. One should note that in the case of risk-free debt, the security price is independent of the marginal investor's belief $\widehat{\theta}$. However, in the case of risky debt, when the required amount of investment funding I is large, the debt price becomes sensitive to the marginal outside investor's belief $\widehat{\theta}$.

[5]One should note that, unlike an equity issue, the straight debt issue has no impact on the price of the firm's existing equity since the firm's marginal equity investor is the same as before the straight debt issue.

2.3 The Case where the Firm issues Convertible Debt alone

We now analyze the case where the firm issues convertible debt alone to raise the required investment amount of I. The terms of the convertible debt security are as follows: each unit has a face value of 1 and is sold at a price of p at time 1; each unit of convertible debt can be converted into x shares of equity at time 2 if the investor chooses to exercise this option. We assume that there are restrictions on the conversion ratio x so that convertible debt will be a truly hybrid security between equity and straight debt.[6]

We assume that the issuing firm has only one share of stock outstanding before issuing convertible debt. To raise the amount I, the firm has to issue a total of I/p units of convertible debt. If investors decide to convert into equity at time 2, then the value of each unit of convertible debt from conversion is $\frac{x}{1+xI/p}V$, where V is the firm's market value at time 2, which is equal to either X^H or X^L. Investors will convert to common stock only if the payoff from conversion is greater than the face value of the convertible debt, 1, that is, if

$$\frac{x}{1+x\frac{I}{p}}V > 1, \tag{5}$$

or equivalently

$$\frac{V}{1+x\frac{I}{p}} > \frac{1}{x}. \tag{6}$$

The quantity on the RHS of the inequality, $\frac{1}{x}$, is the conversion price of the convertible debt, whereas the LHS of the inequality corresponds to the firm value per share after the conversion. The following lemma characterizes the optimal conversion ratio x and the price p of the convertible debt, if the firm issues convertible debt alone in order to raise the required amount of investment financing I.

[6]If the conversion ratio x is too high, new investors holding convertible debt will find it optimal to convert into equity at time 2 regardless of the value of the firm's cash flow. Thus, there will be practically no difference between convertible debt and equity. Similarly, if the conversion ratio x is too low, there will be practically no difference between convertible debt and straight debt. Thus, convertible debt will be a truly hybrid security between equity and straight debt, only if the conversion ratio x is between a lower bound and an upper bound. Existing shareholders can also impose an upper bound on the conversion ratio simply due to their concerns about maintaining the control of the firm.

Lemma 3 *Suppose that* $x > \frac{1}{X^H - I}$ *if* $I \leq X^L$, *and* $x > \frac{\widehat{\theta}}{\widehat{\theta}X^H + (1-\widehat{\theta})X^L - I}$ *otherwise. Suppose also that* $x < \frac{\widehat{\theta}X^H + (1-\widehat{\theta})X^L}{X^L(\widehat{\theta}X^H + (1-\widehat{\theta})X^L - I)}$. *If the firm decides to issue convertible debt alone to raise the required investment amount of* I, *then:*

(i) When outsiders are more optimistic about the firm on average and their beliefs are more dispersed so that $\widehat{\theta} = \theta^m + d - \frac{2dI}{W} \geq \theta^f$, *it is optimal for the firm to set the conversion ratio at* $x = \overline{x}$. *In this case, the firm needs to issue*

$$F = \frac{I}{\overline{p}} \tag{7}$$

units of convertible debt, where the convertible debt price $p = \overline{p}$ *is given by* (A.29).

(ii) When outsiders are more pessimistic about the firm on average and their beliefs are less dispersed so that $\widehat{\theta} = \theta^m + d - \frac{2dI}{W} < \theta^f$, *it is optimal for the firm to set the conversion ratio at* $x = \underline{x}$ *given by* (A.23). *In this case, the firm needs to issue*

$$F = \frac{I}{\underline{p}} \tag{8}$$

units of convertible debt, where the convertible debt price $p = \underline{p}$ *is given by* (A.26).

When the firm issues convertible debt alone in order to raise the required amount of new financing I, it raises these funds from the same group of investors as in the above cases where it issues equity alone or straight debt alone. Thus, the marginal investor in the firm's convertible debt is determined by starting with the outside investor who is most optimistic about the firm's future cash flows and working down the ladder of outsider beliefs until the entire amount I required for investment in the firm is raised by selling convertible debt. Therefore, the belief of the marginal outside investor in the firm's convertible debt is identical to the belief of the marginal investor in the above cases where the firm issues equity or straight debt alone. We therefore denote the belief of the marginal investor in the firm's convertible debt also by $\widehat{\theta}$ which is equal to $\theta^m + d - \frac{2dI}{W}$. Given the price p, the conversion ratio x, and the expected cash flows offered by each unit of the convertible debt, the marginal investor breaks even in return for his investment in the firm.

The above lemma shows that the difference in beliefs between firm insiders and outside investors plays a critical role in the pricing and the design of the convertible debt security. When outsiders are sufficiently more optimistic about the firm's future cash flows on average (i.e., the outsiders' average

belief θ^m is higher) and their beliefs are more dispersed, the marginal outside investor with belief $\widehat{\theta}$ will also be more optimistic about the firm's future cash flows than firm insiders (i.e., $\widehat{\theta} \geq \theta^f$). In this case, we show that it is optimal for firm insiders to set the conversion ratio x at the highest possible value $\overline{x}$, and thereby maximize the equity component of the convertible debt. This makes sense since this equity component will be overvalued by the marginal outside investor relative to firm insiders' belief, and therefore, firm insiders will seek to benefit from capturing the outsiders' optimism on behalf of the existing shareholders by maximizing the equity component of convertible debt. The price of the convertible debt in this case is given by equation (7).[7] On the other hand, when outsiders are less optimistic about the firm's future cash flows on average and their beliefs are less dispersed, the marginal outside investor will also be less optimistic about the firm's future cash flows than firm insiders. In this case, it is optimal for firm insiders to set the conversion ratio at the lowest possible value $\underline{x}$ in order to minimize the equity component of the convertible debt since this component will now be undervalued relative to firm insiders' belief. The price of the convertible debt in this case is given by equation (8).[8]

2.4 The Choice between Equity alone, Straight Debt alone, and Convertible Debt alone

In this subsection, we now assume that the firm has the choice of issuing either equity alone, debt alone, or convertible debt alone in order to finance the project. Thus, we assume that the issue costs are prohibitively expensive so that the firm can not issue a combination of different securities, and the entire investment amount I needs to be raised by issuing only one type of security. The following proposition characterizes the conditions under which the firm chooses to issue each security.

[7] However,we will later show in Proposition 1 that if the firm is unconstrained with regard to its choice of security, so that it can choose among equity, straight debt, and convertible debt, it will always choose to issue equity rather than convertible debt under this scenario.

[8] One should again note that, unlike an equity issue, the convertible debt issue has no impact on the price of the firm's existing equity since the firm's marginal equity investor is the same as before the convertible debt issue.

Proposition 1 ***(The Choice between Equity alone, Straight Debt alone, and Convertible Debt alone)*** *Let* $\widehat{\theta}X^H + (1-\widehat{\theta})X^L > I$ *so that the firm's project has positive NPV based on the marginal outside investor's belief* $\widehat{\theta} = \theta^m + d - \frac{2dI}{W}$. *If the firm can issue only one type of security in order to raise the required amount of* I *for the project from outside investors, then:*

(i) The firm will choose to issue equity alone if outsiders are more optimistic about the firm on average and their beliefs are more dispersed so that the marginal outside investor is more optimistic than firm insiders, i.e., if $\widehat{\theta} = \theta^m + d - \frac{2dI}{W} > \theta^f$;

(ii) The firm will choose to issue straight debt alone if outsiders are more pessimistic about the firm on average and their beliefs are less dispersed so that the marginal outside investor is less optimistic than firm insiders, i.e., if $\widehat{\theta} = \theta^m + d - \frac{2dI}{W} \leq \theta^f$;

(iii) The firm will never choose to issue convertible debt in the absence of issue costs or costs of financial distress, since convertible debt will be dominated by either equity alone or straight debt alone, depending on outsiders' beliefs.

As we discussed the issuance of each particular security to fund the new project under lemmas 1, 2, and 3, the marginal outside investor in the firm's securities is determined by starting with the most optimistic investor willing to invest in the firm and working down the ladder of outsider beliefs until the entire investment amount of I is raised. Therefore, in each case, we showed that the marginal outside investor has the same belief $\widehat{\theta}$ about the firm's future cash flow at time 2: i.e., $\widehat{\theta} = \theta^m + d - \frac{2dI}{W}$, regardless of the particular security the firm chooses to issue at time 1 in order to externally raise the total amount of I. However, since each security has its own unique payoff structure depending on the state of the world at time 2, the expected payoffs of insiders and existing shareholders will be different across all three different securities despite the fact that the marginal outside investor's belief does not depend on the type of security issued by the firm. The above proposition shows that the sensitivity of the value of a particular security to the beliefs of outside investors about the firm's future cash flows is a critical factor affecting the choice of external finance for the firm.

In the case when outside investors are more optimistic about the firm's future cash flows on average, i.e., the average outsider belief θ^m is relatively high, and their beliefs are more dispersed, i.e., the dispersion d in beliefs across outside investors is relatively high, the belief of the most optimistic new

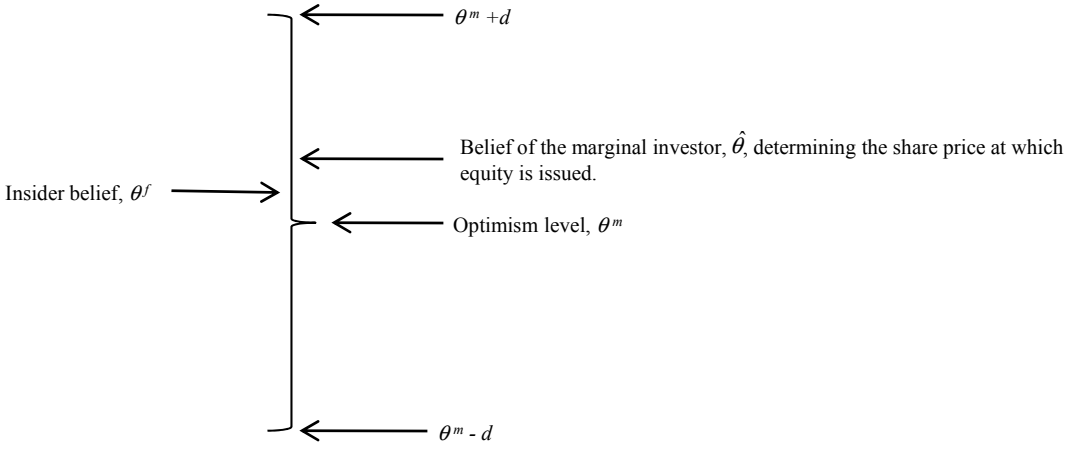

Figure 2: Beliefs of insiders and outsiders in the scenario where the firm chooses to issue equity

investor in the firm's security (given by $\theta^m + d$) is likely to be significantly higher than that of firm insiders, i.e., θ^f. Then, starting with this most optimistic investor willing to invest in the firm and working down the ladder of outsider beliefs until the entire investment amount of I is raised, the belief of the marginal outside investor, $\widehat{\theta}$, should also be more likely to be above that of firm insiders. In this situation, all these securities (equity, straight debt, or convertible debt) will be overvalued relative to firm insiders' belief. However, since equity is the most sensitive security to outsider beliefs, it will also be the most overvalued security based on insiders' beliefs, if the marginal outside investor is more optimistic than firm insiders.[9] Therefore, in this scenario, we show that the firm chooses to issue equity alone instead of the other two securities in order to best capture outside investors' optimism. Figure 2 illustrates the beliefs of insiders and outsiders in the scenario where the firm chooses to issue equity alone.

[9]Note that if we rank each security based on its value sensitivity to outsiders' beliefs about the firm's future cash flows, equity is the most sensitive security, since its payoffs are perfectly positively correlated with the state of the world. Straight debt is the least sensitive security to investor beliefs, since it promises the repayment of a fixed face value F unless the firm defaults in the future. Convertible debt which is a hybrid of straight debt and equity ranks in between the two with respect to its price sensitivity to outsider beliefs.

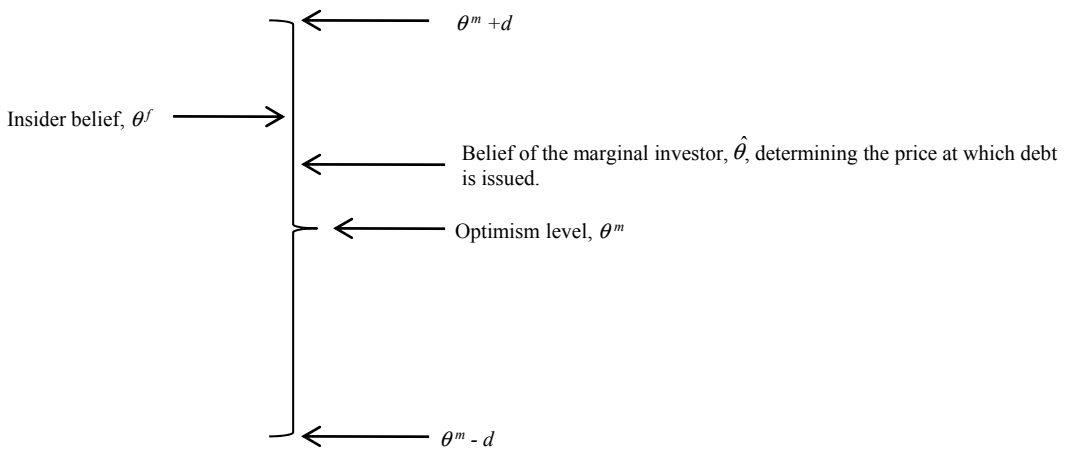

Figure 3: Beliefs of insiders and outsiders in the scenario where the firm chooses to issue debt

On the other hand, when outside investors are more pessimistic about the firm's future cash flows on average, and their beliefs are less dispersed, the belief of the most optimistic outside investor will not be as optimistic as in the scenario discussed in the previous paragraph. In this case, if the marginal investor's belief, $\widehat{\theta}$, is below that of firm insiders, and the firm chooses to sell equity, its equity will be substantially undervalued relative to the insiders' belief. Therefore, the firm will choose to issue straight debt since this security is less sensitive to outsider beliefs than either equity or convertible debt, and therefore the least undervalued. Figure 3 illustrates the scenario (the beliefs of insiders and outsiders) under which the firm chooses to issue straight debt.

In the absence of issue costs and costs of financial distress, the above proposition shows that issuing convertible debt is never optimal for the firm in either of the above two scenarios. When the marginal outside investor is more optimistic than firm insiders, i.e., $\widehat{\theta} \geq \theta^f$, the equity component of convertible debt will be overvalued. In other words, the embedded option to convert into equity and thereby share the upside of the firm will be overvalued by the marginal outside investor who is more optimistic than firm insiders. However, in this case, firm insiders would be even better off by issuing 100% new equity

instead of issuing convertible debt with overvalued equity component. One should note that there is always a positive probability that convertible debt holders will find it optimal not to convert into equity and instead prefer to claim the fixed face value of the convertible debt promised by the firm. Hence, the payoff of the convertible debt is less than perfectly positively correlated with the future cash flows of the firm at time 2, whereas the equity payoff is perfectly positively correlated. Thus, the valuation of equity is even more sensitive to outside investors' beliefs than that of convertible debt. Therefore, if the marginal outside investor is more optimistic than firm insiders, equity will be even more overvalued than convertible debt, and insiders can capture outside investors' optimism better by issuing equity rather than convertible debt.

On the other hand, when the marginal outside investor is more pessimistic than firm insiders, i.e., $\widehat{\theta} < \theta^f$, the equity component of convertible debt will be undervalued. In this case, while firm insiders are better off issuing convertible debt rather than equity (since the undervaluation of equity is more severe than that of convertible debt), they are even better off by issuing straight debt rather than convertible debt. Since straight debt always promises the repayment of a fixed face value no matter how good the state of the world is, its undervaluation based on insiders' belief will be less severe than that of convertible debt which gives the investor the right to participate in the upside potential of the firm by converting to equity. In other words, if the marginal outside investor is more pessimistic about the firm's upside potential than firm insiders, this conversion option will be undervalued based on insiders' belief, and therefore, it will be optimal for the firm to issue straight debt whose promised future payoff is fixed and therefore uncorrelated with the upside potential of the firm. In summary, convertible debt is a dominated security in the absence of issue costs and costs of financial distress. We will later demonstrate in section 3 using our full fledged model that the advantage of convertible debt over straight debt arises from its ability to minimize the expected costs of financial distress of the firm.

2.5 The Case of External Financing by Issuing a Combination of Securities

We now consider the possibility that the firm can issue a combination of debt and equity to raise the necessary financing for the project. In this subsection, we still assume that there are no costs of financial distress and no costs of issuing securities (no underwriting cost, for example).

Proposition 2 ***(The Choice between Equity alone, Straight Debt alone, and a Combination of Straight Debt and Equity)*** *Let $\theta^f < \theta^m + d$.*

(i) The firm will choose to issue equity alone when outsiders are more optimistic about the firm on average, their beliefs are more dispersed, and the required investment amount I is not too large so that the following condition holds: $\widehat{X}^2 - (I + X^f)\widehat{X} + (1-a)IX^f \geq 0$. For a given level of θ^m and d, this condition is satisfied only if $I \leq I_1$.

(ii) The firm will choose to issue a combination of straight debt and equity when outsiders are more pessimistic about the firm on average, their beliefs are less dispersed, and the required investment amount I is sufficiently large so that $\widehat{X}^2 - (I + X^f)\widehat{X} + (1-a)IX^f < 0$. For a given level of θ^m and d, this condition is satisfied only if $I > I_1$:

(a) If $I_1 < I \leq I_2$, the firm will choose to issue a combination of risk-free straight debt and equity.

(b) If $I > I_2$, the firm will choose to issue a combination of risky straight debt and equity.

(iii) It is never optimal for the firm to issue debt alone.

When the average outside investor is very optimistic about the firm's future cash flows and outsiders' beliefs are very dispersed, the marginal outside investor will be willing to pay a very high price for the firm's equity with respect to the insiders' beliefs. In this case, the above proposition shows that it will be optimal for the firm to issue equity alone in order to capture the high degree of optimism of the marginal outside investor. Issuing equity alone clearly dominates issuing debt alone since the value of equity is much more sensitive to the beliefs of outside investors than the value of debt, and therefore, if outside investors are much more optimistic than firm insiders, equity will be much more overvalued than straight debt based on insiders' beliefs. Issuing equity alone in this case also dominates issuing a combination of debt and equity because of the following trade-off the firm faces when issuing a combination of debt and equity. While raising part of the total funding I through debt issuance will increase the equity

price (since less money is raised through equity issuance), the debt price will not be as sensitive to the optimism in outsiders' beliefs as the equity price. When the marginal outside investor has a very optimistic view of the firm even in the case where the entire amount of funding is raised by issuing equity, issue equity alone better captures the optimism of outside investors than issuing a combination of equity and debt. Thus, firm insiders will choose to maximize the overvaluation benefit they capture due to the large difference in equity valuation between insiders and the marginal outside investor.

When the average outside investor is not so optimistic about the firm's future, and outsiders' beliefs are not so dispersed, issuing equity alone to raise the entire funding will hurt the firm's existing shareholders (and insiders), if the marginal outside investor has a lower valuation of the firm than the insiders do. Similarly, if the marginal outside investor's valuation of the firm is only slightly higher than the insiders' valuation of the firm (assuming that the firm issues equity alone), the firm can actually be better off by raising part of the total funding I through debt, and thereby increasing the equity price paid by the marginal equity investor. In such cases, the above proposition shows that it is optimal for the firm to issue a combination of debt and equity to raise the required funding I for the firm's project. Starting with the most optimistic outside investor with belief $\theta^m + d$ and going down the ladder of outsider beliefs, the firm can raise some money $(I - I_D)$ by issuing equity to the most optimistic investors and the rest (I_D) by issuing debt to the less optimistic investors until the entire amount of I is raised. In this way, as long as the most optimistic outside investor is more optimistic than the firm insiders, i.e., $\theta^f < \theta^m + d$, the firm can still capture and benefit from the optimism of the most optimistic outsiders by issuing some equity. On the other hand, by issuing some debt simultaneously, the firm will not be hurt by the views of the less optimistic and downright pessimistic outside investors.

If it is feasible for the firm to issue a combination of debt and equity, issuing debt alone is never optimal since this fails to capture the optimism of those investors with very optimistic beliefs about the

firm. If there exist some very optimistic outside investors who value the firm higher than the insiders, the firm can benefit from the optimism of these outsiders by issuing some equity to them. Thus, even if the average outside investor is not so optimistic about the firm's future prospects, issuing a combination of equity and debt dominates issuing debt alone as long as there exists some heterogeneity in outsiders' beliefs and the most optimistic outside investor is more optimistic than the firm insiders.[10]

Proposition 3 ***(The Choice between Convertible Debt alone and a Combination of Straight Debt and Equity)*** *In the absence of issuing costs, convertible debt is always dominated by a combination of straight debt and equity.*

As showed in Proposition 2, issuing straight debt alone is always dominated by issuing a combination of straight debt and equity. Similarly, the above proposition shows that in the absence of issuing costs, convertible debt is also dominated by a combination of straight debt and equity. When the firm issues a combination of straight debt and equity, it can sell equity to the most optimistic outside investors at a relatively high price and sell debt to the less optimistic outsiders. Starting with the most optimistic outside investor with belief $\theta^m + d$ and going down the ladder of outsider beliefs, the firm can raise some money $(I - I_D)$ by issuing equity to the most optimistic investors and the rest (I_D) by issuing debt to the less optimistic investors until the entire amount of I is raised. In other words, the marginal equity investor will have a more optimistic belief $\widetilde{\theta}$ about the firm than the marginal debt investor with belief $\widehat{\theta}$. The relationship between these two beliefs is given by the following equation:

$$\widetilde{\theta} = \theta^m + d - \frac{2d(I - I_D)}{W} = \widehat{\theta} + \frac{2dI_D}{W}. \tag{9}$$

In contrast, when the firm issues convertible debt, the equity component and the debt component of the security have to be sold to the same group of investors. Thus, the marginal outside investor who is

[10]However, this particular result is true only under the assumption that there are no issuing costs. When issuing costs are significant (as we assume in later sections), we will show that it can be optimal for the firm to issue debt alone as well as equity alone under certain conditions.

pricing the equity component of convertible debt is the same marginal investor who is pricing the debt component of it, so that both components are priced by the marginal investor with belief $\widehat{\theta}$. However, if the firm instead issues a combination of equity and debt, it can sell equity to the more optimistic outside investor with belief $\widetilde{\theta}$ who is willing to pay a higher price than the marginal convertible debt investor with belief $\widehat{\theta}$. Thus, when the firm issues convertible debt, it is unable to achieve the optimal price differentiation between its debt and equity components. Therefore, in the absence of issuance costs, convertible debt is always dominated by a combination of debt and equity.

3 The Model with Costs of Financial Distress and Issuing Costs: The Choice between Equity, Straight Debt, and Convertible Debt

In this section, we introduce two costs into our model: issue costs and costs of financial distress. We denote the costs of issuing a security (e.g., underwriting fees) by C^I, and the expected costs of financial distress by C^B. We analyze how these costs interact with heterogeneous investor beliefs in determining the firm's optimal choice of external financing among three different securities: equity, straight debt, and convertible debt.

3.1 The Choice between Equity alone, Straight Debt alone, and Convertible Debt alone

First, we analyze the case of prohibitively expensive issue costs where the firm can issue only one type of security to finance its investment. We assume that if the face value F of a debt security (straight or convertible) is strictly greater than the firm's cash flow X^L in the bad state, it faces an expected financial distress cost of $C^B > 0$. Otherwise, if $F \leq X^L$, the firm's expected cost of financial distress is 0. The following proposition characterizes the conditions under which the firm issues equity alone,

debt alone, or convertible debt alone.

Proposition 4 ***(The Choice between Equity alone, Straight Debt alone, and Convertible Debt alone)***

(i) When outside investors are very optimistic about the firm's future cash flows on average, and the dispersion in outsiders' beliefs is very high so that

$$\theta^m + d - \frac{2dI}{W} \geq \theta^f, \tag{10}$$

it is optimal for the firm to issue equity.

(ii) When outside investors are less optimistic about the firm's future cash flows on average and the dispersion in outsiders' beliefs is low so that

$$\theta^m + d - \frac{2dI}{W} < \theta^f, \tag{11}$$

the firm's optimal security choice is as follows:

a) If the required investment amount I is small so that $I \leq X^L$, it is optimal for the firm to issue risk-free straight debt.

b) If the required investment amount I is large so that $I > X^L$, it is optimal for the firm to issue risk-free convertible debt with total face value $F = X^L$.

The intuition behind the above proposition is as follows. When the average outside investor is much more optimistic about the firm's future cash flows than firm insiders, and outsiders' beliefs are very dispersed, the marginal outside investor will be more optimistic than firm insiders. In this situation, it is optimal for the firm to issue equity alone. Equity dominates both straight debt and convertible debt from the point of view of firm insiders since it best allows the firm to take advantage of the optimism among outsiders and thus sell a security that is most overvalued relative to the insiders' valuation based on their own belief. Furthermore, issuing equity alone minimizes the firm's costs of financial distress (i.e., they will be zero).

On the other hand, when the average outside investor is not so optimistic or downright pessimistic about the firm's future prospects and the dispersion in outsider beliefs is low (so that the marginal outside investor is less optimistic than firm insiders), equity is no longer the optimal security to issue.

This is because, in this case, equity (or any other security with an equity component) will be undervalued relative to the belief of firm insiders. The firm will then issue either risk-free straight debt or risk-free convertible debt depending on the size of investment I.

If the size of the investment is small, i.e., $I \leq X^L$, so that the firm can issue risk-free straight debt, the choice between straight debt and convertible debt depends on the following trade-off. On the one hand, the convertible debt has an equity component embedded in it, which will be undervalued relative to the insiders' belief in this situation (unlike risk-free straight debt, whose valuation is totally insensitive to outsiders' beliefs): we will call this effect as the "undervaluation effect." On the other hand, the option to convert to equity embedded in the convertible debt is also valuable to outsiders so that it reduces the face value of the debt to be offered to them (in return for a given amount of financing), thereby reducing the probability of the firm going into financial distress and therefore, the expected financial distress cost incurred by the firm: we will call this second effect as the "embedded option effect." In particular, in our model, the firm can always optimally set the total face value of the convertible debt such that it avoids bankruptcy with probability 1 altogether. However, if the firm can issue risk-free straight debt, then it will also have no costs of financial distress either. On the other hand, risk-free convertible debt with its embedded equity options will be undervalued with respect to the risk-free straight debt. Therefore, the undervaluation effect will dominate the embedded option effect, and the firm will prefer to issue risk-free straight debt rather than convertible debt.

If the size of the investment is large, i.e., $I > X^L$, the firm cannot issue risk-free straight debt to raise the entire amount of I. In this case, the embedded option effect of the convertible debt will favor issuing risk-free convertible debt to issuing risky straight debt. As we show in Lemma 2, risky straight debt is sensitive to the marginal investor's belief, and it is more undervalued than risk-free straight debt. In addition, it will also face financial costs of distress of C^B. However, we also show in the above

proposition that, while the firm can minimize the costs of financial distress toward 0 by reducing the face value with the presence of embedded equity options in convertible debt, risky straight debt will have the same undervaluation costs as convertible debt as long as the face value of the convertible debt is greater than or equal to the cash flow X^L in the bad state. Even though the embedded equity options of the convertible debt will be more undervalued by the marginal investor, the straight debt component of the convertible debt will be less undervalued as the face value of convertible debt decreases. These two effects cancel each other out, and therefore, risky straight debt has the same total undervaluation as the risky convertible debt for any face value $F \geq X^L$ (or the barely risk-free convertible debt with face value $F = X^L$). We also know from Lemma 3 that the firm has no incentive to reduce its face value below X^L, since this would increase the undervaluation of convertible debt. Thus, since the firm will optimally set the face value F of convertible debt to X^L if $I > X^L$ and reduce C^B to 0, the embedded option effect will dominate the undervaluation effect, and the firm will prefer to issue risk-free convertible debt with face value $F = X^L$ rather than risky straight debt.

In summary, the above proposition shows that the belief of the marginal investor relative to that of firm insiders determines a "pecking order" of external financing under heterogeneous beliefs. Thus, when the average outsider is very optimistic, and the dispersion in outsider beliefs is relatively high, so that the marginal outside investor is more optimistic than firm insiders, the firm always chooses to issue equity alone to raise external financing. In other words, if the marginal outside investor is more optimistic than firm insiders, the firm can capture the benefit of this optimism by issuing the most belief-sensitive security, i.e., equity, rather than less belief-sensitive securities like convertible debt or straight debt. However, when the average outside investor is less optimistic, and the dispersion in outsider beliefs is lower, so that the marginal investor is less optimistic than firm insiders, the firm's optimal choice of security depends on the trade-off between security undervaluation and costs of financial

distress. The firm chooses to issue risk-free convertible debt to raise external financing, if the required investment level I is considerably large, and therefore, the firm faces default risk. Even though risk-free convertible debt has some undervaluation (less than equity, and same as risky straight debt), it saves the firm financial costs of distress. Finally, if the size of required investment amount is small, the firm can also avoid costs of financial distress by issuing risk-free straight debt, which has the extra benefit of having no undervaluation at all when outsiders are more pessimistic than firm insiders. Thus, if the the amount of required external financing is small and the marginal outside investor is more pessimistic than firm insiders, the firm can avoid both undervaluation costs and costs of financial distress by issuing the most belief-insensitive security, i.e., the risk-free debt.

3.2 The Choice between Equity alone, Straight Debt alone, Convertible Debt alone, and a Combination of Straight Debt and Equity

We relax the assumption made in the previous subsection that the issuing cost C^I is always so prohibitively large that the firm finds it optimal to issue only one type of security: in other words, we will analyze situations where the firm may find it optimal to issue a combination of securities to raise the required amount of financing. We start with the simplest case where the menu of securities available to the firm consists of equity, straight debt, or a combination of equity and straight debt. The following proposition characterizes the firm's choice of external financing between equity alone, straight debt alone, and a combination of straight debt and equity when the firm faces issue costs and costs of financial distress.

Proposition 5 ***(The Choice between Equity alone, Straight Debt alone, and a Combination of Straight Debt and Equity)*** *Let $\theta^f < \theta^m + d$.*

(i) The firm will choose to issue equity alone when outsiders are more optimistic about the firm on average, their beliefs are more dispersed, and the required investment amount I is not too large so that the following condition holds: $\widehat{X}^2 - (I + X^f)\widehat{X} + (1 - a)IX^f \geq 0$. For a given level of θ^m and d, this condition is satisfied only if $I \leq I_1$.

(ii) The firm will choose to issue a combination of straight debt and equity when outsiders are more pessimistic about the firm on average, their beliefs are less dispersed, and the required investment amount I is sufficiently large so that $\widehat{X}^2 - (I + X^f)\widehat{X} + (1-a)IX^f < 0$. For a given level of θ^m and d, this condition is satisfied only if $I > I_1$:

(a) If $I_1 < I \leq I_3$, the firm will choose to issue a combination of risk-free straight debt and equity.

(b) If $I > I_3$, the firm will choose to issue a combination of risky straight debt and equity if the issuing cost is low ($C^I \leq \overline{C}^I$), and it is optimal for the firm to issue debt alone if the issuing cost is high ($C^I > \overline{C}^I$).

The threshold investment level I_3 is greater than I_2 given in Proposition 2, and increasing in the expected cost of financial distress C^B.

Similar to the intuition behind Proposition 2, when the average outside investor is very optimistic about the firm's future cash flows and the dispersion in outsider beliefs is high, the marginal equity investor will be much more optimistic about the firm than firm insiders. Hence, the equity price determined by the marginal outside investor will be much higher than the insiders' equity valuation. In this case, the firm will issue equity only in order to fully capture the greater optimism of outside investors. Conversely, when the average outside investor is less optimistic and the dispersion in outsider beliefs is lower, the marginal outside investor will value the firm only slightly higher or lower than firm insiders. Then, the firm will choose between straight debt alone and a combination of equity and debt based on the following trade-off. On the one hand, issuing a combination of debt and equity allows the firm to raise some money by selling equity to most optimistic investors and the rest by selling debt to less optimistic investors. In this way, the firm can capture the optimism of those few very optimistic investors in the market by selling equity to them at a relatively high price. By issuing some debt simultaneously, the firm will not be hurt by the views of the less optimistic and downright pessimistic outside investors. This is the case since the pricing of straight debt is much less sensitive to the beliefs of outside investors than the pricing of equity. On the other hand, issuing a combination of debt and equity also means that the firm has to pay issuing costs on two tranches of securities instead of just one. Therefore, the firm incurs more issuing costs when it issues a combination of debt and equity. The

firm will choose to issue straight debt alone when the issue costs are very high and it will choose to issue a combination of debt and equity when the issue costs are relatively low compared to the price differentiation benefits of the debt-equity combination.

We now include convertible debt in the menu of securities available to the firm as well. Given that we have already analyzed the firm's choice between equity alone, debt alone, and convertible debt alone in the previous subsection, we will now confine ourselves to analyzing the optimality of the firm issuing convertible debt versus a combination of debt and equity.

Proposition 6 ***(The Choice between Convertible Debt alone and a Combination of Straight Debt and Equity)*** *It is optimal for the firm to issue convertible debt when the dispersion d in outsiders' beliefs is low and the issuing cost C^I is high, and it is optimal for the firm to issue a combination of straight debt and equity when the dispersion in outsiders' beliefs d is high and the issuing cost C^I is low.*

Issuing a combination of straight debt and equity means that the firm can sell its securities at higher prices because the firm can sell equity to the most optimistic outside investors at a relatively high price and sell debt to the less optimistic outsiders. In contrast, when the firm issues convertible debt, the equity component and the debt component of the security have to be sold to the same group of investors, and therefore the firm is unable to capture the optimism among the most optimistic outside investors. The benefits of issuing a combination of debt and equity over issuing convertible debt increases with the dispersion in outsider beliefs, since more heterogeneous outsider beliefs imply the existence of a greater mass of very optimistic outside investors to whom the firm can sell equity at a relatively high price compared to the insiders' beliefs. On the other hand, issuing convertible debt rather than a combination of debt and equity can save the firm issuing costs. The issue cost saving benefit of the convertible debt increases with the issue cost C^I. When the dispersion in outsiders' beliefs is low and the issuing cost C^I is high, the cost saving benefit of convertible debt outweighs the valuation benefit of the debt-equity combination, and it is optimal for the firm to issue convertible debt. Conversely, when the dispersion in outsiders' beliefs is high and the issuing cost is low, the benefit of the debt-

equity combination outweighs the benefit of convertible debt, and it is optimal for the firm to issue a combination of debt and equity.

4 The Pecking Order of External Financing under Heterogeneous Beliefs

In this section, we discuss the pecking order implications of our basic model (section 2) and our extended model (section 3) for the security issuance of a firm under heterogenous beliefs.

4.1 The Pecking Order of External Financing under Heterogeneous Beliefs in the Basic Model

In our basic model with no costs of financial distress and issue costs, when the firm is allowed to issue only one type of security, we showed that it is optimal to issue either equity alone or straight debt alone depending on the marginal belief of the outside investor relative to the belief of firm insiders. If the marginal outside investor is more optimistic than firm insiders, the firm optimally chooses to issue the most belief-sensitive security, i.e., equity, in order to capture outsiders' optimism rather than straight debt or convertible debt which are less belief-sensitive securities, and therefore less overvalued than equity. On the other hand, if the marginal outside investor is more pessimistic than firm insiders, the firm optimally chooses to issue the least belief-sensitive security, i.e., straight debt rather than equity or convertible debt which are more belief-sensitive securities, and therefore more undervalued than straight debt. Thus, the belief of the marginal investor relative to that of firm insiders determines a "pecking order" of external financing under heterogeneous beliefs.

When we allow the firm to issue a combination of securities, our basic model still implies a "pecking order" of external financing under heterogeneous beliefs. If the marginal outside investor is much more

optimistic than firm insiders, the firm issues the most belief-sensitive security (equity only) to capture outsiders' optimism. If outside investors are moderately more optimistic or less optimistic than firm insiders on average, and the dispersion in their beliefs is not too high, we showed that the firm can benefit from issuing different securities to separate investor clienteles with different beliefs about firm value. If the required investment amount is not too large, the firm optimally issues a combination of risk-free straight debt and equity. If the amount of external financing is large, the firm optimally issues a combination of risky straight debt and equity. Risky straight debt is more undervalued than risk-free debt, but at the same time, it is less undervalued than equity.

Finally, in our basic model where the only market imperfection is heterogenous beliefs, convertible debt is dominated by either straight debt or equity (or a combination of these securities) depending on the belief of the marginal investor relative to that of firm insiders. This is because convertible debt is moderately belief-sensitive, and unlike a combination of straight debt and equity, it cannot be issued to separate investor clienteles with different beliefs.

4.2 The Pecking Order of External Financing under Heterogeneous Beliefs in the Model with Costs of Financial Distress and Issuance Costs

In our extended model with costs of financial distress and issuance costs, we also showed that the belief of the marginal investor relative to that of firm insiders determines a "pecking order" of external financing under heterogeneous beliefs. Recall the case when the firm is allowed to issue only one type of security in order to raise the required investment amount. When the average outsider is very optimistic, and the dispersion in outsider beliefs is relatively high, so that the marginal outside investor is more optimistic than firm insiders, the firm always chooses to issue equity alone to raise external financing. In other words, if the marginal outside investor is more optimistic than firm insiders, the firm can capture the benefit of this optimism by issuing the most belief-sensitive security, i.e., equity, rather than less

belief-sensitive securities like convertible debt or straight debt. However, when the average outside investor is less optimistic, and the dispersion in outsider beliefs is lower, so that the marginal investor is less optimistic than firm insiders, the firm's optimal choice of security depends on the trade-off between security undervaluation and costs of financial distress. The firm chooses to issue risk-free convertible debt in equilibrium, if the required investment level I is considerably large, and therefore, the firm faces default risk. Even though risk-free convertible debt has some undervaluation (less than equity, and same as risky straight debt), it saves the firm financial costs of distress. Finally, if the size of required investment amount is small, the firm can also avoid costs of financial distress by issuing risk-free straight debt, which has the extra benefit of having no undervaluation at all when outsiders are more pessimistic than firm insiders. Thus, if the the amount of required external financing is small and the marginal outside investor is more pessimistic than firm insiders, the firm can avoid both undervaluation costs and costs of financial distress by issuing the most belief-insensitive security, i.e., the risk-free debt. When we allow the firm to issue a combination of securities in our extended model, the same intuition goes through, and we obtain similar results.

5 The Price Impact of Security Issues

In this section, we investigate the price impact of equity, straight debt, and convertible debt issues on the current stock price of the firm at the time of a security issue. The price impact of a security issue is measured as the abnormal return to the firm's equity from the price prevailing before the external security issue (not the announcement date) to the price prevailing after the issue. Since the market already is aware that a security issue has been announced, one would expect a price impact of zero in the absence of heterogeneity in investor beliefs. The price impact of a security issue should take place as soon as the issue is effective. However, in this paper, we model the effective date and the

announcement date together. If we separate the two, there should be no announcement effect in our setting, since investors do not update their beliefs based on others' actions (insiders actions do not convey any information to outsiders in our setting, since there is no information asymmetry in our model). In the presence of heterogeneous beliefs, insiders and outside investors agree to disagree about the likelihood of future events (realization of future cash flows).

Proposition 7 ***(Price Impact of Security Issues)*** *(i) If the firm issues equity at time 0, there is a negative impact on the stock price*

$$\Delta PE^{Equity} = -\left(1 - \frac{I}{\widehat{\theta}X^H + (1-\widehat{\theta})X^L}\right)\frac{2dI}{W}(X^H - X^L) < 0. \tag{12}$$

(ii) If the firm issues straight debt or convertible debt, there is no impact on the stock price

$$\Delta PE^{Debt} = \Delta PE^{Convertible} = 0. \tag{13}$$

(iii) The greater the dispersion in outsiders' beliefs, d, the greater the price impact of an equity issue

$$\frac{\partial|\Delta PE^{Equity}|}{\partial d} > 0. \tag{14}$$

When the firm issues equity, it must sell the equity to investors who are less optimistic about the firm's value than current shareholders (since the current shareholders have limited wealth). Hence, the valuation of the new marginal equity investor (i.e., the stock price just after the equity issue) will be lower than the valuation of the marginal equity investor before the equity issue. Therefore, the equity price decreases, yielding a negative price impact. On the other hand, if straight debt or convertible debt is issued, the equity price will remain at the same level as before the security issue. This is because the marginal investor who holds equity in the firm remains the same in these cases, so that the valuation of the marginal equity holder is unaffected, resulting in a zero price impact of the straight debt issue or

convertible debt issue on the firm's equity.[11, 12, 13] This proposition also shows that the negative price impact of an equity issue will be larger in absolute value as the dispersion in outsiders' beliefs increases. If outsiders' beliefs are more dispersed (d is greater), the distance the firm has to go down the ladder of outside investors' beliefs (until the entire investment amount of I is raised) increases, yielding a more negative price impact of an equity issue.

6 Long-Run Stock Returns following Security Issues

In this section, we will analyze the long-run stock returns of firms following equity, straight debt, and convertible debt issues that have been studied extensively in the empirical literature. Here we will make the additional assumption that, in the long run, the dispersion in outside investors' beliefs substantially goes down due to the arrival of new public information about the firm. This new information is hard and credible, and can be publicly collected from the firm's annual reports and earnings announcements. Thus, due to the arrival of this new information, we assume that investor beliefs about the firm's cash flows become less heterogeneous over time. In particular, we assume that in the long run (i.e., just before the realization of cash flows at time 2), the dispersion in outsider beliefs decreases from d to $d(1-\delta)$ where δ is the percentage reduction in the dispersion of outside investors' beliefs about the firm's cash flow. Note that for a firm that issues equity alone at time 1 to raise the investment amount I, the marginal equity investor's belief at time 1 is equal to $\widehat{\theta} = \theta^m + d\left(1 - \frac{2I}{W}\right)$. If the dispersion in investor beliefs about the firm goes down by amount of δd by time 2, the marginal equity investor's belief just before at time 2 will be equal to $\widehat{\theta}_2 = \theta^m + d(1-\delta)\left(1 - \frac{2I}{W}\right)$, and the market value of the

[11] However, if the firm has outstanding debt, the debt issue will have a negative impact on the price of the firm's debt, through a mechanism similar to that generating a negative price impact of an equity issue on the firm's outstanding equity.

[12] Note that the mechanism generating a differential price impact of an equity issue versus a debt issue on a firm's outstanding equity is completely different from that generating differences in announcement effect of equity and debt issues on the firm's equity in asymmetric information models such as Myers and Majluf (1984).

[13] Note that, empirically, the price impact of a security issue is quite different from an announcement effect in the abnormal return measured on the day of the announcement of the security issue (before the issue becomes effective), while the price impact is the abnormal return measured on the day the security issues actually came into effect.

firm will be given by

$$V_2 = \widehat{\theta}_2 X^H + (1 - \widehat{\theta}_2) X^L. \tag{15}$$

Proposition 8 ***(Long-Run Stock Returns following Equity Issues)*** *The long run stock return of firms issuing equity is given by*

$$LR^{Equity} = \frac{PE_2^{Equity} - PE_1^{Equity}}{PE_1^{Equity}} = \frac{-\delta d(X^H - X^L)\left(1 - \frac{2I}{W}\right)}{\widehat{\theta} X^H + (1 - \widehat{\theta}) X^L} < 0. \tag{16}$$

The above proposition states that the long-run stock return of firms following equity issues will always be negative. The key to understanding the intuition behind the above result is to recall that the firm's equity at time 1 is priced by the marginal outside investor whose belief $\widehat{\theta} = \theta^m + d\left(1 - \frac{2I}{W}\right)$ is determined by going down the ladder of investor beliefs (starting with the most optimistic outsider belief $(\theta^m + d)$) until the entire amount of I is raised. Thus, the higher the dispersion of beliefs d at the time of security issue (time 1), the higher the marginal equity investor's belief $\widehat{\theta}$ at time 1, and therefore, the higher the firm's share price at the time of the equity issue. If the dispersion in investor beliefs decreases over time by an amount of δd, this implies that the existing marginal equity holder of the firm will become less optimistic about the firm and, therefore, the stock price will fall in the long-run (by time 2) corresponding to the decline in the dispersion in outsiders' beliefs. This is the case, since if there is a fall in the dispersion d around the mean belief θ^m and the belief rankings of all investors remains the same with respect to each other, all investors with a belief above θ^m at the time of equity issue will update their beliefs in the long run so that they become less optimistic about the firm.[14] Thus, the long-run stock returns of firms subsequent to equity issues will always be negative.

Proposition 9 ***(Comparative Statics on Long-Run Stock Returns following Equity Issues)*** *The long-run stock returns of equity issuers is decreasing in the percentage reduction in the dispersion in outsiders' beliefs, and the dispersion of outsiders' beliefs d at the time of equity issue ; i.e.,* $\frac{\partial LR^{Equity}}{\partial \delta} < 0$ *and* $\frac{\partial LR^{Equity}}{\partial d} < 0$.

[14] Similarly, all investors with beliefs below θ^m at the time of security issue will become more optimistic as the spread around the mean belief level decreases over time. Note that since $W > 2I$, the marginal equity investor's belief $\widehat{\theta}$ at time 1 is always greater than θ^m.

The reduction in the dispersion in outsiders' beliefs will be greater as more hard information about the firm arrives after the equity issue, and investors update their beliefs about the firm using this new public information accordingly. Hence, the marginal equity investor will become less optimistic about the firm, and his valuation of the firm will reflect the beliefs of more pessimistic investors to a greater extent as the dispersion in outside investors' beliefs falls over time. Since the long-run (time-2) stock price of the firm will reflect the beliefs of pessimistic investors to a greater extent in the long run, the above proposition implies that the long-run stock returns of equity issuers will be more negative as the percentage reduction δ in the dispersion in outsiders' beliefs is higher. Since the total downward adjustment δd in the marginal equity holder's belief is proportional to the initial level of dispersion d in outsiders' beliefs, the above proposition shows that the fall in stock price in the long run is increasing in the initial dispersion in outsider beliefs, d.

The initial level of optimism of the marginal investor can significantly affect the subsequent reduction in the dispersion in outside investors' beliefs. In particular, if the marginal outside equity investor is much more optimistic than the average outside investor initially, the magnitude of the subsequent downward adjustment in marginal investor beliefs will be much larger as a result of the arrival of new information about the firm over time. Thus, one can reasonably expect that the higher the marginal equity investor's belief $\widehat{\theta}$ at the time of equity issue, the greater will be the percentage reduction δ in the dispersion of investor beliefs over time as well, and therefore, the greater the subsequent fall in stock prices in the long run. We earlier showed that the marginal investor's belief $\widehat{\theta}$ at the time of equity issue is increasing in the initial dispersion in outsider beliefs, d, and the average level of investor optimism, θ^m. Thus, if there is a positive relationship between the initial level of optimism of the marginal investor and the subsequent percentage fall δ in the dispersion in investor beliefs, this will also result in a negative relationship between the average long-run stock return of equity issuers and the

average belief of outsiders, θ^m at the time of the equity issue.[15]

Proposition 10 ***(Long-Run Stock Returns following Straight Debt and Convertible Debt Issues)*** *The long-run stock return of firms issuing straight debt or convertible debt is given by*

$$LR^{Debt} = LR^{Convertible} = \frac{-\delta d}{\theta^m + d} < 0. \tag{17}$$

The intuition behind the above proposition is as follows. Unlike in the case of an equity issue, issuing straight debt or convertible debt does not affect the marginal investor in the firm's equity. Therefore, the stock price of the firm immediately after a debt (either straight or convertible) issue will reflect only the beliefs of the current marginal investor in the firm's equity (since the marginal investor in the firm's equity prior to the debt issue will be the same as the marginal investor in the firm's equity after the debt issue). Further, the long-run (time-2) stock price of these firms will reflect the belief of the same marginal equity investor who is expected to have an updated belief of $\theta^m + d(1 - \delta)$ just before time 2. Therefore, since the marginal equity investor becomes less optimistic in the long run, the long-run stock return following a debt issue (or a convertible debt issue) will also be negative.[16]

We now compare the average long-run stock returns of firms issuing equity, straight debt, and convertible debt. Given insiders' belief and market conditions (i.e., the mean belief of outsiders and the dispersion in outsider beliefs) it can clearly be optimal for any given firm to issue only one kind of security, namely, equity, straight debt, or convertible debt. Therefore, we can only make comparisons across the average long-run stock returns of samples of firms issuing each security. The following proposition compares the average long-run stock return across different types of security issues.

[15] If the percentage fall δ is also increasing in the dispersion in outsiders' beliefs d at the time of equity issue, the initial level of dispersion d in outsiders' beliefs at the time of equity issue will have an indirect negative effect on the long-run stock returns in addition to the direct negative effect show in the above proposition.

[16] Note that in determining the long-run stock returns following a straight debt or a convertible debt issue, the relevant marginal investor is the marginal investor in the firm's equity at the time of the debt issue (since the marginal investor in the firm's equity does not change as a result of the debt issue). In contrast, the marginal investor determining firms' long-run stock returns following an equity issue is the marginal investor holding the firm's equity immediately after the equity issue (since the marginal investor in the firm's equity after the equity issue is different from the marginal investor in the equity prior to the equity issue).

Proposition 11 ***(Comparison of Long-Run Stock Returns across Security Issues)*** *Let* $\theta^f > \theta^m$*, and suppose that* (A.65) *holds. Then, the long-run stock return following a debt issue (issue of straight debt or convertible debt) will be greater than that following an equity issue; i.e.,* $LR^{Debt} = LR^{Convertible} > LR^{Equity}$*.*

We showed in Proposition 1 and Proposition 4 that a firm prefers to issue equity rather than debt only if the marginal outside investor is more optimistic than firm insiders. If the marginal outside investor is more pessimistic than firm insiders, the firm issues convertible debt or straight depending on the size of required investment I and the costs of financial distress. Recall that the marginal outside investor is more optimistic than firm insiders only if the average level of beliefs, and the dispersion in outside investors' beliefs are sufficiently high. Thus, for a given level of average investor beliefs, we know that the dispersion in outsider's beliefs at the time of a security issue should be higher for equity-issuing firms than the dispersion in outsiders' beliefs about debt-issuing firms. We also showed in Propositions 9 and 10 that the long-run stock return subsequent to a security issue is decreasing in the initial level of dispersion in outside investors' beliefs. More generally, the more optimistic the marginal equity investor at time of a security issue, the more negative the long-run stock return after investors update their beliefs with the arrival of new information over time. Since the marginal equity holders of firms that issue equity are more optimistic than the marginal equity holders of firms that issue debt, the average long-term stock return following a debt issue is less negative than the average long-term stock return following an equity issue.[17]

7 Empirical Implications

We now highlight some of the testable implications of the model and their relationship to the existing empirical literature.

[17]From Proposition 10, we know that the long-run stock returns of firms issuing straight debt or convertible debt are the same. This implies that the same intuition also applies to a firm that issues convertible debt, and therefore, on average, the long-run stock return of a convertible-issuing firm is less negative than that of an equity-issuing firm.

(i) Relationship between investor optimism, dispersion in investor beliefs, and the choice of equity versus debt: Our model predicts that the greater the level of optimism (average belief) among outsiders, and the greater the dispersion in outsider beliefs (or both), the more likely the firm is to choose to issue equity rather than debt. This is because the belief of the marginal investor in the firm's equity is more likely to be above that of firm insiders if either the level of optimism, or dispersion, or both, among outsiders is higher. Evidence supporting this prediction is provided by Chemmanur, Nandy, and Yan (2008).

(ii) Relationship between investment amount and the choice of equity versus debt: Our model predicts that, the greater the investment amount to be raised by the firm, the less likely it is to issue equity rather than debt. This is because, since each investor has limited wealth to invest in the firm, the beliefs of the marginal investor is more likely to fall below that of insiders as the amount raised by the firm is greater. Evidence supporting this prediction is provided by Chemmanur, Nandy, and Yan (2008).

(iii) The price impact of an equity issue: The price impact refers to the return to a firm's equity upon the actual issue (not the announcement) of a security (in our context, debt or equity). The price impact will be given by the change in share price of the firm's equity (or return) from the price prevailing before the security issue to the price prevailing after the issue. In our setting, the price prevailing before an equity issue will be determined by the beliefs of the marginal investor holding the firm's equity prior to the issue. Since current shareholders have limited wealth, when new equity is issued it will have to be sold to new investors who are less optimistic about the firm's long-term prospects, so that the beliefs of the marginal investor after the equity issue will be less optimistic compared to that before the equity issue, resulting in a lower share price after the equity issue: i.e. the price impact of an equity issue will be negative. Further, the above fall in share price upon an equity issue will be greater as the dispersion in outsider beliefs is greater: i.e., greater the dispersion, more negative the price impact.

Evidence supporting this prediction is provided by Chemmanur, Nandy, and Yan (2008).

(iv) The price impact of a debt or convertible debt issue: Consider now the case where the firm chooses to issue debt rather than equity. Clearly, since no new equity was issued, the beliefs of the marginal equity investor before the firm's debt issue and the marginal investor holding the firm's equity after the debt issue will be the same in this case. In other words, the price of the firm's equity will be unchanged on the day of the debt issue, so that the price impact on equity of a debt issue will be zero. By a similar argument, the price impact of a convertible debt issue will be zero as well.

(v) The long-run stock returns to a firm's equity following an equity issue: We define long-run stock return as the one year, two year, or three year stock return starting from the closing price on the day of the actual equity issue. In other words, long-term stock returns exclude the price impact of a security issue. Consider first the case where a firm issues equity. In analyzing long-term stock returns. Recall that, in our analysis of long-term stock returns following security issues, we make the additional assumption that the dispersion in outside investors' beliefs about the firm will be reduced in the long run as more hard information is revealed about the firm's cash flow generating process. This means that, in a setting of heterogeneous beliefs and short sale constraints, the long-term stock returns to a firm issuing equity will always be negative. This is the case since, in equilibrium, since the marginal holder of the firm's equity will have less optimistic beliefs about the firm in the long run as the dispersion in beliefs is reduced. Consequently, the long-term stock returns following an equity issue will be negative. Further, since the beliefs of the marginal investor in the firm's equity (and hence the valuation of the firm's equity on the day of the equity issue) will be increasing in the optimism of outsider beliefs and in the initial dispersion of these beliefs, the long-term fall in the firm's share price is likely to be greater (i.e., the long-run post-issue stock return will be more negative) as the level of average optimism and the dispersion in outsider beliefs at the time of equity issue is greater. Evidence supporting this prediction

is provided by Chemmanur, Nandy, and Yan (2008).

(vi) The long-run stock returns to a firm's equity following a debt or convertible debt issue: We showed that, at the time of a security issue, the marginal shareholders of a firm that issues straight debt or convertible debt are less optimistic than the marginal shareholders of a firm that issues equity. This implies that the long-term stock return upon a debt issue (convertible or straight debt) will be greater (less negative, or more positive) than the corresponding long run stock returns upon an equity issue as the dispersion in outside investors' beliefs decreases in the long run. Evidence supporting this prediction is provided by Chemmanur, Nandy, and Yan (2008).

8 Conclusion

In this paper, we have analyzed a firm's financing decision in an environment of heterogeneous beliefs and short sales constraints. We studied a setting in which the insiders of a firm, owning a certain fraction of its equity, choose between equity, debt, or convertible debt to raise additional financing to implement a positive net present value project. The insiders' objective was to maximize their long-run wealth conditional on their own beliefs about their firm's future prospects. Market participants, each of whom have limited wealth, had heterogeneous beliefs about the firm's long-run value. We analyzed two different economic settings: one in which there are no issue costs or costs of financial distress, and another in which these two costs are significant. We showed that, in the absence of these two costs, the average belief of outsiders ("optimism") and the dispersion in outsider beliefs are the crucial determinants of the firm's security choice. When outsider beliefs are highly optimistic relative to firm insiders and the dispersion in outsider beliefs is high, the firm issues equity alone; when outsider beliefs are less optimistic (and less dispersed), the firm issues a combination of equity and debt. We further showed that neither straight debt alone nor convertible debt alone is optimal in the above setting.

Finally, we showed that, once the two costs are significant, the firm issues equity when outsider beliefs are optimistic and highly dispersed; they issue debt when outsider beliefs are quite pessimistic and least dispersed; and convertible debt when outsider beliefs are between the above two extremes. In addition to our predictions for security choice, we also developed several testable predictions for the price impact of equity, debt, and convertible debt issues and for the firm's long-run stock returns following the issuance of these securities.

References

[1] Allen, F., and D. Gale. 1999. Diversity of Opinion and Financing of New Technologies. Journal of Financial Intermediation 8, 68-89.

[2] Asquith, P., Pathak, P.A., Ritter, J.R., 2005. Short Interest, Institutional Ownership, and Stock Returns. Journal of Financial Economics 78 (2), 243-276.

[3] Baker, M., Wurgler J., 2006. Investor sentiment and the cross-section of stock returns, Journal of Finance 61 (4), 1645-1680.

[4] Biais, B., Bossaerts, P., 1998. Asset prices and trading volume in a beauty contest, Review of Economic Studies 65, 307-340.

[5] Boot, A.W.A., R. Gopalan, and A. V. Thakor, 2006, The entrepreneurs choice between private and public ownership, Journal of Finance 61, 803836.

[6] Brav, A., Heaton, J.B., 2002. Competing Theories of Financial Anomalies, Review of Financial Studies 15, 575-606.

[7] Cao, H., Ou-Yang, H., 2004. Differences of opinion and options, Working paper, University of North Carolina and Duke University.

[8] Chemmanur, T., Nandy, D., Yan, A., 2008. Capital Structure and Security Issuance under Heterogeneous Beliefs, Working paper, Boston College.

[9] Chen, J., Hong, H., Stein, J.C., 2002. Breadth of Ownership and Stock Returns. Journal of Financial Economics 66, 171-205.

[10] Diether, K.B., Malloy, C.J., Scherbina, A., 2002. Differences of Opinion and the Cross Section of Stock Returns. Journal of Finance 57 (5), 2113-2141.

[11] Dittmar, A., Thakor, A., 2007. Why Do Firms Issue Equity? Journal of Finance 62 (1), 1-54.

[12] Duffie, D., Gârleanu, N., Pedersen, L.H., 2002. Securities lending, shorting, and pricing. Journal of Financial Economics 66, 307-339.

[13] Garmaise, M.J., 2001, Rational beliefs and security design, Review of Financial Studies 14, 11831213.

[14] Harris, M., Raviv, A., 1991. The theory of capital structure. Journal of Finance 46, 297-355.

[15] Harris, M., Raviv, A., 1993. Differences of opinion make a horse race. Review of Financial Studies 6, 473-506.

[16] Harrison, M., Kreps, D., 1978. Speculative investor behavior in a stock market with heterogeneous expectations. Quarterly Journal of Economics 92, 323-336.

[17] Jarrow, R., 1980. Heterogeneous expectations, restrictions on short sales and equilibrium asset prices, Journal of Finance 35, 1105–1114.

[18] Kandel, E., Pearson, N., 1995. Differential interpretation of public signals and trade in speculative markets, Journal of Political Economy 103, 831-872.

[19] Kreps, D., 1990, A Course in Microeconomics (Princeton University Press, Princeton).

[20] Kurz, M., 1994, On the structure and diversity of rational beliefs, Economic Theory 4, 877900.

[21] Kyle, A., Lin, T., 2003. Continuous trading with heterogeneous beliefs and no noise trading, Working paper, Duke University.

[22] Mayshar, J., 1982. On divergence of opinion and imperfections in capital markets, American Economic Review 73, 114–128.

[23] Miller, E.M., 1977. Risk, Uncertainty, and Divergence of Opinion. Journal of Finance 32 (4), 1151-1168.

[24] Morris, S., 1995. The Common Prior Assumption in Economic Theory. Economics and Philosophy 11, 227-253.

[25] Morris, S., 1996. Speculative Investor Behavior and Learning. Quarterly Journal of Economics 3 (4), 1111-1133.

[26] Nagel, S., 2005. Short sales, institutional investors and the cross-section of stock returns. Journal of Financial Economics 78 (2), 277-309.

[27] Scheinkman, J., Xiong, W., 2004. Heterogeneous Beliefs, Speculation and Trading in Financial Markets. Working Paper, Princeton University.

[28] Varian, H., 1989. Differences of opinion in financial markets, in Courtenay Stone (ed.) Financial Risk: Theory, Evidence and Implications, Kluwer Academic Publishers, Boston.

[29] Viswanathan, S., 2000, Strategic trading, heterogeneous beliefs and short constraints, Working paper, Duke University.

Appendix: Proofs of Propositions

Proof of Lemma 1:

If the equity is traded at a price of PE_1^{Equity} per share at time 1 when the firm issues equity, all investors whose valuation higher than PE_1^{Equity} will be willing to buy. Denote $\widehat{\theta}$ as the belief of the marginal investor, whose valuation of the equity equals the market price PE_1^{Equity}. Because potential outside investors have a total wealth of W, and they are uniformed distributed over the interval with length $2d$, each investor has a wealth of $\frac{W}{2d}$. Because the firm needs to raise an amount I from investors in the interval $[\widehat{\theta}, \theta^m + d]$, we have

$$\int_{\widehat{\theta}}^{\theta^m+d} \frac{W}{2d} d\theta = I. \tag{A.1}$$

The above equation means that the total wealth of those who buy the new issues equals the amount the firm wants to raise, I. Solve for $\widehat{\theta}$, we have

$$\widehat{\theta} = \theta^m + d - \frac{2dI}{W}. \tag{A.2}$$

The market price of new shares sold should be determined by the marginal investor's valuation of the shares, i.e.,

$$PE_1^{Equity} = \frac{[\widehat{\theta}X^H + (1-\widehat{\theta})X^L]}{1+E_1}, \tag{A.3}$$

where the left side is the market price of each share of equity and the right side is the marginal investor's valuation of each share of equity. Further, the amount raised by the firm is I, which means

$$PE_1^{Equity} \times E_1 = I. \tag{A.4}$$

Solving for equations (A.3) and (A.4) leads to

$$PE_1^{Equity} = \widehat{\theta}X^H + (1-\widehat{\theta})X^L - I \tag{A.5}$$

and

$$E_1 = \frac{I}{\widehat{\theta}X^H + (1-\widehat{\theta})X^L - I} \tag{A.6}$$

The expected payoff to the firm's current shareholders is

$$EU^{Equity} = (1 - \frac{I}{\widehat{\theta}X^H + (1-\widehat{\theta})X^L})[\theta^f X^H + (1-\theta^f)X^L]. \tag{A.7}$$

Q.E.D.

Proof of Lemma 2

Suppose the firm needs to issue F units of straight debt to raise the amount I. Because potential outside investors have a total wealth of W, and they are uniformed distributed over the interval with length $2d$, the marginal investor in the firm's debt is also $\widehat{\theta} = \theta^m + d - \frac{2dI}{W}$, similar to the argument in the proof of lemma 1.

First, assume that $X^H > I > X^L$, i.e., the debt is risky. The payoff to each unit of straight debt is 1 in the good state and $\frac{X^L}{F}$ in the bad state, so the market price of each unit of debt, which is determined by the marginal investor's valuation of the debt, is given by

$$PD_1 = \widehat{\theta} + (1-\widehat{\theta})\frac{X^L}{F}. \tag{A.8}$$

The firm has to sell I/PD_1 units of straight debt to raise an amount of I. Since we already assume that the firm

issues a total of F units of debt, we must have

$$F = \frac{I}{PD_1} = \frac{I}{\widehat{\theta} + (1-\widehat{\theta})\frac{X^L}{F}}$$

or equivalently,

$$F = \frac{I - (1-\widehat{\theta})X^L}{\widehat{\theta}} \tag{A.9}$$

The payoff to the equity holders of the firm is 0 in the bad state and $X^H - F$ in the good state, so the expected payoff to current shareholders of the firm is

$$EU^{Debt} = \theta^f (X^H - \frac{I - (1-\widehat{\theta})X^L}{\widehat{\theta}}). \tag{A.10}$$

Now, assume that the firm can issue risk-free straight debt, i.e., $I \leq X^L$. The payoff to each unit of straight debt is 1 in every state, so the market price of each unit of risk-free debt is

$$PD_1 = \widehat{\theta} + (1-\widehat{\theta}) = 1. \tag{A.11}$$

The firm has to sell I/PD_1 units of risk-free straight debt to raise an amount of I. Thus, $F = I/PD_1 = I$. The payoff to the equity holders of the firm is $X^L - I$ in the bad state and $X^H - F$ in the good state, so the expected payoff to current shareholders of the firm is

$$EU^{RiskFreeDebt} = \theta^f X^H + (1-\theta^f)X^L - I. \tag{A.12}$$

Q.E.D.

Proof of Lemma 3: We impose the condition that not all investors prefer to convert to equity at time 2 if the bad state with low cash flow (X_L) is realized. Otherwise, there would be no difference between convertible debt and equity. Thus, we have the following restriction on the conversion ratio x and the price p:

$$x\frac{X^L}{1 + x\frac{I}{p}} < 1, \tag{A.13}$$

which translates into the following no-conversion condition:[18]

$$F = \frac{I}{p} > X^L - \frac{1}{x}. \tag{A.14}$$

In addition, we impose the restriction that all convertible debt investors prefer to convert to equity at time 2 if the good state with high cash flow (X_H) is realized. Otherwise, there would be no difference between convertible debt and straight debt. Thus, we have the following restriction on the conversion ratio x and the price p:

$$F = \frac{I}{p} < X^H - \frac{1}{x}. \tag{A.15}$$

Combining these two conversion conditions we obtain the following restriction:

$$X^L < \frac{I}{p} + \frac{1}{x} < X^H. \tag{A.16}$$

First consider the case when there is no default of the convertible at time 2. Then the following condition

[18]This condition also guarantees that none of the convertible debt investors has the incentive to convert to equity at time 2 if the cash flow is equal to X^L, i.e., it is equivalent to this condition: $x\frac{\left[X^L - \left(\frac{I}{p} - 1\right)\right]}{1+x} < 1$.

has to be satisfied:

$$F = \frac{I}{p} \leq X^L. \tag{A.17}$$

Thus, in the case of default-free convertible, the valuation of the marginal investor for the convertible security at time 1 is given by

$$p = \widehat{\theta}\frac{px}{p+Ix}X^H + (1-\widehat{\theta}). \tag{A.18}$$

If we solve this equation for the conversion ratio x, we get

$$x = \frac{p^2 - (1-\widehat{\theta})p}{\widehat{\theta}X^H p + (1-\widehat{\theta})I - pI}. \tag{A.19}$$

It is easy to show that x is increasing in p. Note also that if the firm wants to issue risk-free convertible debt (i.e., if $F \leq X^L$), the conversion condition (A.15) in the good state will be satisfied only if $p > 1$, or equivalently, if $F < I$. The expected payoff to the current shareholders is

$$EU^{RiskFreeConvertible} = \theta^f \frac{p}{p+Ix}X^H + (1-\theta^f)(X^L - \frac{I}{p}) \tag{A.20}$$

If we plug in the value of x from (A.19), we have

$$EU^{RiskFreeConvertible} = \theta^f \frac{p}{p+Ix}X^H + (1-\theta^f)(X^L - \frac{I}{p}) \tag{A.21}$$

$$= \theta^f X^H + (1-\theta^f)X^L - \frac{\theta^f}{\widehat{\theta}}I + \frac{(\theta^f - \widehat{\theta})I}{\widehat{\theta}p} \tag{A.22}$$

If $\widehat{\theta} < \theta^f$, it is optimal to set p as low as possible, i.e., $x = \underline{x}$. Given the no default condition in (A.17) and the conversion condition (A.15) in the good state of the world, the lowest possible value of p, which we denote by $\underline{p}$, can be determined. If the firm can issue risk-free straight debt (if $I \leq X^L$), this lower bound on the price p of risk-free convertible is equal to 1, and $\underline{p}$ must be strictly greater than 1. In other words, the total face value $F = \frac{I}{p}$ of the risk-free convertible must be slightly less than I. Otherwise, if $I > X^L$, the total face value F of the risk-free convertible issue must be less than or equal to X^L. Therefore, given the no-default condition (A.17), $\underline{p}$ is equal to $\frac{I}{X^L} > 1$. If we plug in these lower bounds of p in (A.19), we obtain the following optimal conversion ratio x, i.e., $\underline{x}$:

$$\underline{x} = \begin{cases} \frac{I-(1-\widehat{\theta})X^L}{X^L[\widehat{\theta}X^H+(1-\widehat{\theta})X^L-I]} & : \quad I > X^L, \\ \frac{1}{X^H-I} + \epsilon & : \quad I \leq X^L. \end{cases} \tag{A.23}$$

where ϵ is an arbitrarily small positive number. Consider the case when $I > X^L$. Note that since the optimal conversion ratio $x = \underline{x}$ from (A.23) implies that $p = \underline{p} = \frac{I}{X^L}$ in this case, it follows from (A.22) that

$$EU^{RiskFreeConvertible} = \frac{\theta^f}{\widehat{\theta}}(\widehat{\theta}X^H + (1-\widehat{\theta})X^L - I) = EU^{Debt}. \tag{A.24}$$

Note that in this case, the total face value F of the risk-free convertible debt issue is optimally set to $\frac{I}{P} = X^L$, i.e., the firm's cash flow at the bad state.

Similarly, in the case where $I \leq X^L$, we have $p = \underline{p} > 1$ for convertible debt. Therefore, it follows from (A.22) that

$$EU^{RiskFreeConvertible} < \theta^f X^H + (1-\theta^f)X^L - I = EU^{RiskFreeDebt}. \tag{A.25}$$

In both cases, the price $p = \underline{p}$ of the convertible debt as a function of $\underline{x}$ is given by

$$\underline{p} = \frac{(1-\widehat{\theta}) - \underline{x}I + \underline{x}\widehat{\theta}X^H + \sqrt{(1-\widehat{\theta} - \underline{x}I + \underline{x}\widehat{\theta}X^H)^2 + 4(1-\widehat{\theta})\underline{x}I}}{2} \tag{A.26}$$

Thus, we showed that risk-free straight debt always dominates risk-free convertible debt, if the marginal investor is more pessimistic than firm insiders. If the required investment I is greater than X^L, we also showed that risky straight debt generates the same expected payoff to firm insiders as the risk-free convertible debt ($F = X^L$), when the marginal investor is more pessimistic than firm insiders.

If $\widehat{\theta} \geq \theta^f$, it follows from (A.22) that it is optimal to set p as high as possible, that is, $x = \overline{x}$. Given the no-conversion condition from (A.14) and the value of x from (A.19), the highest possible value of p should be less than $\frac{\widehat{\theta}X^H + (1-\widehat{\theta})X^L}{X^L}$. If we plug in this upper bound of p in (A.19), we obtain the following upper bound on $\overline{x}$:

$$\overline{x} < \frac{\widehat{\theta}X^H + (1-\widehat{\theta})X^L}{X^L(\widehat{\theta}X^H + (1-\widehat{\theta})X^L - I)} \tag{A.27}$$

Note that since we have $p < \frac{\widehat{\theta}X^H + (1-\widehat{\theta})X^L}{X^L}$, it follows from (A.22) that

$$EU^{RiskFreeConvertible} < \theta^f X^H + (1-\theta^f)X^L - I\frac{\theta^f X^H + (1-\theta^f)X^L}{\widehat{\theta}X^H + (1-\widehat{\theta})X^L} = EU^{Equity}. \tag{A.28}$$

Thus, if the marginal investor is more optimistic than firm insiders, issuing equity only is preferred to issuing risk-free convertible debt. The price $p = \overline{p}$ of the convertible debt as a function of $\overline{x}$ is given by

$$\overline{p} = \frac{(1-\widehat{\theta}) - \overline{x}I + \overline{x}\widehat{\theta}X^H + \sqrt{(1-\widehat{\theta} - \overline{x}I + \overline{x}\widehat{\theta}X^H)^2 + 4(1-\widehat{\theta})\overline{x}I}}{2} \tag{A.29}$$

Note that if $I > X^L$ and x is set so low that $x < \frac{I-(1-\widehat{\theta})X^L}{X^L[\widehat{\theta}X^H + (1-\widehat{\theta})X^L - I]}$, convertible debt will have default at time 2 if the bad state is realized, i.e., we will have $F = \frac{I}{p} > X^L$. Thus, in the case of risky convertible debt, the valuation of the marginal investor for the convertible security at time 1 will be given by

$$p = \widehat{\theta}\frac{px}{p+Ix}X^H + (1-\widehat{\theta})\frac{X^L}{I/p} \tag{A.30}$$

which leads to

$$\frac{x}{p} = \frac{I - (1-\widehat{\theta})X^L}{I[\widehat{\theta}X^H + (1-\widehat{\theta})X^L - I]} \tag{A.31}$$

From (A.31) and the conversion condition (A.15) in the good state, it follows that the conversion ratio x must be higher than $\frac{\widehat{\theta}}{\widehat{\theta}X^H + (1-\widehat{\theta})X^L - I}$. Equivalently, the total face value of the risky convertible debt issue must be strictly less than the total face value of the risky straight debt issue, i.e., $F < \frac{I-(1-\widehat{\theta})X^L}{\widehat{\theta}}$. The firm's expected payoff from issuing risky convertible debt is

$$EU^{Convertible} = \theta^f\frac{p}{p+Ix}X^H = \theta^f\frac{1}{1+Ix/p}X^H \tag{A.32}$$

Plug in the value of $\frac{x}{p}$ from above, we have

$$EU^{Convertible} = \frac{\theta^f}{\widehat{\theta}}(\widehat{\theta}X^H + (1-\widehat{\theta})X^L - I) = EU^{Debt}, \tag{A.33}$$

no matter what the conversion ratio x is. Note that the firm's expected payoff from issuing risky convertible debt

alone is equal to its expected payoff from issuing risky straight debt alone. (A.33) and (A.28) together also imply that issuing equity alone dominates issuing risky convertible as well, if the marginal investor is more optimistic than firm insiders. Q.E.D.

Proof of Proposition 1: From equations (A.7) and (A.10) we have

$$\begin{aligned} EU^{Equity} - EU^{Debt} &= (1 - \frac{I}{\widehat{\theta}X^H + (1-\widehat{\theta})X^L})[\theta^f X^H + (1-\theta^f)X^L] \\ &\quad -\theta^f(X^H - \frac{I - (1-\widehat{\theta})X^L}{\widehat{\theta}}). \end{aligned}$$

The firm will prefer equity to straight debt iff $EU^{Equity} > EU^{Debt}$, which is equivalent to

$$\frac{\widehat{\theta}X^H + (1-\widehat{\theta})X^L - I}{\widehat{\theta}X^H + (1-\widehat{\theta})X^L}[\theta^f X^H + (1-\theta^f)X^L] > \theta^f \frac{\widehat{\theta}X^H + (1-\widehat{\theta})X^L - I}{\widehat{\theta}}, \tag{A.34}$$

when $\widehat{\theta}X^H + (1-\widehat{\theta})X^L - I > 0$, it is equivalent to $\theta^f \geq \widehat{\theta}$.

When $\theta^f \geq \widehat{\theta}$, we have proved that convertible debt is dominated by straight debt (see equation (A.24)). When $\theta^f < \widehat{\theta}$, we have proved that convertible debt is dominated by equity (see equation (A.28)). Therefore, convertible debt is never optimal. Q.E.D.

Proof of Proposition 2 First, we suppose that the firm issues $I_D \in [0, X^L]$ units of debt, and each unit of debt has face value of 1. Since the debt is safe, the price of the debt is 1. The valuation of the equity to agent θ is $\theta X^H + (1-\theta)X^L - I_D$.

The firm needs to raise $I - I_D$ by issuing equity. The marginal investor $\widetilde{\theta}$ is characterized by

$$\int_{\widetilde{\theta}}^{\theta^m + d} \frac{W}{2d} d\theta = I - I_D$$

or equivalently

$$\widetilde{\theta} = \theta^m + d - \frac{2d(I - I_D)}{W} = \widehat{\theta} + \frac{2dI_D}{W} \tag{A.35}$$

The marginal investor values the equity value of the whole firm at $\widetilde{\theta}X^H + (1-\widetilde{\theta})X^L - I_D$. Suppose the firm needs to issue x shares of new equity to raise $I - I_D$, then we have

$$\frac{x}{1+x} = \frac{I - I_D}{\widetilde{\theta}X^H + (1-\widetilde{\theta})X^L - I_D}$$

or equivalently

$$x = \frac{I - I_D}{\widetilde{\theta}X^H + (1-\widetilde{\theta})X^L - I}. \tag{A.36}$$

The expected payoff to the insiders will be

$$EU^{Combi} = \frac{1}{1+x}[\theta^f X^H + (1-\theta^f)X^L - I_D] = \frac{\widetilde{\theta}X^H + (1-\widetilde{\theta})X^L - I}{\widetilde{\theta}X^H + (1-\widetilde{\theta})X^L - I_D}[\theta^f X^H + (1-\theta^f)X^L - I_D]$$

The objective of the insiders is to choose the optimal amount of debt, I_D^*, to be issued:

$$\begin{aligned} \underset{I_D \in [0, X^L]}{Max} \quad & \frac{\widetilde{\theta}X^H + (1-\widetilde{\theta})X^L - I}{\widetilde{\theta}X^H + (1-\widetilde{\theta})X^L - I_D}[\theta^f X^H + (1-\theta^f)X^L - I_D] \\ s.t. \quad & \widetilde{\theta} = \widehat{\theta} + \frac{2dI_D}{W}. \end{aligned}$$

We have the following FOC to this maximization problem:

$$\frac{\partial EU^{Combi}}{\partial I_D} = \frac{-\widehat{X}^2 - 2aI_D\widehat{X} + a(1-a)I_D^2 + X^f\widehat{X} + I\widehat{X} - (1-a)IX^f}{[\widehat{X} - (1-a)I_D]^2} = 0,$$
$$= \frac{a[\widehat{X} - (1-a)I_D]^2 - [\widehat{X} - (1-a)X^f][\widehat{X} - (1-a)I]}{(1-a)[\widehat{X} - (1-a)I_D]^2} = 0.$$

Thus, the optimal amount of debt to be raised is

$$I_D^* = \frac{a\widehat{X} - \sqrt{a[\widehat{X} - (1-a)X^f][\widehat{X} - (1-a)I]}}{a - a^2}, \tag{A.37}$$

where

$$\widehat{X} = \widehat{\theta}X^H + (1-\widehat{\theta})X^L,\ X^f = \theta^f X^H + (1-\theta^f)X^L,\ a = \frac{2d(X^H - X^L)}{W} > 0.$$

In particular, we find that equity alone dominates debt-equity combination when $I_D^* \leq 0$, or equivalently,

$$\widehat{X}^2 - (I + X^f)\widehat{X} + (1-a)IX^f \geq 0. \tag{A.38}$$

Note that if the firm issues safe debt, it holds that $I_D \leq X_L < I$ by assumption. Thus, in the case of safe debt, we have $I - I_D > 0$, and therefore issuing safe debt alone to raise I is infeasible.

Now we consider the case when it is optimal for the firm to issue risky debt. Suppose that the firm wants to raise amount $I - I_D$ by issuing equity, and amount I_D by issuing debt. The debt is risky so that $I_D > X^L$. The face value of debt D (number of units of debt the firm has to issue) can be determined by the following equation

$$I_D = \widehat{\theta}D + (1-\widehat{\theta})X^L \tag{A.39}$$

The face value of debt is therefore

$$D = \frac{I_D - (1-\widehat{\theta})X^L}{\widehat{\theta}} \tag{A.40}$$

The marginal equity investor is characterized by his belief $\widetilde{\theta} = \widehat{\theta} + \frac{2dI_D}{W}$. Investors in the range $[\widehat{\theta}, \widetilde{\theta}]$ will be purchasing debt, and the price of debt will be determined by the marginal debt investor with belief $\widehat{\theta}$. The total market value of equity in this case is $\widetilde{\theta}[X^H - D]$. Suppose the firm needs to issue x shares of new equity to raise $I - I_D$, then we have

$$\frac{x}{1+x} = \frac{I - I_D}{\widetilde{\theta}[X^H - D]} \tag{A.41}$$

or equivalently

$$x = \frac{I - I_D}{\widetilde{\theta}[X^H - D] - (I - I_D)}. \tag{A.42}$$

Therefore, the expected payoff of the insiders is equal to

$$EU^{Combi} = (1 - \frac{I - I_D}{\widetilde{\theta}(X^H - D)})\theta^f(X^H - D). \tag{A.43}$$

The firm has to choose the optimal split between debt and equity, i.e.,

$$\underset{I_D \in [X^L, I]}{Max}\ (1 - \frac{I - I_D}{\widetilde{\theta}(X^H - D)})\theta^f(X^H - D)$$
$$\text{s.t.}\ \ \widetilde{\theta} = \widehat{\theta} + \frac{2dI_D}{W}\ \text{ and }\ D = \frac{I_D - (1-\widehat{\theta})X^L}{\widehat{\theta}}.$$

The solution to this maximization problem is

$$I_D^* = \frac{\sqrt{\widehat{\theta}W(\widehat{\theta}W + 2dI)} - \widehat{\theta}W}{2d}. \tag{A.44}$$

Note that it is never true that $I_D^* \geq I$ where I_D^* is given by (A.44). Therefore, issuing risky debt alone is always dominated by issuing a combination of debt and equity. It is optimal for the firm to issue risky debt only if $I_D^* > X^L$. Given (A.44), this leads to $I > 2X^L + \frac{2d(X^L)^2}{\widehat{\theta}W}$ as a necessary condition for the optimality of issuing risky debt. Q.E.D.

Proof of Proposition 3: When the firm issues convertible debt, let us assume that the firm chooses the conversion ratio optimally so that the face value of the convertible debt is F^*. The price of the convertible debt (both the equity component and the straight debt component) is determined by the valuation of the marginal investor $\widehat{\theta}$. Now assume that the firm issues F^* units of straight debt and issues equity to raise the remaining amount, $I - PD_1 \times F^*$. The price of straight debt is determined by the valuation of the marginal investor $\widehat{\theta}$, while the price of equity is determined by the marginal investor $\widetilde{\theta} = \widehat{\theta} + \frac{2d \times PD_1 \times F^*}{W} > \widehat{\theta}$. That is, the firm can sell the equity at a higher price if it issues a debt-equity combination than if it sells convertible debt only. Q.E.D.

Proof of Proposition 4: When outsiders' average belief is optimistic, and dispersion is high, so that $\theta^f \leq \widehat{\theta}$, we have shown in the proof of Lemma 3 that $E^{Convertible} < X^f - I\frac{X^f}{\widehat{X}} = E^{Equity}$. Therefore, equity will dominate convertible debt. We have already shown that equity dominates debt in Proposition 1. Hence, it is optimal for the firm to issue equity in this case.

When $\theta^f > \widehat{\theta}$, the choice is between convertible debt and straight debt and which one of these is optimal depends on the required investment financing I. If the firm can issue risk-free straight debt, the costs of financial distress will be 0 for both securities. Lemma 3 shows that risk-free convertible debt will be more undervalued than risk-free straight debt. Therefore, the firm will choose to issue risk-free straight debt alone in order to minimize the undervaluation cost. If the firm's straight debt issue has to be risky since $I > X^L$, we know from Lemma 3 that the undervaluation of the risky straight debt is the same as the undervaluation of the risk-free convertible debt with total face value $F = X^L$. Since the firm can minimize the cost of financial distress to 0 by setting $F = X^L$, it will optimally choose to issue risk-free convertible debt with face value $F = X^L$ rather than risky straight debt. Q.E.D.

Proof of Proposition 5: The proof of part (i) is similar to that of Proposition 2. To prove part (ii), note that the firm now has to incur the issue cost for each tranche of security. When outsiders' average belief is low and the dispersion in outsider beliefs is low so that $\widehat{X}^2 - (I + X^f)\widehat{X} + (1-a)IX^f < 0$, it is not optimal for the firm to issue equity alone because equity will be undervalued by outsiders. The choice is between a equity-debt combination and debt alone. We have proved in Proposition 2 that issuing equity-debt combination dominates debt alone without issue cost. With issue cost, the firm faces a trade-off. It is optimal for the firm to issue a combination of equity and debt when the issuing cost is low ($C^I \leq \overline{C}^I$) following the proof in proposition 2. Conversely, the firm will choose to issue straight debt alone when the issue cost is high ($C^I > \overline{C}^I$) because the additional issue cost associated with the equity-debt combination will outweigh the benefits. Q.E.D.

Proof of Proposition 6: As we have proved in Proposition 3 that a combination of equity and straight debt will lead to higher selling prices for the firm's security prices than convertible debt alone. This is because the firm can sell equity and debt to different groups of investors while the firm is forced to sell the convertible debt to the same group of investors. This benefit of equity-debt combination increases with the dispersion in outsiders' beliefs, d. However, issuing equity-debt combination instead of convertible debt leads to additional issue costs. The firm will choose the optimal security based on this trade-off. The firm will choose equity-debt combination when the dispersion in outsiders' beliefs, d, is high and the issue cost, C^I, is low. Otherwise, the firm will choose to issue convertible debt. Q.E.D.

Proof of Proposition 7: The marginal investor's belief on the firm value is $\theta^m + d$ at time 0. Investors anticipate that the firm will issue E_1 shares of new equity at time 1 to raise the amount I. The price of stock

before issuance of equity is therefore

$$PE_0^{Equity} = \frac{1}{1+E_1}[(\theta^m + d)X^H + (1-\theta^m - d)X^L] \quad (A.45)$$

$$= (1 - \frac{I}{\widehat{\theta}X^H + (1-\widehat{\theta})X^L})[(\theta^m + d)X^H + (1-\theta^m - d)X^L] \quad (A.46)$$

After the issuance of equity, the total number of shares outstanding is $1 + E_1$, and the marginal investor is now $\widehat{\theta}$ and the equity price per share is

$$PE_1^{Equity} = \frac{1}{1+E_1}[\widehat{\theta}X^H + (1-\widehat{\theta})X^L] = \widehat{\theta}X^H + (1-\widehat{\theta})X^L - I \quad (A.47)$$

Therefore, the stock price will go down immediately after more shares come to the market. The price impact is

$$\Delta PE^{Equity} = PE_1^{Equity} - PE_0^{Equity} = -(1 - \frac{I}{\widehat{\theta}X^H + (1-\widehat{\theta})X^L})\frac{2dI}{W}(X^H - X^L) < 0. \quad (A.48)$$

If the firm issues debt, the face value of the debt is $F = \frac{I-(1-\widehat{\theta})X^L}{\widehat{\theta}}$ and the equity price will be

$$PE_0^{Debt} = PE_1^{Debt} = (\theta^m + d)(X^H - \frac{I-(1-\widehat{\theta})X^L}{\widehat{\theta}}) \quad (A.49)$$

at both time 0 and time 1. Therefore, the price impact of debt issuance is zero.

Note that

$$|\Delta PE^{Equity}| = (1 - \frac{I}{[\theta^m + d(1-\frac{2I}{W})](X^H - X^L) + X^L})\frac{2dI}{W}(X^H - X^L) \quad (A.50)$$

Since we have assumed that $1 - \frac{2I}{W} > 0$, both $(1 - \frac{I}{[\theta^m + d(1-\frac{2I}{W})](X^H - X^L) + X^L})$ and $\frac{2dI}{W}(X^H - X^L)$ increase in d, therefore, we have

$$\frac{\partial |\Delta PE^{Equity}|}{\partial d} > 0 \quad (A.51)$$

which means the price impact of equity increases with the dispersion in outsiders' beliefs. Q.E.D.

Proof of Proposition 8: We showed in the proof of Proposition 7 that the stock price at issuance is

$$PE_1^{Equity} = \frac{\widehat{\theta}X^H + (1-\widehat{\theta})X^L}{1 + \frac{I}{\widehat{\theta}X^H + (1-\widehat{\theta})X^L - I}}. \quad (A.52)$$

where $\widehat{\theta} = \theta^m + d\left(1 - \frac{2I}{W}\right)$. The stock price at time 2 is

$$PE_2^{Equity} = \frac{\widehat{\theta}_2 X^H + (1-\widehat{\theta}_2)X^L}{1 + \frac{I}{\widehat{\theta}X^H + (1-\widehat{\theta})X^L - I}} \quad (A.53)$$

where $\widehat{\theta}_2 = \theta^m + d(1-\delta)\left(1 - \frac{2I}{W}\right)$. The long-run stock return is therefore

$$LR^{Equity} = \frac{PE_2^{Equity} - PE_1^{Equity}}{PE_1^{Equity}} = \frac{-\delta d\left(1 - \frac{2I}{W}\right)(X^H - X^L)}{\widehat{\theta}X^H + (1-\widehat{\theta})X^L} < 0. \quad (A.54)$$

Q.E.D.

Proof of Proposition 9: Partially differentiating LR^{Equity} from (A.54), we obtain:

$$\frac{\partial LR^{Equity}}{\partial \delta} = \frac{-d\left(1-\frac{2I}{W}\right)\left(X^H - X^L\right)}{\widehat{\theta}X^H + (1-\widehat{\theta})X^L} < 0, \tag{A.55}$$

$$\frac{\partial LR^{Equity}}{\partial d} = \frac{-(1+\delta)\left(1-\frac{2I}{W}\right)\left(X^H - X^L\right)}{(\widehat{\theta}X^H + (1-\widehat{\theta})X^L)^2} < 0. \tag{A.56}$$

Q.E.D.

Proof of Proposition 10: The equity price at the time of straight debt issue (time 1) is

$$PE_1^{Debt} = (\theta^m + d)(X^H - \frac{I - (1-\widehat{\theta})X^L}{\widehat{\theta}}), \tag{A.57}$$

and the equity price at time 2 is

$$PE_2^{Debt} = (\theta^m + d(1-\delta))(X^H - \frac{I - (1-\widehat{\theta})X^L}{\widehat{\theta}}). \tag{A.58}$$

The long-run stock return subsequent to the straight debt issue is therefore

$$LR^{Debt} = \frac{PE_2^{Debt} - PE_1^{Debt}}{PE_1^{Debt}} = \frac{-\delta d}{\theta^m + d} < 0. \tag{A.59}$$

For convertible debt, at the time of convertible debt issue (time 1), we have

$$PE_1^{Convertible} = (\theta^m + d)\frac{1}{1+xF}X^H, \tag{A.60}$$

and the equity price at time 2 is

$$PE_2^{Convertible} = (\theta^m + d(1-\delta))\frac{1}{1+xF}X^H. \tag{A.61}$$

The long-run stock return subsequent to the convertible debt issue is therefore

$$LR^{Convertible} = \frac{PE_2^{Convertible} - PE_1^{Convertible}}{PE_1^{Convertible}} = \frac{-\delta d}{\theta^m + d} < 0. \tag{A.62}$$

Q.E.D.

Proof of Proposition 11: We assume that d is uniformly distributed in the interval $[0, 1-\theta^m]$. The firm will issue equity at time 1 only when $\theta^f < \widehat{\theta}$, i.e., when $d > \frac{\theta^f - \theta^m}{1-\frac{2I}{W}}$. Therefore, the expected long-run stock return of equity issuance is

$$\begin{aligned}
\overline{LR}^{Equity} &= \int_{\frac{\theta^f-\theta^m}{1-\frac{2I}{W}}}^{1-\theta^m} \frac{-\delta x\left(1-\frac{2I}{W}\right)\left(X^H - X^L\right)}{\widehat{\theta}X^H + (1-\widehat{\theta})X^L} \frac{1}{1-\theta^m - \frac{\theta^f-\theta^m}{1-\frac{2I}{W}}} dx, \\
&= \frac{-\delta(X^H - X^L)}{1-\theta^f - (1-\theta^m)\frac{2I}{W}} \int_{\frac{\theta^f-\theta^m}{1-\frac{2I}{W}}}^{1-\theta^m} \frac{x}{X^m + \left(1-\frac{2I}{W}\right)(X^H - X^L)x} dx,
\end{aligned}$$

$$\overline{LR}^{Equity} = \frac{-\delta(X^H - X^L)}{1-\theta^f-(1-\theta^m)\frac{2I}{W}}\left[\frac{x}{\left(1-\frac{2I}{W}\right)(X^H-X^L)} - \frac{X^m \ln(X^m+\left(1-\frac{2I}{W}\right)(X^H-X^L)x)}{\left(1-\frac{2I}{W}\right)^2(X^H-X^L)^2}\right]_{\frac{\theta^f-\theta^m}{1-\frac{2I}{W}}}^{1-\theta^m},$$

$$= -\frac{\delta}{\left(1-\frac{2I}{W}\right)}\left[1 - \frac{X^m}{(1-\theta^f-\frac{2I}{W}(1-\theta^m))(X^H-X^L)}\ln\left(\frac{X^m+(1-\frac{2I}{W})(X^H-X^L)(1-\theta^m)}{X^m+(1-\frac{2I}{W})(X^H-X^L)\frac{\theta^f-\theta^m}{1-\frac{2I}{W}}}\right)\right]. \tag{A.63}$$

The expected long-run stock return following debt issues are

$$\overline{LR}^{Debt} = \frac{\left(1-\frac{2I}{W}\right)}{\theta^f-\theta^m}\int_0^{\frac{\theta^f-\theta^m}{\left(1-\frac{2I}{W}\right)}}\frac{-\delta x}{\theta^m+x}dx = -\frac{\left(1-\frac{2I}{W}\right)}{\theta^f-\theta^m}\delta\left[x-\theta^m\ln(\theta^m+x)\right]_0^{\frac{\theta^f-\theta^m}{\left(1-\frac{2I}{W}\right)}},$$

$$= -\frac{\left(1-\frac{2I}{W}\right)}{\theta^f-\theta^m}\delta\left[\frac{\theta^f-\theta^m}{\left(1-\frac{2I}{W}\right)} - \theta^m\ln\left(1+\frac{\theta^f-\theta^m}{\theta^m\left(1-\frac{2I}{W}\right)}\right)\right],$$

$$= -\delta\left[1-\frac{\theta^m(1-\frac{2I}{W})}{\theta^f-\theta^m}\ln\left(1+\frac{\theta^f-\theta^m}{\theta^m(1-\frac{2I}{W})}\right)\right]. \tag{A.64}$$

Note that $\overline{LR}^{Equity} < \overline{LR}^{Debt}$ if and only if the following parameter condition is satisfied:

$$\frac{1}{1-\frac{2I}{W}} - 1 + \frac{\theta^m(1-\frac{2I}{W})}{\theta^f-\theta^m}\ln\left(1+\frac{\theta^f-\theta^m}{\theta^m(1-\frac{2I}{W})}\right) -$$

$$\frac{X^m}{\left(1-\frac{2I}{W}\right)(1-\theta^f-\frac{2I}{W}(1-\theta^m))(X^H-X^L)}\ln\left(\frac{X^m+(1-\frac{2I}{W})(X^H-X^L)(1-\theta^m)}{X^m+(1-\frac{2I}{W})(X^H-X^L)\frac{\theta^f-\theta^m}{1-\frac{2I}{W}}}\right) > 0. \tag{A.65}$$

Q.E.D.