The Conditional CAPM with Estimation-risk and Learning: Theory and Evidence

Praveen Kumar Sorin M. Sorescu Rodney Boehme Bartley R. Danielsen

September 24, 2006

1We thank an anonymous referee for very helpful comments. We also thank Beth Allen, Suman Bannerjee, Charles Cuny, Richard Green, Ed Green, Ro Gutirrez, Mark Flannery, Ravi Jagannathan, Ronni Michaely, Maureen O’Hara, Ramon Rabinovitch, Michael Rebello, Bhaskar Swaminathan, and seminar participants at Texas A&M and Tulane for useful comments or discussions on issues examined in this paper. All shortcomings are our own responsibility.

2Praveen Kumar is from the University of Houston; Sorin Sorescu is from Texas A&M University; Rodney Boehme is from Wichita State University; and, Bartley Danielsen is from the University of North Carolina-Greensboro.
Abstract

We derive — and test the empirical implications of — the conditional CAPM as a security market equilibrium when returns are multi-variate normal, but investors face estimation-risk with respect to the mean returns and learn in a Bayes-rational manner from a sequence of noisy signals of uncertain precision. The conditional CAPM augments the standard CAPM so that the expected return of assets is driven not only by their intrinsic market-risk, but also by their sensitivity to the estimation-risk on the market portfolio. Our empirical tests find that innovations in market volatility, the firm’s publicly traded age, and innovations in the dispersion in analyst forecasts at the firm-level explain the cross-section of average returns — in the direction predicted by the theory — even after controlling for size, book-to-market equity, market beta, and momentum. Our model also predicts that corporate events or disclosures will have announcement effects because of changes in the required rate of return — due to event-induced effects on estimation-risk — rather than revisions in expected cash flows; we find empirical support for this prediction, as well.

Keywords: Estimation-risk; Bayesian Learning; Conditional CAPM; Announcement effects.

JEL Numbers: D83, D92, E22
1 Introduction

While the textbook (unconditional) capital asset pricing model (CAPM) assumes a linear and stable relationship between market risk and excess returns, there is accumulating empirical evidence that market betas are time-varying [e.g., Harvey (1980), Ferson and Harvey (1991, 1993) and Ferson and Korajcyk (1995)]. In response to this evidence — which is not really surprising given the strong assumptions on probability distribution, attitudes toward risk, and the informational structure underlying the CAPM — some have suggested replacing the unconditional CAPM with the conditional CAPM [e.g., Jagannathan and Wang (1996)].

However, unless the conditional CAPM is derived from a more primitive specification of stochastically evolving return distributions or information sets or attitudes toward risk, ad-hoc representations of beta dynamics are likely to be misspecified. In such situations, the conditional CAPM may present no empirical advantage over the unconditional CAPM [see, e.g., Ghysels (1998)].

In this paper, we derive — and test the empirical implications of — the conditional CAPM as a security market equilibrium from a complete specification of investor uncertainty and a stochastically evolving information environment. In our model, returns are multi-variate normal, but investors face estimation-risk with respect to the mean returns and learn in a Bayes-rational manner from a sequence of noisy but informative signals. A crucial aspect of the model is that investors are uncertain about the conditional higher moments of the unknown parameters.

In equilibrium, there is a single-factor representation of risk-premia that are proportional to the conditional risk-premium on the market portfolio. However, the loading on the market risk in the conditional CAPM augments the standard beta by the information-dependent conditional covariance matrix of the unknown mean returns. That is, the expected return of assets is driven not only by their intrinsic systematic (or market) risk, but also by their sensitivity to the estimation-risk on the market portfolio; the latter is time-varying and information dependent, even if the former is held fixed. In addition to the standard beta, assets also have a conditional estimation-risk beta — in effect. Significantly, such a conditional CAPM does not emerge if investors are not uncertain about the conditional higher moments of the mean returns.

While the unconditional CAPM assumes that investors have complete information on the parameters of the return generating processes, in reality, investors are typically uncertain about these parameters. That is, in addition to facing risk that is intrinsic to production and investment, investors also face estimation-risk because of subjective parameter uncertainty. Indeed, a vigorous
literature acknowledges this and examines portfolio optimization and capital market equilibrium in the presence of estimation-risk. However, investors are rarely passive about estimation-risk and use a variety of information sources—announcements and disclosures, analyst reports, observed returns and so on—to learn about the unknown parameters.

But investors will generally also be uncertain about the precision (or quality) of these information sources. Take, for example, the case of newly public firms: there is generally parameter uncertainty due to the absence of a public record of economic performance [see, e.g., Clarkson and Thompson (1990)]; in addition, there is uncertainty about the information content of management’s (strategic) announcements and reliability of analyst predictions. After all, if investors have incomplete information on the return generating processes of new firms, then it is difficult to see how they will know the information content of analyst estimates or payoff-relevant announcements by firms. Uncertainty about information quality matters—it is quite clear that capital market outcomes are quite sensitive to the precision (or quality) of the information available to investors [e.g., Veronesi (2000) and Easley and O’Hara (2004)]. Our contribution is to point out that if estimation-risk and uncertain information quality are simultaneously present, then investors will be faced with a random and time-varying conditional covariance matrix of expected returns, and that this will result endogenously in a conditional CAPM with market betas that vary with the arrival of new payoff relevant information.

The conditional CAPM we derive is empirically rich, because it predicts the cross-section of expected returns will change in response to new information that affects the priced estimation-risk of assets. Specifically, changes in the market estimation-risk will have a greater influence on stocks with higher estimation-risk betas, ceteris paribus. We use this insight to empirically study the effect of innovations in stock market volatility and the publicly traded age of the firm on the cross-section of average stock returns, controlling for the risk factors forwarded in the literature. This is because increases in market volatility proxy for increases in market estimation-risk, while younger firms will have higher estimation-risk betas compared to more seasoned firms [cf. Clarkson and Thompson (1990)]. In a related vein, our model predicts that equilibrium expected returns

---

1In a single-period model, Klein and Bawa (1976) and Bawa et al. (1979) examine the impact of estimation-risk on portfolio allocation problems. Barry and Brown (1985) consider the effect of differential information and estimation-risk on equilibrium returns. Kandel and Stambaugh (1996) allow estimation errors in a multi-period setting, but assume that the investors only have a single-period horizon. Brennan (1998) and Brennan and Xia (2001) consider multi-period models where the investment opportunity set is a constant, but there is parameter uncertainty. Barberis (2000) and Xia (2001) examine the effect of estimation-risk on the asset allocation of long-horizon investors.
for stocks will be higher not only in more volatile market environments, but also when there is greater volatility in firm-specific information. We therefore also examine whether innovations in the dispersion of analyst forecasts at the level of the firm can explain the cross-section of average returns, controlling for the known risk factors.

We find that innovations in market volatility and the firm’s age explain the cross-section of average returns — in the direction predicted by the theory — even after controlling for size, book-to-market equity, market beta, and momentum; i.e., the standard risk factors used in the recent literature. Moreover, market volatility innovations have greater influence on the returns of younger firms compared to more seasoned firms. We also find that innovations in the dispersion of analyst forecasts at the level of the firm explain the cross-section of average returns, in the direction predicted by the theory, even after controlling for the announced risk factors.

These results are of independent interest for the asset pricing literature because they indicate the existence of priced risk factors not currently emphasized. More interestingly, these factors are suggested by an asset pricing model of estimation-risk and learning, but to our knowledge can not be easily reconciled by other existing asset pricing theories.

Our framework also implies that corporate events that affect the asset’s estimation-risk will change the asset’s loading on market risk. Thus, we propose a new mechanism for announcement effects on security returns, namely, a change in the required return because of the event’s effect on the conditional estimation-risk, rather than a revision in the market’s estimation of future cash flows that is typically assumed in the event studies literature. Our empirical analysis, based on the initiation of cash payouts to shareholders through stock repurchases during 1988-2000, supports these predictions.

More generally, information-dependent higher conditional moments imply that good (or bad) news that elevates (or depresses) investors’ conditional assessment of expected returns will generally have an ambiguous effect on equilibrium asset prices. In particular, the stock’s own variability of signals and the covariance of signals from related stocks, will significantly influence the direction and size of announcement effects. Furthermore, assets’ sensitivity to news events itself is dependent on the information history. These insights present a powerful agenda for both theoretical and empirical research relating to event studies.

Our analysis is facilitated by a specification of prior beliefs and likelihood functions for signals such that the investors’ posterior beliefs belong to the family of elliptical distributions. Conse-
quently, there is a mean-variance representation of the investors’ attitudes toward risk [Chamberlin (1983) and Meyer (1987)], even when the intrinsic return uncertainty is supplemented by estimation and information quality risk. Our model is therefore of independent interest because we show that elliptical distributions continue to facilitate tractability in studying security market equilibrium with estimation-risk and uncertain information quality.

Finally, and part from the contributions to the conditional CAPM and empirical asset-pricing literatures, our paper adds significantly to the literature on estimation-risk (see footnote 1). The role of estimation-risk in asset pricing has long intrigued financial economists. While early work conjectured that estimation-risk would be “diversified away,” Barry and Brown (1985) assume that the CAPM holds and then show that estimation-risk can affect equilibrium returns if investors have heterogeneous beliefs. Our analysis derives the conditional CAPM with estimation-risk in equilibrium, and also indicates that heterogeneity of beliefs is not necessary for estimation-risk to be priced in equilibrium.

In the remaining paper, Section 2 describes the basic model, while Section 3 derives the conditional CAPM. Section 4 develops the empirical implications of the model and specifies the test design. Section 5 presents the results of the empirical tests, and Section 6 discusses especially promising applications and extensions of the model. Section 7 summarizes and concludes.

2 The Model

2.1 Returns and Investor Preferences

We consider a canonical “single-period” economy with $J$ perfectly divisible risky assets, where $J$ is a large number. Asset rates of return, \( \mathbf{r} = (r_1, \ldots, r_J)' \), are multi-variate normal, with a mean vector \( \mathbf{\mu} = (\mu_1, \ldots, \mu_J) \) and the covariance matrix \( \mathbf{\Sigma} = [\sigma_{ij}] \). (The precision matrix is denoted \( \mathbf{\Omega} = \mathbf{\Sigma}^{-1} \).) There is also a riskless asset, the “\((J + 1)\)” asset, with a known rate of return \( r_f \). All assets are traded in competitive markets without transactions costs and taxes.

There are $Z$ investors, $z = 1, \ldots, Z$, who each maximize the expected utility of the end-of-period-wealth and exhibit constant absolute risk-aversion; i.e., investor $z$’s von-Neumann-Morgenstern expected utility function is, \( U^z(W^z) = - \exp(-\phi^z W^z) \), \( \phi^z > 0 \). Investors’ investable wealth is denoted by \( \bar{W}^z \), $z = 1, \ldots, Z$. Letting, \( \mathbf{\mathbf{x}}^z = (x_1^z, \ldots, x_J^z)' \), denote the vector of portfolio weight of risky assets for investor $z$, the random end-of-the-period wealth is given by, \( W^z = \bar{W}^z [1 + r_f + \mathbf{x}^z' (\mathbf{r} - r_f)] \),
where \( r_f = (r_f \cdot 1) \), when \( 1 \) is the \( J \times 1 \) unit vector.

### 2.2 Parameter Uncertainty and Learning

We assume that investors do not know — or are uncertain about — the mean return vector \( \mu \). (However, \( \Sigma \) is common knowledge.) Investors have homogenous prior beliefs about \( \mu \), and update on these beliefs based on commonly observed signals that are informative with respect to \( \mu \).² Specifically, all investors receive \( n \) signal vectors, \( S = (s_1, \ldots, s_n) \), \( s_t = (s_{1t}, \ldots, s_{Jt}) \), \( 1 \leq t \leq n \), where the individual signals are generated by information innovations according to,

\[
s_{it} = \mu_i + \epsilon_{it}, \quad i = 1, \ldots, J; \quad t = 1, \ldots, n. \tag{1}
\]

For each security \( i \), the information innovations \( \epsilon_{it} \) are i.i.d. normally distributed with mean zero and precision \( \gamma_{ii} \).

Economic intuition suggests that the information innovations will be correlated across assets. Take, for example, the case where the signals belong to firms in the same industry grouping; or are being generated by analysts applying the same methodology across assets; or are derived from a common macroeconomic innovation, such as a change in monetary policy. The correlation matrix of the information innovations signal-errors is given by the (positive-definite) precision matrix \( \Gamma = [\gamma_{ij}] \). The likelihood function \( f(S|\mu, \Gamma) \) is thus multivariate normal with mean vector \( \mu \) and the precision matrix \( \Gamma \).

For the reasons mentioned at the outset, investors faced with estimation risk on mean returns are unlikely to know precisely the parameters governing the joint distribution of signal errors. We therefore assume that investors are uncertain about \( \Gamma \). Effectively, then, there is investor uncertainty about the parameters that determine the likelihood function \( f(S|\mu, \Gamma) \). The assumption of investor uncertainty about the second moments of the signal errors (or precision), i.e., \( \Gamma \) distinguishes our analysis from the literature.

---

²These signals are presumably received from informed insiders through accounting based disclosures such as earning announcements [e.g., Aharony and Swary (1980)], financial policy choices such as dividend initiations or shares repurchase decisions [e.g., Pettit (1972), Vermalaen (1981) and Asquith and Mullins (1983)], and other announcements such as mergers and acquisitions related developments [e.g., Loughran and Viju (1997) and Andrade et al., (2001)]. These signals could also be received from outside informed agents, such as analysts, whose recommendation or projections can sometimes be informative [e.g., Brav and Levahy (2003)]. Finally, these signals could be based on macro- or market-level events that influence the mean returns of risky assets — for example, monetary, technological, and commodity supply shocks.
We will assume that investors' prior beliefs on the joint distribution of \((\mu, \Gamma)\) belong to the Normal-Wishart family, i.e., there exists a vector \(\nu_0 = (\nu_{0,1}, \ldots, \nu_{0,J})\); a symmetric and positive-definite matrix, \(\theta_0 = [\theta_{0,ij}]\); and, scalars \(y\) and \(\alpha\) such that the prior beliefs are,

\[
q(\mu, \Gamma) = N_J(\mu, y \Gamma) W_i J(\Gamma | \alpha, \theta),
\]

where \(N_J(\mu, y \Gamma)\) is the \(J\)-dimensional multi-variate normal distribution with the mean vector \(\mu\) and precision matrix \(y \Gamma\). Meanwhile, \(W_i J(\Gamma | \alpha, \theta)\) is the \(J\)-dimensional Wishart distribution with the parameters \(\alpha\) and \(\theta_0\), so that \(E(\Gamma | \alpha, \theta_0) = \alpha (\theta_0)^{-1}\).

### 2.3 Conditional Moments

The inference process for the annunciated system of prior beliefs and likelihood function is [see, e.g., Bernardo and Smith (1994, page 541)]:

**Proposition 1** Let \(S\) be a signal with the mean vector \(s = (n)^{-1} \sum_{i=1}^{n} s_i\) and the covariance matrix \(D = (n)^{-1} \sum_{i=1}^{n} (s_i - \bar{s})(s_i - \bar{s})'\). Put,

\[
\nu(S) = \frac{y \nu_0 + n \bar{s}}{y + n}; \quad \theta(S) = \theta_0 + \frac{1}{2} D + \frac{1}{2} \frac{ny(\nu_0 - \bar{s})(\nu_0 - \bar{s})'}{y + n}; \quad \alpha_n = \alpha + \frac{n}{2}. \tag{2}
\]

Then, the marginal posterior beliefs of \((\mu, \Gamma)\) are:

\[
q(\mu | S) = t_K(\mu | \nu(S), (y + n)\alpha_n(\theta(S))^{-1}, 2\alpha_n) \quad \& \quad q(\Gamma | S) = W_i K(\Gamma | \alpha_n, \theta(S)), \tag{3}
\]

where, \(t_K\) is the \(K\)-dimensional multivariate (Student’s) \(t\)-distribution.

The multivariate \(t\)-distribution belongs to the family of elliptical distributions (E-D) [see, Owen and Rabinovitch (1983) and Fang et al. (1990)]. Hence, the posterior beliefs on \(\mu\) also belong to the E-D family, and this substantially aids tractability.

Proposition 1 shows that information innovations influence investors’ posterior beliefs through their effect on the (signal) mean vector \(s\) and the covariance matrix \(D\). In particular, the conditional mean and covariance matrix of \(\mu\) are, \(E(\mu | S) = \nu(S)\) and \(\Psi(\mu | S) = \theta(S)[(2\alpha_n - 2)(y + n)]^{-1}\), respectively. Both the first and the second conditional moments are therefore random, and depend on the (realization of) the information history. Moreover, investors’ prior beliefs regarding information quality affect the conditional covariance matrix.
While the signal generation process given in (1) is a natural one and extensively used in the literature, our analysis would apply for any signal generation process where the likelihood function is normal. However, (1) has the appealing “signal calibration” property that higher signal values are positively associated with—or more likely to be realized with—higher values of the mean returns. That is, \( f(S|\mu, \Gamma) \) satisfies the Monotone Likelihood Ratio Property (MLRP) [see, Milgrom (1981)]: if \( S \) and \( S' \) are two given signals with mean vectors \( \bar{s} \) and \( \bar{s}' \), respectively, then, \( E(\mu|S) > E(\mu|S') \) if \( \bar{s} > \bar{s}' \).

2.4 Learning and Portfolio Optimization

In the standard case, with no parameter uncertainty, constant absolute risk-aversion and the joint normality of returns together substantially simplify the analysis of investors’ portfolio optimization problem. It turns out that the combination of constant absolute risk-aversion along with posterior beliefs that belong to the E-D family also facilitates analysis of the effect of parameter uncertainty with unknown information quality on equilibrium asset prices. This is because we exploit an especially useful property of the E-D family: if random variables are jointly elliptically distributed, then risk orderings from the expected utility criterion are identical to those derived from the more convenient mean-variance criterion [Chamberlin (1983) and Meyer (1987)].

Take any investor \( z = 1, ..., Z \), and fix his risky asset portfolio, \( x^z \). Let, \( w^z \) denote the dollar return on the risky assets earned in excess of the risk-free return, i.e., \( w^z = W^z x^z(r - r_f) \). Then, given the signals \( S \), the first two conditional moments of \( w^z \) are,

\[
E(w^z | S) = \hat{W}^z [x^z' (\nu(S) - r_f)]
\]
\[
\Delta(w^z | S) = (\hat{W}^z)^2 x^z' \Psi(S) x^z
\]

where, \( \Psi(S) = [\psi_{ij}] \) is the conditional covariance matrix of \( \mu \). Thus, the typical investor faces two types of risk: the intrinsic market risk represented by the covariance matrix of returns, \( \Sigma \), and the estimation-risk represented by the information-dependent covariance matrix, \( \Psi(S) \).

**Theorem 2** Maximizing \( EU^z \left( W^z | S \right) \) is equivalent to maximizing,

\[
\hat{U}^z(S, x^z) = - \exp \left( - \phi^z \hat{W}^z [1 + r_f (1 - x^z' 1) - \frac{\phi^z \hat{W}^z}{2} x^z' \Sigma x^z] \right) G^z \left( E(w^z | S), \Delta(w^z | S) \right),
\]

where, \( G^z \) is a positive-valued function that is decreasing (increasing) in its first (second) argument.
$G^z$ is the mean-variance representation of the part of the investors’ expected utility function that is subject to estimation-risk. Importantly, $G^z$ completely represents the effects of investor parameter uncertainty with respect to both expected returns and information quality. And because any risk-averse expected utility function is increasing in the mean and decreasing in variance of random variables drawn from the E-D family [e.g., Meyer (1987)], $G^z$ is increasing in the first conditional moment and decreasing in the second conditional moment of payoffs. We note that even with homogeneous conditional expectations, $G^z$ depends on the investor’s type because of investors’ heterogeneous attitudes toward risk, i.e., coefficients of absolute risk-aversion.

We emphasize that under the specified system of prior beliefs and likelihood functions in this model, the mean-variance representation of estimation-risk would apply for any strictly concave expected utility function. However, under the joint assumptions of constant absolute risk-aversion and multi-variate normality of asset returns, the typical investor’s conditional objective function is multiplicatively separable in the known parameters and the first and second conditional moments of the posterior distribution of the unknown parameters.

An important point is that even if the covariance matrix of returns ($\Sigma$) is known, as is the case in our model, the conditional covariance matrix of $\mu$ affects equilibrium asset prices. By contrast, if investors are uncertain about the expected returns and learn through the signals in (1), but the precision of the signals is known, then the conditional second moments of $\mu$ will be deterministic or not information-dependent [see, e.g., DeGroot (1970, page 175)]. Consequently, the empirical covariance matrix will play no role in updating on the unknown parameters.

**Corollary 3** Suppose that $\Gamma$ is common knowledge and investors’ prior on $\mu$ is normal with mean $\nu_0$ and precision matrix $\tau$. Then, conditional on observing signals $S = (s_1, \ldots, s_n)$, with the sample means $\bar{s} = (\bar{s}_1, \ldots, \bar{s}_J)$, investor $z$ maximizes,

$$
\max_{\{x_j\}_{j=1}^J} \left[ \sum_{j=1}^J x_j^z (\nu_{j,n}^z - r_f) + -\frac{\sigma^z}{2} \sum_{i=1}^J \sum_{j=1}^J x_j^z x_i^z \sigma_{ji} \right],
$$

where, $\nu_n = (\tau + n\Gamma)^{-1}(\tau\nu_0 + n\Gamma\bar{s}) = \{\nu_{j,n}\}$ is the conditional expected return vector.
3 Equilibrium Conditional Risk-Premia

In the usual fashion, we derive the equilibrium conditional risk-premia by aggregating the optimal portfolio weights of individual investors. Then, let \( Y^z_j(S, x^z) \equiv -\frac{G^z_j}{\phi^z} \), where \( G^z_j \) denotes the derivative of \( G^z \) with respect to the \( j \)th argument. Conditional on \( S \), \( Y^z_1 \) and \( Y^z_2 \) represent the proportional marginal expected utility of increasing the first and second moments of \( w^z \), respectively; thus, \( Y^z_1 > 0 \) and \( Y^z_2 < 0 \). The optimal portfolio weights are:

\[
[\Sigma - Y^z_2(S, x^z)\Psi(S)] x^z \tilde{W}^z = \left[ \frac{Y^z_1(S, x^z)}{\phi^z} \right] (\nu(S) - r_f).
\]  

(5)

To understand better the effects of estimation-risk on portfolio optimization, we compare (5) with the corresponding optimality condition with complete information (i.e., the standard case), viz.,

\[
\Sigma x^z \tilde{W}^z = \left( \frac{1}{\phi^z} \right) (\mu - r_f).
\]  

(6)

Thus, estimation-risk augments the typical investor’s risk. Specifically, the augmentation factor is \((-Y^z_2(\Psi(S)))\), which we interpret as the effect of the conditional second moments of \( \mu \) on the investor’s marginal conditional expected utility.

To illustrate the effects of estimation-risk on the investors’ portfolio optimization problem, let us examine the case where \( J = 2 \), so that there are two risky assets (\( j = 1, 2 \)). Now examine the welfare effects of a small portfolio rebalancing from the risk-less asset toward, say, asset 1. This adjustment increases portfolio risk on two accounts: first, it increases portfolio return risk; in addition, it increases the investor’s exposure to estimation-risk. In terms of marginal expected utility, the total increase in risk exposure is proportional to,

\[
\sigma_{11} + \sigma_{12} - Y^z_2(S, x^z)[\psi_{11}(S) + \psi_{12}(S)],
\]  

(7)

where, \( \psi_{ij}(S) = \text{Cov}(\mu_i, \mu_j|S) \), \( i, j = 1, 2 \). Note that in the absence of estimation-risk, only the increased risk due to the covariance of \( r_1 \) with the return on the risky assets, i.e., \( \sigma_{11} \) and \( \sigma_{12} \), will be relevant. For a given portfolio weight vector \( x^z \), we will henceforth denote the augmented risk covariance matrix, conditional on \( S \), by \( \Phi^z(S, x^z) \equiv \Sigma - Y^z_2(S, x^z)\Psi(S) \).\(^3\)

\(^{3}\Phi^z \) depends on the initial portfolio and is investor specific, even though the underlying beliefs and information are homogeneous, because the marginal expected utility function is investor-specific and depends on the initial portfolio.
Using the individual investor portfolio optimization condition, (5), we can solve for the optimal investments in the risky assets:

\[ x^z W^z = (|\Phi^z|)^{-1} \left[ \frac{Y^z_1}{\phi^z} \Omega - \frac{Y^z_2}{\phi^z} (\Psi(S))^{-1} \right] (\nu(S) - r_f) \]  

(8)

(Here, $|\Phi^z|$ is the determinant of the augmented risk covariance matrix $\Phi^z$, and we recall that $\Omega = \Sigma^{-1}$.)

Then, aggregating (8) across investors, we obtain the equilibrium market portfolio weights, $x^M = (x^M_1, ..., x^M_J)^t$,

\[ x^M = [T_M(S)\Omega + T_{M^*}(S)(\Psi(S))^{-1}] (\nu(S) - r_f). \]  

(9)

Here,

\[ T_M(S) \equiv \sum_z \frac{Y^z_1(S, x^z)}{W^z \Phi^z |\phi^z|} \quad \& \quad T_{M^*}(S) \equiv \sum_z \frac{Y^z_1(S, x^z)}{Y^z_2(S, x^z) W^z \Phi^z |\phi^z|.} \]  

(10)

We interpret the scalars $T_M$ and $T_{M^*}$ as the market’s conditional risk-tolerance with respect to asset’s intrinsic (return) risk and estimation-risk, respectively. Intuitively, for any given risk-premium vector, the weights of assets in the market portfolio are positively associated with the precision of the return risk (i.e., elements of $\Omega$) and the precision of the estimation-risk (i.e., elements of $(\Psi(S))^{-1}$).

Furthermore, the effect of return and estimation precision on the optimal market portfolio weights depends on the investors’ intrinsic risk-tolerance (i.e., $1/\phi^z$) and the marginal rate of substitution of total risk for portfolio excess returns (i.e., $Y^z_1/Y^z_2$).

We now express the equilibrium conditional risk-premia, $\nu(S) - r_f$, in terms of the assets’ contribution to the augmented risk of return on the market portfolio, namely, $r^M = x^M \cdot r$. Assets’ contribution to the intrinsic risk of $r^M$ is given by the covariance vector, $C_M = (\text{Cov}(r^M, r_1), ..., \text{Cov}(r^M, r_J))^t$, where, $\text{(Cov}(r^M, r_1), ..., \text{Cov}(r^M, r_J))^t = \left( \sum_{i=1}^J x^M_i \sigma_{1i}, ..., \sum_{i=1}^J x^M_i \sigma_{Ji} \right)^t$.

But we also need to incorporate each asset’s contribution to the estimation-risk of the market portfolio ($M$). To do this, we imagine another portfolio, say $M^*$, with the same dimensionality as $M$. Specifically, $M^*$ is composed of $J$ risky assets with returns $r^* = (r^*_1, ..., r^*_J)$ and portfolio weights $x^*_i, i = 1, ..., J$. Further, the covariance matrix of $r^*$ is $\Psi(S)$. We interpret $M^*$ as an aggregate (or market) estimation-risk portfolio and, for each $j = 1, 2$, we interpret $r^*_j$ as asset $j$’s return projection in this (aggregate) estimation-risk portfolio.

By construction, the return on the aggregate estimation-risk portfolio is, $r^{M^*} = x^M \cdot r^*$. Therefore, the contribution of each asset in $M^*$ to the risk of $r^{M^*}$ is given by the covariance vector,
\( C_{M'}(S) = \left( \text{Cov}(r^{M'}, r'_1 | S), ..., \text{Cov}(r^{M'}, r'_J | S) \right)' \), where,

\[ \left( \text{Cov}(r^{M'}, r'_1 | S), ..., \text{Cov}(r^{M'}, r'_J | S) \right)' = \left( \sum_{i=1}^{J} x_i^M \psi_{1i}(S), ..., \sum_{i=1}^{J} x_i^M \psi_{Ji}(S) \right)' \].

The first (or second) component of \( C_{M'}(S) \) gives the contribution of asset 1 (or asset 2) to the estimation-risk of the market portfolio, because \( M' \) is formed by using the same portfolio weights as the market portfolio.

Next, we prove an important result: the equilibrium risk-premia of assets are a weighted average of their contribution to the intrinsic and estimation-risk of the market portfolio, as defined above.

**Proposition 4** In equilibrium, there exist positive scalars \( L_{M}(S) \) and \( L_{M'}(S) \) such that,

\[ \nu(S) - r_f = L_{M}(S)C_M + L_{M'}(S)C_{M'}(S). \] (11)

In fact, \( L_{M} \propto T_{M}/|\Sigma| \) and \( L_{M'} \propto M^*/|\Psi(S)| \). Because \(|\Sigma| > 0 \) (and similarly for \(|\Psi(S)|\)), the weight of the market’s risk-tolerance with respect to intrinsic- or estimation-risk on equilibrium returns is reduced if the assets’ own risks are high relative to the covariance risk.

It now remains to obtain the conditional CAPM representation of equilibrium risk-premia. We define the conditional market risk-premium as, \( R_{M}(S) \equiv x^{M'}(\nu(S) - r_f) \). Then pre-multiplying both sides of (11) with \( x^{M'} \) yields,

\[ R_{M}(S) = L_{M}(S)\sigma_M^2 + L_{M'}(S)\sigma_{M'}^2(S), \] (12)

where, in the usual way, \( \sigma_Q^2 \) represents the variance of the return on portfolio \( Q \). Substituting (12) in (11) then gives:

**Theorem 5** In equilibrium, the asset risk-premia, conditional on the signal history \( S \), are given by:

\[ \nu(S) - r_f = \left[ \frac{L_{M}(S)C_M + L_{M'}(S)C_{M'}(S)}{L_{M}(S)\sigma_M^2 + L_{M'}(S)\sigma_{M'}^2(S)} \right] R_{M}(S) \]

\[ \equiv (\beta(S) + \beta^*(S)) R_{M}(S). \] (13)
Theorem 5 clarifies the aspect of estimation-risk of risky assets that is priced in equilibrium. In the standard CAPM, the equilibrium excess returns are proportional to the covariance of asset returns with the return on the market portfolio: \( \mu - r_f \propto C_M \). However, with estimation-risk on \( \mu \) and Bayesian learning, the equilibrium conditional risk-premia are also positively associated with the covariance of the return on the asset’s estimation-risk projection (\( r^*_j \)) with the return on the aggregate estimation-risk portfolio (\( r^{M^*} \)).

Differently put, Theorem 5 states that in equilibrium the risk-premium of an asset, conditional on the signals \( S \), can be decomposed into the risk-premium due to the asset’s market-risk and the risk-premium due to the asset’s priced estimation-risk. The risk-premium due to market-risk is given by the product of the conditional beta and the conditional market risk-premium, consistent with the usual rendition of the conditional CAPM [see, e.g., Harvey (1989)].

And the risk-premium due to the asset’s priced estimation-risk is given by the product of the conditional estimation-risk beta — defined as \( \beta^*_j(S) \) in (13) — and the conditional market risk-premium. In turn, \( \beta^*_j(S) \) depends on (elements of) the conditional covariance matrix \( \Psi(S) \), i.e., \( \psi_{ij}(S), j = 1, \ldots, J \). For expositional ease, we will use, \( \tilde{\beta}_i(S) \equiv (\beta_i(S) + \beta^*_i(S)) \), to denote the augmented conditional beta of the asset.

Thus, the extension of the CAPM to incorporate estimation-risk is in conformance with one’s basic intuition on equilibrium asset pricing, in that the risk-premium of an asset will not be completely determined by its intrinsic systematic risk, i.e., \( \text{Cov}(r_j, r^{M}) \); but, it will also be influenced by its priced estimation-risk. And the priced estimation-risk is a type of systematic risk because it is determined by the covariance of the return on the asset’s estimation-risk projection (\( r^*_j \)) with the return on the aggregate estimation-risk portfolio (\( r^{M^*} \)). It is noteworthy that one can not generally compare the conditional beta in (13) with the standard beta, i.e., introducing estimation-risk, for the fixed \( \Sigma \), can either increase or decrease the conditional beta relative to the standard beta. This is because the conditional beta is calculated by augmenting both the numerator and the denominator of the standard beta.

Finally, the derivation of the estimation-risk based conditional CAPM, derived from an equilibrium learning model, reiterates the importance of the assumption that investors face random higher (conditional) moments of unknown mean returns. Again, imagine (as in Section 2.4) that

\(^4\)Note that while the covariance matrix of returns, i.e., \( \Sigma \), is common knowledge, the standard beta is still conditioned on \( S \), because the asset’s market covariance risk is normalized by aggregate volatility that depends on \( S \) (cf. (13)).
\( \Gamma \) is common knowledge and investors’ prior on \( \mu \) is normal with mean \( \nu_0 \) and precision matrix \( \tau \). Then, conditional on the signals \( S = ((s_{11}, ..., s_{J1}), ..., (s_{1n}, ..., s_{Jn})) \), the posterior covariance matrix of \( \mu \) is deterministic, independent of \( S \), and a function of the sample size \( n \) alone. (Let us denote this by \( \Psi(n) \).) A review of the steps leading to Theorem 5 then shows that the equilibrium asset risk-premia are themselves independent of the information, \( S \). That is, parameter uncertainty will augment the standard beta by the deterministic conditional second moments, \( \Psi(n) \). And the augmented beta will converge to the standard beta deterministically as the signal size, \( n \), gets large. In sum, the conditional CAPM, with risk-premia that are information-dependent does not obtain if information quality is known.

4 Empirical Implications and Test Design

In this section, we highlight the empirical implications of the conditional CAPM given in Theorem 5. We then develop the empirical tests of these implications, using a variety of empirical proxies — both at the firm- and the market-level. The next section then reports and analyzes the results of the tests developed here.

4.1 Information Innovations and the Cross-Section of Returns

While we have set-up our model in a static framework for tractability and comparison with the standard CAPM, the (Bayesian) learning-based conditional CAPM derived above is empirically rich, because it predicts the cross-section of equilibrium returns will change in response to new information that effects the priced estimation-risk of assets. Specifically, we can develop empirically meaningful correlations between certain types of news (or information innovations) and the cross-section of asset returns — in effect, our model suggests announcement affects on the cross-section of returns.

The transmission mechanism from information innovations and the cross-section of returns is clear from (13). To fix ideas, consider news or information innovations that alter the signal history from, say, \( S \) to \( S' \). Intuitively, then, assets whose estimation-risk beta, i.e., \( \beta^*(\cdot) \), is more affected by the new information will also exhibit greater changes in their equilibrium risk-premium, other things held fixed. Consequently, the cross-section of returns will also be altered in a predicted manner. For example, if the information innovation tends to increase the estimation-risk beta of a certain class of assets, then compared with the pre-information-innovation cross-section, in the
post-information-innovation cross-section these assets will cross-sectionally exhibit higher expected returns relative to other assets. Below, we will use this insight to develop empirical predictions on the effect on the cross-section of returns of information innovations both at the macro- (or market-) and the firm-level.

### 4.2 Information Spreads and the Cross-Section of Returns

The fine structure of our Bayesian learning-based model also imposes restrictions on the relationship between the cross-section of returns at any given time and observable characteristics of the signal history \( (S) \) of assets (up-to that point in time). It is clear from Proposition 1 that the first two own sample signal moments of the assets, i.e., \( \bar{s}_i = (1/n) \sum_{t=1}^{n} s_{it} \) and \( \overline{s}_i^2 = (1/n) \sum_{t=1}^{n} (s_{it} - \bar{s}_i)^2 \) influence the posterior mean and variance of \( \mu_i \). In particular, the posterior variance of \( \mu_i \) is increasing in the sample variance \( (\overline{s}_i^2) \), holding fixed the other sample moments. We will say that the signal history \( S_i^* = (s_{i1}^*, ..., s_{it}^*) \) second-order stochastically dominates the signal history \( S_i^{**} = (s_{i1}^{**}, ..., s_{it}^{**}) \) if \( \overline{s}_i^{2*} > \overline{s}_i^{2**} \), but \( \overline{s}_i^* = \overline{s}_i^{**} \) and \( \overline{s}_{ij}^* = \overline{s}_{ij}^{**}, \ i \neq j \); that is, \( S_i^* \) is a spread on \( S_i^* \).

Intuitively, we expect that own signal spreads will increase the estimation-risk premium on an asset, because they increase the posterior second moment (or uncertainty) regarding the unknown mean return. The following result makes this intuition precise.

**Proposition 6** For any security, \( i = 1, ..., J \), the conditional estimation-risk beta \( \beta_i^*(S^{**}) > \beta_i^*(S^{**}) \) if \( S^* \) and \( S^{**} \) are such that \( S_i^* \) second-order stochastically dominates \( S_i^{**} \).

Of course, signal variance **per se** has no role in the equilibrium risk-premia defined by the standard CAPM. But we reiterate that signal variance would also not influence equilibrium risk-premia in models with estimation-risk in which the posterior second (or higher) moments are known or are not random (see Corollary 3). From an empirical perspective, the power of Proposition 6 is that — it allows refutable predictions on the relation of the cross-section of equilibrium returns to suitable empirical proxies for signal or information variance.

### 4.3 Announcement Effects on Firms’ Cost of Capital

The learning-based conditional CAPM derived above also suggests announcement effects on the cost of capital of (individual) firms. Events that impact the stock’s estimation-risk components, namely \( \psi_{ij} \), will also affect the stock’s market risk loading and therefore influence its equilibrium
risk-premium. Unlike the usual event-study story that interprets announcement effects of corporate events through changes in expected cash flows of stocks, in our model the transmission mechanism for such announcement effects is changes in the market’s estimate of the firm’s cost of capital.

Specifically, consider a corporate event relating to some firm $i$, occurring at some date $\tau$. We denote by $S^-_i$ and $S^+_i$ the signal histories immediately prior and posterior to the event, respectively. The event reduces estimation-risk for stock $i$ if: $\psi_{ii}(S^+_i) < \psi_{ii}(S^-_i)$, $\psi_{ij}(S^+_i) \leq \psi_{ij}(S^-_i)$, for each $i \neq j$, and $\psi_{jk}(S^+_i) = \psi_{jk}(S^-_i)$, $j, k \neq i$. Therefore, events that reduce the estimation-risk for asset $i$ also reduce their conditional estimation-risk beta, i.e., $\beta^*_i(S^+_i) < \beta^*_i(S^-_i)$.

4.4 Test Design

To test for estimation-risk related announcement effects on the cross-section of stock returns, we look for types of news or events that differentially affect the estimation-risk betas of stocks. The requirement of differential impact is important for empirical content, because the post-information cross-section must look predictably and measurably and pre-information cross-section.

The definition of the estimation-risk beta ($\beta^*_i$) suggests that stocks whose estimation-risk is more highly correlated with the estimation-risk of the aggregate or market-portfolio will exhibit greater changes in expected returns in periods when the aggregate estimation-risk itself changes. This is because the estimation-risk of such stocks is more sensitive to changes in the market-wide estimation risk. We hypothesize that the aggregate estimation-risk increases following periods of high stock market return volatility. That is, the aggregate estimation-risk is positively related to innovations in market volatility. Therefore, in periods following exceptionally high market volatility, stocks with the greatest parameter uncertainty should exhibit the highest magnitude of increases in expected returns.

As we have mentioned before, there is considerable empirical evidence in the literature that newly public firms typically are most susceptible to estimation-risk [see, e.g., Clarkson and Thompson (1990)]. Thus, our model predicts that, other things held fixed, changes in market volatility, i.e., market volatility innovations, will have greater influence on the expected return of younger versus more seasoned firms. We use monthly series of innovations in market volatility, computed from daily data. Using the daily equally-weighted and value-weighted market index from CRSP,
for the period 1964-2005, we compute the market volatility innovation in month \( t \), i.e.,

\[
\delta(\sigma_M)_t = \sigma_M - \sigma_M_{t-1}, \quad \sigma_M = \sqrt{\frac{1}{K-1} \sum_{k=1}^{K-1} (R_{m,t+k+1} - R_{m,t+k})^2 / K}
\]

when, \( R_{m,t+k} \) is the daily return on the market portfolio on day \( k \) of month \( t \) and \( K \) represents the number of trading days in the month [see, Moise(2006)].

We then test the effects of innovations in market volatility on the cross-section of returns by examining the firm-specific stock price responses to innovations in market volatility, in terms of the age of the firm. Specifically, for each firm, \( i \), we estimate the following regression:

\[
R_{i,t} - R_{f,t} = \alpha_i + \pi_i \delta(\sigma_M)_t + a_i \text{age}_{i,t} + \eta_i \delta(\sigma_M)_t \text{age}_{i,t} + \beta_i (R_{m,t} - R_{f,t}) + s_i \text{SMB}_t + h_i \text{HML}_t + u_i \text{UMD}_t
\]

where, \( R_{i,t} \) is the monthly rate of return for firm \( i \) during month \( t \) (or “at time \( t \)), \( R_{f,t} \) is the rate of return on a one-month Treasury bill at time \( t \), \( R_{m,t} \) is the value-weighted return of the stock market at time \( t \). Furthermore, \( \text{age}_{i,t} \) is the natural logarithm of firm’s \( i \) age at time \( t \), \( \text{SMB}_t \) is the difference in returns between portfolios of large stocks and portfolios of small stocks, \( \text{HML}_t \) is the difference in returns between portfolios of stocks with high book-to-market ratios and those with low book-to-market, and \( \text{UMD}_t \) is the difference in the returns of stocks with high (positive) momentum and those with low (negative) momentum. That is, we control for market-risk, the Fama and French (1992) factors, and the momentum factor, in examining the effect of market volatility innovations on the cross-section of returns.

We note that \( R_{i,t} \) is the rate of return for firm \( i \) at time \( t \) contemporaneous to the volatility innovations. But our model predicts that positive market innovation during month \( t \) will increase the expected return at the end of the month, i.e., end-of-the-month stock prices will fall, holding other things held fixed. Therefore, our model predicts that \( \pi_i < 0 \). Furthermore, if younger firms have greater estimation-risk, then controlling for the annunciated priced risk-factors, the prediction is that returns will be negatively related to age, ceteris paribus; i.e., \( a_i < 0 \). Finally, our model predicts that the influence of market volatility innovations will be more (less) for younger (seasoned) firms. But because market volatility innovations have a negative influence on contemporaneous excess returns, the prediction is that \( \frac{\partial^2(R_{i,t} - R_{f,t})}{\partial(\sigma_M)_t \partial \text{age}_{i,t}} > 0 \). Thus, we predict that \( \eta_i > 0 \).

Turning, next, to the relationship between signal volatility or information spreads and the cross-
section of returns, we interpret signals as analysts’ forecasts of expected earnings per share. Hence, signal volatility (or the information spread) is measured through the dispersion of opinion using the coefficient of variation for analysts’ annual forecasts estimated from I/B/E/S data, now part of Thompson Financial. The coefficient of variation is estimated by dividing the I/B/E/S reported standard deviation of analyst earnings/share forecasts for the current fiscal year end (I/B/E/S Fiscal Year period “1”) by the absolute value of the mean earnings/share forecast, as listed in the I/B/E/S Summary History file. This proxy is typically used in the literature [e.g., Diether et al. (2002)].

The empirical prediction from our model is that the current returns will be negatively related to innovations in the coefficient of variation for analysts’ annual forecasts. Calendar time portfolios are formed each month \((t)\) of the calendar according to contemporaneous changes in the dispersion of analyst forecast occurring between month \(t-1\) and month \(t\). Portfolios are both equally weighted and value weighted according to the firm’s market capitalization in the previous month. Changes in dispersion are divided into five equal quintiles. Each month of the calendar, a firm is placed into one of five different calendar time portfolios according to the value of its contemporaneous change in dispersion quintile. Firms that experienced the most positive innovation in dispersion of analyst forecast are placed in the portfolio corresponding to quintile 5. Firms with the most negative innovation are placed in quintile 1. The hedge portfolio at the bottom of the table takes long positions in the quintile 5 portfolio and short positions in quintile 1. The abnormal returns for each quintile portfolio are the intercept \(\alpha\) from the following four-factor regression:

\[
R_{q,t} - R_{f,t} = \alpha + \beta(R_{m,t} - R_{f,t}) + s \text{ SMB}_t + h \text{ HML}_t + u \text{ UMD}_t \tag{15}
\]

where, \(R_{q,t}\) is the rate of return of the portfolio corresponding to quintile \(q\) at time \(t\). The abnormal returns of the hedge portfolio are then estimated with the following regression:

\[
R_{1,t} - R_{5,t} = \alpha + \beta(R_{m,t} - R_{f,t}) + s \text{ SMB}_t + h \text{ HML}_t + u \text{ UMD}_t \tag{16}
\]

where, \(R_{5,t}\) is return of the portfolio corresponding to quintile 5 and \(R_{5,t}\) corresponds to quintile 1. And our model predicts that the abnormal returns in (16) will be positive because firms in quintile 5 (quintile 1) will have relatively low (high) returns, controlling for the specified priced risk-factors. We also examine the firm-specific stock price responses to contemporaneous innovations in
dispersion of analyst forecast. For each firm, \( i \), we estimate the following regression:

\[
R_{i,t} - R_{f,t} = \alpha_i + \pi_i \delta(\text{disp}_{i,t}) + \beta_i (R_{m,t} - R_{f,t}) + s_i \text{SMB}_t + h_i \text{HML}_t + u_i \text{UMD}_t
\]  

(17)

where, \( R_{i,t} \) is the rate of return for firm \( i \) at time \( t \), and \( \delta(\text{disp}_{i,t}) \) is the change in dispersion of analyst forecast for firm \( i \), between time \( t - 1 \) and time \( t \). Thus, our model predicts that \( \pi_i < 0 \).

Finally, to test the predictions on the announcement effects on firms’ cost of capital, we focus market behavior around payout initiation by firms that occurs through share repurchases. Initiation of cash disbursements to shareholders is a watershed event in the life-cycle of the firm, and is duly noted as such by investors. While much of the literature has interpreted the initiation announcement as affecting investors’ expectations of future cash flows, a plausible case can be made that the initiation of cash disbursements also signals a decline in the systematic (cash flow) risk of the firm [see, e.g., Grullon et al. (2001)].\(^5\)

Now, Theorem 5 implies that if there is estimation-risk, then the betas estimated from the standard market model already incorporate the asset’s priced estimation-risk. Thus, the model predicts that immediately following the said event, the stock’s observed loading on the market factor will fall. For an event at date \( \tau \), we therefore estimate the pre- and post-event market factor loadings for the stock by using the market model [see, e.g., Brown and Warner (1985)],

\[
R_{i,t} = \alpha_i + \beta_i R_{M,t} + \epsilon_{i,t}
\]  

(18)

for \( t = \tau - T, \ldots, \tau \) and \( t = \tau, \ldots, \tau + T \), respectively. The pre- and post-event estimated betas are denoted \( \hat{\beta}_i(S_t^-) \) and \( \hat{\beta}_i(S_t^+) \), respectively. Based on the model, the hypothesis is that if there is an event that reduces the estimation-risk of asset \( i \), then \( \hat{\beta}_i(S_t^+) < \hat{\beta}_i(S_t^-) \).

We can also derive some other refutable predictions by imposing further structure on the estimation-risk effects of the corporate event. The equilibrium return representation (13) suggests the statistical model,

\[
R_{i,t} = \alpha_{i,t} + \beta_{i,t} S_t + \epsilon_{i,t}
\]  

(19)

Here, \( S_t \) is the signal history up-to date \( t \) and \( \epsilon_{i,t} \) are disturbance terms with \( E[\epsilon_{i,t} | S_t] = 0 \), \forall i,
In general, the matrix of second-order moments, \( E[\epsilon_{i,t} \epsilon_{j,t-k} | S_t] \) (denoted \( V(S_t) \)) may be non-diagonal. We will let \( \text{Var}_i(S_t) = E[\epsilon_{i,t}^2 | S_t] \), and denote by Std. Error(\( \hat{\beta}_{i,t} \)), the standard error of the beta estimate in the market model.

It turns out that if the estimation-risk reducing event also reduces the specification error in (19), which is plausible since part of the specification error is because of the fact that the parameters of the equilibrium return distribution are unknown, then the post-event standard errors of the beta estimates will be lower than their pre-event counterparts. Then suppose that \( \text{Var}_i(S_{t+}^i) < \text{Var}_i(S_{t-}^-) \) but the event leaves other elements of \( V(S_t^-) \) unaffected. Then, the hypothesis is that \( \text{Std. Error}(\hat{\beta}_i(S_{t+}^i)) < \text{Std. Error}(\hat{\beta}_i(S_{t-}^-)) \).

### 5 Tests and Results

#### 5.1 Information Innovations and the Cross-Section of Returns

In Table 1, we present the results of the cross-sectional averages of the firm-specific coefficients \( \pi_i, a_i, \) and \( \eta_i \), in the regression equation (14). We also report the \( t\)-statistics computed in the cross-section. For robustness, we also estimate alternative specifications, using a model that does not include the UMD factor.

The results in table 1 support the model’s predictions. Using the full regression model specified in (14), we find that the estimated \( \pi_i < 0, a_i < 0, \) and \( \eta_i > 0 \), at a high degree of significance. These estimates are robust to alternative model specifications. In words, following periods of increased market volatility, stock prices drop on average (controlling for the market-risk, the Fama-French, and momentum risk-factors); And, the prices of younger firms — with higher estimation-risk — fall more than seasoned firms. Thus, the cross-section of returns is altered by an increase in the aggregate estimation-risk along the lines predicted by our model.

The results in Table 1 imply that innovations in market volatility, firm age, and the interaction of volatility and the age of the firm are significant in explaining the cross-sectional differences in asset returns, even after controlling for the commonly used four- or three-factor asset pricing models. These results are therefore of independent interest for the asset-pricing literature, and their motivation through an equilibrium model of estimation-risk and learning is noteworthy.
5.2 Information Spreads and the Cross-Section of Returns

In Table 2, we report the average contemporaneous returns sorted in quintiles of analyst dispersion changes (cf. (15)), and the abnormal returns on the hedge portfolio (cf. (16)). For robustness, we also provide estimates using a three-factor model that does not include the UMD factor.

Consistent with the model’s predictions, the contemporaneous results fall significantly when we go from the quintiles with negative or low increase in analyst forecast dispersion to quintiles with the greatest increase in analyst dispersion. This result is robust to controlling with either the three- or the four-factor asset pricing model. The difference between the average return in the third or fourth quintile and the average return in the highest quintile of analyst dispersion innovations is especially pronounced.

In Table 3, we report the results of the firm-specific responses to innovations in analyst forecast dispersion. Reinforcing the results of Table 2, and consistent with the theoretical predictions, the cross-sectional average of the estimates of $\pi_i$, the coefficient for the innovation in analyst dispersion (i.e., $\delta(\text{disp}_{i,t})$) is negative and highly significant. Again, this result is robust to controlling for either the three- or four-factor asset pricing model. Consequently, the results of Tables 2 and 3 indicate that innovation in analyst dispersion is a significant determinant of cross-sectional differences in returns, even after controlling for the risk factors currently employed in the literature. This result is therefore also of independent interest to the asset pricing literature.

5.3 Announcement Effects on Firms’ Cost of Capital

We define our event to be the first announcement of share repurchases by firms that have not initiated dividends prior to the announcement. That is, the event at hand is the first cash payout to shareholders by firms. Our sample of 1604 firms (or events) is derived from the SDC Shares Repurchases database (from Thomson Financial) during 1988-2000.\textsuperscript{6} For each sample firm $i$, we estimate the market model (18) independently for the 36-month period prior to, and the 36-month period after, the month of the shares repurchase announcement.

Table 4 reports the mean and median Cumulative Abnormal Returns (CARs) for a three-day window centered on the repurchase announcement (event) day for the entire sample of 1604 firms.

\textsuperscript{6}We focus on shares repurchases as the payout initiation method, rather than dividends, because the sample of dividend initiations (for comparable sample period) is typically smaller than the sample at hand. One of the reasons for smaller dividend initiation samples is that firms in many industries, for example high-technology industries, typically do not pay dividends.
In the usual fashion, these CARs are estimated by summing the relevant daily returns of the event firm during the announcement window and subtracting corresponding return of the CRSP equally weighted market index, as explained in Brown and Warner (1985). Payout initiations through repurchases have significant price effects and hence appear to have substantial information content.

Table 5 reports the mean and median pre- and post-event beta estimates. Both the mean and median post-event betas are substantially lower than their pre-event benchmarks, and the differences are highly significant. Indeed, the median post-event beta is about twenty-five percent lower than the median pre-event beta. Thus, the evidence here supports an empirical prediction of the model. Table 5 also shows the mean and median standard errors of the pre- and post-event betas. The standard errors of the beta estimates have fallen significantly after the event; the median standard error falls by about twenty-two percent; and, the null hypothesis of no change in the standard errors after the event can be rejected at a high degree of confidence. Thus, this evidence here supports another empirical prediction of the model.

The results in Table 5 are of substantial interest because they show that a pivotal event in firms’ careers—the initiation of cash payouts to shareholders—may significantly reduce the stock’s loading on the market factor by reducing investors’ estimation-risk. Our analysis indicates that announcement effects of events that reduce (priced) estimation-risk could arise because of event-induced variations in the required rate of returns for the asset. One important issue for future empirical research is to examine the robustness of our results—by considering short versus long-term effects and by examining other type of events that influence estimation-risk. Another is to develop alternative proxies for the signal variance and covariance by identifying firm-related variables whose volatility is positively correlated with the signal volatility in investors’ learning models.

6 Extensions and Relation to the Literature

In this section, we indicate other empirical predictions — potentially testable — that can derived from the model. We also link the model to empirical results known in the literature.

6.1 Patterns in Estimated Betas

The decline in beta following estimation-risk reducing events, documented above, should in fact occur over the life-cycle of the firm as more information gets developed on the unknown parameters.
As we noted above, the estimated coefficient, say $\hat{\beta}_j$, from the market model (18) is actually an estimate of the augmented beta, $\beta^*_j(S)$. However, if the estimation-risk for the asset declines toward zero, i.e., $\psi_{ij}(S)$ tend to zero for each $i = 1, \ldots, J$, then $\beta^*_j(S) \rightarrow \text{Cov}(r_j, r_M)/\sigma^2_M$, i.e., the standard beta. Now, if the standard beta is a stable parameter and if for a given firm $i$, $\psi_{ij}(S_t)$ declines for each $j$, then we get the prediction that the estimated betas (from the market model) will be negatively related to the age of the firm. This prediction is consistent with the results reported by Clarkson and Thompson (1990) who find that the estimated betas decline in the months following the firm’s IPO.

However, not all events reduce estimation-risk. Certain events, like restatements of previously filed financial statements may actually increase estimation-risk. Following such events, the model at hand—with information-dependent higher conditional moments—predicts that the estimated beta may actually rise. Indeed, Anderson and Yohn (2002) find negative CARs following revenue restatements and also a decrease in the earnings response coefficients, which is consistent with higher second moments of the posterior distribution of the unknown parameter. However, they do not directly measure the change in estimated betas. But Hribar and Jenkins (2003) do study the effects of restatements on the equity cost of capital and find that in their sample this cost rises between 7 to 19 percent in the month following the restatement.

By contrast, stochastically evolving estimated betas are inconsistent with models where the conditional higher moments are non-random or not dependent on information history (cf. Section 3.3). Differently put, if the estimated betas in stock portfolios (where the measurement errors are diversified away) show stochastic variation, i.e., do not decline secularly, then this is prima facie evidence in support of information- or path-dependent higher conditional moments. We also note that the empirical results described above are complementary to the results we report in Section 4.1: betas estimated from the market model are sensitive to variations in the (priced component of) the estimation-risk of stocks.

### 6.2 Non-linear Beta Effects

Jagannathan and Wang (1996) argue that bias introduced by using the unconditional CAPM, when in fact the conditional CAPM is true, may possibly explain some well-known deviations from the predictions of the standard CAPM. We derive some new predictions regarding the deviations from the standard CAPM in cross-sectional regressions, a topic that continues to receive much attention.
in the literature [e.g., Lewellen and Shanken (2002)].

For convenience we maintain the assumption that the covariance of the asset’s return with market’s return, $\text{Cov}(r_j, r_M)$, is a known constant. Then, using iterated expectations, we can write the conditional CAPM (13) in unconditional form as,

$$E(r_j) = r_f + [E(\beta_j^*(S))E(R_M(S))] + \text{Cov}(\beta_j^*(S), R_M(S)).$$

(20)

Hence, the measured excess or abnormal returns on asset $j$, $r_e^j = \text{Cov}(\beta_j^*(S), R_M(S))$. Thus, if we use the standard CAPM to predict expected returns, then we will underestimate the returns of those stocks whose augmented beta is positively correlated with the market risk-premium, because the abnormal returns for such a stock will be positive.

Now, let $c_{j,M} \equiv \text{Cov}(\frac{L_M}{\sigma_M + \sigma_{M^*}}, R_M(S))$, and $c_{j,M^*} \equiv \text{Cov}(\frac{L_{M^*} \text{Cov}(r_j, r_{M^*})}{\sigma_{M^*} + \sigma_{M^*}}, R_M(S))$. Then,

$$\text{Cov}(\beta_j^*(S), R_M(S)) = \text{Cov}(r_j, r_M)c_{j,M} + c_{j,M^*}.$$ 

(21)

Ignoring, for the moment, the second term in (21), we note that $r_e^j$ is proportional to $\text{Cov}(r_j, r_M)$, and the sign of the proportionality factor depends on the sign of $c_{j,M^*}$. Thus, if we were to use estimates of beta, $\hat{\beta}_j$, derived in some fashion from historical data, and run the regression,

$$r_j = r_f + b_1 j \hat{\beta}_j + b_2 j (\hat{\beta}_j)^2 + \epsilon_j,$$

(22)

then (21) implies that for stocks with low estimation-risk, the sign of $b_{2j}$ is the same as the sign of $c_{j,M^*}$. This is an interesting implication because the non-linear relationship between expected returns and the beta is an important empirical issue [e.g., Fama and Macbeth (1973)].

While the sign of $c_{j,M^*}$ is theoretically ambiguous, it is likely to be positive in practice. We recall that $L_M$ is proportional to the aggregated ratio, $\sum_z G^z_1(S)/\phi^2$ (cf. (10)), where $G^z_1$ is the marginal (posterior) indirect expected utility of wealth for a typical investor, and hence is negatively correlated with the market risk-premium for an individual risk-averse investor. However, the covariation of $L_M$ with the market risk-premium is likely to be small because consumption will generally be cross-sectionally negatively correlated across agents due to risk-sharing and participation in financial markets [see, e.g., Constantinides and Duffie (1996)]. In fact, if we proxy $L_M$ by the marginal expected utility of aggregate consumption in a representative agent economy, then empirically $\text{Cov}(T_M, R_M(S))$ is a low number because the variation in aggregate consumption...
has historically been quite low [see, e.g., Campbell (2003)]. Meanwhile, the volatility of market returns is also known to be higher in recessions [Schwert (1989)], and \((\sigma_M^2 + \sigma_{M*}^2)\) will be negatively correlated with the market risk-premium. In fact, the increase in the volatility of the returns on the market during recessions is quite pronounced. Thus, the available empirical evidence suggests that the sign of \(c_{j,M*}\) is positive because the numerator is essentially flat while the denominator is significantly negatively correlated with the market risk-premium.

Next, we can give empirical content to the second term in (21) by noting that the marginal rate of substitution between expected returns and estimation-risk, which is measured by \(L_{M*}\), will be small for assets with low estimation-risk, such as seasoned securities. Thus, for such assets, the second term in (21) will be small independent of the size (or sign) of \(c_{j,M*}\). Hence, for seasoned securities, the deviations from the standard CAPM will be driven by the first term in (21).

Based on the previous discussion (regarding the first term in (21)), we can therefore derive the empirical prediction that the estimated coefficient, \(b_2j\) in (22), will be more significant for seasoned firms than for newly listed firms. This prediction could be tested by sorting the estimated coefficient \(b_2\) on portfolios formed by various proxies for "seasoned firms:" for example, age since initial listing, the number of quarters since the first cash disbursement, the number of quarters since the initiation of analyst coverage, and the dispersion in analyst forecasts.

### 6.3 Post-event Performance Drifts

Seemingly abnormal equity price “drifts” in periods immediately following certain corporate events are long-known and have recently drawn intense scrutiny. Such drifts have been documented following unexpected earnings changes,\(^7\) dividend initiations [Michaely et al. (1995) and Boehme and Sorescu (2002)], stock repurchases [Ikenberry et al. (1998)], and seasoned equity offerings [Loughran and Ritter (1991)]. These stylized facts have generated a debate about the validity of market efficiency. A behavioral-based literature has attempted to interpret these drifts as reflecting irrational investors with cognitive biases [see, e.g., Daniel et al. (1998), Odean (1998), and Hong and Stein (1999)]. But Fama (1998) argues that the observed drifts could be due to chance, biased methodologies, or misspecified asset pricing models. However, Fama does not provide an alternative equilibrium asset pricing model that could be consistent with the observed drifts.

\(^7\)First reported by Ball and Brown (1968), post-earnings-announcement drifts remain one of the most intensively studied topics in accounting research [see, e.g., Bernard and Thomas (1989), Kothari (2001), and Chordia and Shivakumar (2005)].
While a full development remains outside the scope of the current paper, post-event equity drifts can be consistent with the equilibrium asset pricing with information dependent conditional higher moments of unknown expected returns. The basic insight here is that the equilibrium price-sensitivity of a security to new payoff-related information is itself stochastic (cf. Section 2.6); in particular, the (asset) price response to unanticipated information can magnify, after events. This is because the pricing effects of information innovations depend on the information history at the time of the innovation. Thus, following a corporate event that changes (axiomatically) the information history in a major way, equilibrium asset prices can exhibit greater response to relatively minor signals compared to an analogous response prior to the event.

Specifically, if the signals (cf. Equation (3)) are positively serially correlated, rather than being i.i.d., then the post-event signals will be positively correlated with the signal realization at the time of the event. Therefore, firms with unanticipated positive (negative) news experience positive (negative) drifts post-event. Indeed, the SRC will increase as signal variability increases, with the attendant implication that the post-announcement drifts will be more pronounced following events that widely deviate from market expectations. Interestingly, the post-earnings-announcement drift literature empirically examines an “earnings response coefficient,” and links it to firm- and ownership related variables [e.g., Ball et al. (1993) and Kothari (2001)]. However, time-variation in the SRC is not emphasized in the empirical literature and remains an interesting area for future research.

6.4 The Interaction of Estimation-risk and Shorting Constraints

Miller (1977) argues that with heterogeneous beliefs and binding short-sale constraints, stocks will be over-valued, relative to the friction-free equilibrium, because asset prices will reflect only the beliefs of the optimistic investors. An attendant implication of this argument is that if short sale constraints are binding, then a negative correlation exists between risk-adjusted returns and dispersion of beliefs. A growing empirical literature is in fact generating results that appear to support Miller. For example, on the basis of the observed positive relationship between return volatility and the dispersion of I/B/E/S forecasts [e.g., Peterson and Peterson (1982)], Diether et al. (2002) use the dispersion in analyst forecasts as a proxy for the dispersion in investor beliefs. They then find a negative relationship between analyst dispersion and future returns. And Gebhardt et al. (2001) find similar results when they use analyst forecast dispersion as a risk proxy for estimating the cost of capital.
To economize on space and notational burden, we have not explicitly considered shorting constraints in the foregoing analysis, but we have analyzed the interaction of shorting restrictions and estimation-risk. The so-called Miller effect will be muted for stocks with high estimation-risk, ceteris paribus, because the optimistic or infra-marginal buyers are less aggressive in the presence of parameter (or structural uncertainty). Thus, the empirical prediction is that controlling for the intensity of the shorting constraint, the (negative) effect of dispersion in investor opinion, at the margin, should be less pronounced for securities with higher estimation-risk. This implication is testable through a straightforward modification of the empirical methodology used by an active empirical research literature on the effect of shorting constraints on asset prices.

Specifically, Asquith et al. (2005) test Miller’s hypothesis by examining whether there is evidence for premium- or over-pricing of stocks with high shorting interest (a proxy for high shorting demand) and low institutional ownership (a proxy for low lendable supply of shares). They indeed find that such stocks underperform (on an equally weighted basis) during 1988-2002, i.e., they are initially over-priced. Our model predicts that, for high shorting-interest and low institutional ownership stocks, underperformance will be negatively related to (proxies for) estimation-risk.

7 Summary and Conclusions

It is a truism that investors are uncertain about the parameters of the return generating processes and use a variety of information sources to learn about these parameters. However, parameter uncertainty typically goes hand-in-hand with uncertainty about information quality (or precision). We analyze security market equilibrium when investors are simultaneously learning about the unknown parameters and information quality. Investors in our model hold Normal-Wishart beliefs and face random higher conditional moments of the unknown mean returns. We find that this type of uncertainty has substantial theoretical and empirical implications.

Even in a CAPM-world, the loading on the market risk must be augmented to include estimation-risk, based on the conditional covariance matrix of the mean returns. The conditional CAPM that results has both times-series and cross-sectional implications for asset returns. Specifically, we empirically study the effect of innovations in stock market volatility and dispersion in an athe publicly traded age of the firm on the cross-section of average stock returns, controlling for the risk factors forwarded in the literature. We find that innovations in market volatility and the firm’s age explain

\(^8\)Details are available from the authors on request.
the cross-section of average returns — in the direction predicted by the theory — even after controlling for size, book-to-market equity, market beta, and momentum; i.e., the standard risk factors used in the recent literature. Moreover, market volatility innovations have greater influence on the returns of younger firms compared to more seasoned firms. We also find that innovations in the dispersion of analyst forecasts at the level of the firm explain the cross-section of average returns, in the direction predicted by the theory, even after controlling for the announced risk factors.

Another intriguing implication, that we empirically test and find support for, is that announcement effects may occur through reductions in the asset risk-premium, rather than revisions in expected cash flow payoffs that is generally assumed. More generally, disclosure-related events that affect asset returns will be path-dependent, depending for example on the sample signal covariance. Moreover, the deviation from the standard CAPM in cross-sectional or unconditional tests of the CAPM will be affected by the estimation-risk component of stocks.

Our analysis provides a rich agenda for future theoretical and empirical work. Besides the previous comments on the need to control for signal covariance, the development of a methodology that attempts to decompose announcement effects into expected cash flows and lower cost of capital effects is especially promising. Our framework can also be extended to analyze post-event drifts. In a world with parameter uncertainty and learning, tests of the unconditional CAPM are misspecified. While this point is well known, the literature has been hampered by the paucity of conditional CAPM representations that arise from structural models. Ours is a structural conditional CAPM model, based on estimation-risk, and is empirically rich. For example, it suggests using estimation-risk based proxies in the set of conditioning variables to test the superiority of the conditional CAPM over the unconditional version [see, e.g., Ghysels (1998)]. And because of the specification bias, variables correlated with the unmodeled estimation-risk may be correlated with observed returns.
Appendix

Proof of Theorem 2: Using the law of iterated expectations,

\[
EU^z (W^z | S) = E_{\{\mu_1, \ldots, \mu_J\}} \left[ E(-\phi^z W^z | (\mu_1, \ldots, \mu_J)) | S \right].
\]

(23)

Now, conditional on \((\mu_1, \ldots, \mu_J)\), \(W^z\) is normally distributed with mean and variance equal to,

\[
\bar{W}^z [1 + rf + x^z (\mu - rf)] \& (\bar{W}^z)^2 x^z \Sigma x^z,
\]

respectively. Then, using the moment-generating function for the multivariate normal distribution and the properties of the exponential function, we can re-write (23) as:

\[
E_{\{\mu_1, \ldots, \mu_J\}} \left[ E(-\phi^z W^z | (\mu_1, \ldots, \mu_J)) | S \right] =
\]

\[
E_{\{\mu_1, \ldots, \mu_J\}} \left[ -\exp \left( -\phi^z \bar{W}^z (1 + rf + x^z (\mu - rf) - \frac{\phi^z}{2} \bar{W}^z x^z \Sigma x^z) \right) \right] | S
\]

\[
= -\exp \left( -\phi^z \bar{W}^z [1 + rf (1 - x^z 1)] \right) \exp \left( \frac{1}{2} \phi^z \bar{W}^z x^z \Sigma x^z \right) E_{\{\mu_1, \ldots, \mu_J\}} \left[ \exp \left( -\phi^z \bar{W}^z x^z \mu \right) | S \right]
\]

Here,

\[
E_{\{\mu_1, \ldots, \mu_J\}} \left[ \exp \left( -\phi^z \bar{W}^z x^z \mu - rf \right) \right] | S] = \int \exp \left( -\phi^z \bar{W}^z \sum_{j=1}^J x^z_j \mu_j \right) dq(\mu | S).
\]

But the distribution of \(\sum_{j=1}^J x^z_j \mu_j\) belongs to the E-D family since \((\mu_1, \ldots, \mu_J)\) have the multivariate \(t\)–distribution and linear combinations of these variables also have the \(t\)–distribution [see, e.g., Owen and Rabinovitch (1983)]. And since the CARA expected utility function is a strictly increasing and concave utility function, it follows that there exists a mean-variance representation of estimation-risk, namely the function \(G^z \left( E(w^z | S), \Delta(w^z | S) \right)\), where \(E(w^z | \cdot)\) and \(\Delta(w^z | \cdot)\) are given by (4).

Proof of Corollary 3: Follows immediately from Theorem 1 and DeGroot [1970, page 175].

Proof of Proposition 4: For notational ease, we suppress the dependence of the quantities on \(S\).

Put, \(X_M \equiv T_M / |\Sigma|\) and \(X_M^* \equiv T_{M^*} / |\Phi|\) (where \(|B|\) denotes the determinant of the square matrix
B). Also, let,
\[
\Lambda = \begin{bmatrix}
X_M \sigma_{22} + X_M \psi_{22} & -(X_M \sigma_{12} + X_M \psi_{12}) \\
-(X_M \sigma_{12} + X_M \psi_{12}) & X_M \sigma_{11} + X_M \psi_{11}
\end{bmatrix}
\]
Then, we can directly compute,
\[
[T_M \Omega + T_{M^*} (\Psi)^{-1}]^{-1} = \frac{X_M}{|\Lambda|} \Sigma + \frac{X_{M^*}}{|\Lambda|} \Psi.
\] (24)
Then pre-multiplying both sides of (9) by \( [T_M \Omega + T_{M^*} (\Psi)^{-1}]^{-1} \) yields,
\[
L_M \Sigma x^M + L_M \Psi x^M = L_M C_M + L_M C_{M^*}.
\] (25)

**Proof of Theorem 5:** From (12), we get,
\[
\frac{R_M(S)}{L_M(S) \sigma_M^2 + L_{M^*}(S) \sigma_{M^*}^2(S)} = 1.
\] (26)
Hence, from (11),
\[
\nu(S) - r_f = L_M(S) C_M + L_{M^*}(S) C_{M^*}(S)
\]
\[
= \frac{L_M(S) C_M + L_{M^*}(S) C_{M^*}(S)}{L_M(S) \sigma_M^2 + L_{M^*}(S) \sigma_{M^*}^2(S)} R_M(S).
\] (27)

**Proof of Proposition 6:** Since the expected utility function is exponential and the posterior beliefs belong to the elliptical family of distributions following a mean-varying spread, the composition of the two-fund separation (cf. Theorem 2) is stable as well following a mean-varying spread. Let, \( x_i^{**} \) and \( x_i^{*} \) be the portfolio weights in asset \( i \) with the signal history \( S_i^S \) and \( S_i^{**} \), respectively. Straightforward calculations from Proposition 1 show that, \( \frac{\partial \psi_i}{\partial \mu_i} > 0 \). Hence, given the definition of an information spread, where \( \tilde{s}_i^{**} > \tilde{s}_i^S \), but \( \tilde{s}_j^S = \tilde{s}_j^{**} \) and \( \tilde{s}_{ij}^S = \tilde{s}_{ij}^{**} \), \( i \neq j \), it follows that \( \psi_{ii}^{**} > \psi_{ii}^S \), but \( \psi_{ij}^{**} = \psi_{ij}^S \), \( i \neq j \). (Of course, elements of the covariance matrix \( \Sigma \) are fixed throughout by assumption.) Hence, we have a mean-varying spread on the posterior distribution of the rate return for asset \( i \), i.e., \( r_i \). Therefore, \( x_i^{**} < x_i^{*} \), for every \( z = 1, ..., Z \), [see, e.g., Mitchell
and Douglas (1997)]. It can then be shown from the derivation in (5)-(13) that the conditional expected return, \( \nu_i(S_{t}^{**}) > \nu_i(S_{t}^*) \).


Economics, 24, 289-317.
Table 1: Firm-specific stock price responses to contemporaneous innovations in market volatility

For each firm, $i$, we estimate the following regression:

$$ R_{i,t} - R_{f,t} = \alpha_i + \pi_i \delta(\sigma_M) + a_i \text{age}_{i,t} + \eta_i [\delta(\sigma_M) \times \text{age}_{i,t}] + \beta_i (R_{m,t} - R_{f,i}) + s_i \text{SMB}_t + h_i \text{HML}_t + u_i \text{UMD}_t $$

where $\delta(\sigma_M)$ is the innovation in the volatility of the value-weighted market index at time $t$, $\text{age}_{i,t}$ is the natural logarithm of firm’s age at time $t$, $\delta(\sigma_M) \times \text{age}_{i,t}$ is the interaction of the previous two terms, $R_{i,t}$ is the rate of return for firm $i$ at time $t$, $R_{f,t}$ is the rate of return on a one-month Treasury bill at time $t$, $R_{m,t}$ is the value-weighted return of the stock market at time $t$, SMB$_t$ is difference in returns between portfolios of large stocks and portfolios of small stocks, HML$_t$ is the difference in returns between portfolios of stocks with high book-to-market ratios and those with low book-to-market, and UMD$_t$ is the difference in the returns of stocks with high (positive) momentum and those with low (negative) momentum. For robustness, alternative specifications are also estimated using a model that does not include the UMD factor.

Shown below are the cross-sectional averages of firm specific coefficients $\pi_i$, $a_i$, and $\eta_i$, with t-statistics computed in the cross-section.

<table>
<thead>
<tr>
<th>Model estimated</th>
<th>Cross-sectional average of $\pi_i$, the coefficient of $\delta(\sigma_M)$</th>
<th>Cross-sectional average of $a_i$, the coefficient of $\text{age}_{i,t}$</th>
<th>Cross-sectional average of $\eta_i$, the coefficient of $[\delta(\sigma_M) \times \text{age}_{i,t}]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full model including UMD</td>
<td>Point estimate: -3.76</td>
<td>t-stat: -6.42***</td>
<td>Point estimate: -0.0065</td>
</tr>
<tr>
<td>Reduced model without UMD</td>
<td>Point estimate: -2.93</td>
<td>t-stat: -5.04***</td>
<td>Point estimate: -0.0059</td>
</tr>
</tbody>
</table>

***, Statistically different from zero at the one percent significance level.
Table 2: Returns to portfolios formed on contemporaneous innovations in dispersion of analyst forecasts

Calendar time portfolios are formed each month ($t$) of the calendar according to contemporaneous changes in the dispersion of analyst forecasts occurring between month $t-1$ and month $t$. Portfolios are both equally weighted and value weighted according to the firm’s market capitalization in the previous month. Changes in dispersion are divided into five equal quintiles. Each month of the calendar, a firm is placed into one of five different calendar time portfolios according to the value of its contemporaneous change in dispersion quintile. Firms that experienced the most positive innovation in dispersion of analyst forecasts are placed in the portfolio corresponding to quintile 5. Firms with the most negative innovation are placed in quintile 1. The hedge portfolio at the bottom of the table takes long positions in the quintile 5 portfolio and short positions in quintile 1. The abnormal returns for each quintile portfolio are the intercept ($\alpha$) from the following four-factor regression:

$$R_{q,t} - R_{f,t} = \alpha + \beta (R_{m,t} - R_{f,t}) + s \text{SMB}_t + h \text{HML}_t + u \text{UMD}_t$$

where $R_{q,t}$ is the rate of return of the portfolio corresponding to quintile $q$ at time $t$, $R_{f,t}$ is the rate of return on a one-month Treasury bill at time $t$, $R_{m,t}$ is the value-weighted return of the stock market at time $t$, SMB$_t$ is difference in returns between portfolios of large stocks and portfolios of small stocks, HML$_t$ is the difference in returns between portfolios of stocks with high book-to-market ratios and those with low book-to-market, and UMD$_t$ is the difference in the returns of stocks with high (positive) momentum and those with low (negative) momentum.

The abnormal returns of the hedge portfolio are estimated with the following regression:

$$R_{1,t} - R_{5,t} = \alpha + \beta (R_{m,t} - R_{f,t}) + s \text{SMB}_t + h \text{HML}_t + u \text{UMD}_t$$

where $R_{5,t}$ is return of the portfolio corresponding to quintile 5 and $R_{1,t}$ corresponds to quintile 1. For robustness, alternative specifications are also estimated using a three-factor model that does not include the UMD factor.

<table>
<thead>
<tr>
<th>Model estimated</th>
<th>Dispersion change quintile</th>
<th>Portfolio weight</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Equally-weighted</td>
<td>Value-weighted</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Abnorm. Ret. (%)</td>
<td>t-stat</td>
<td>Abnorm. Ret. (%)</td>
</tr>
<tr>
<td><strong>Four-factor</strong></td>
<td>1 (decreases)</td>
<td>0.43%</td>
<td>3.63***</td>
<td>0.02%</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.92%</td>
<td>8.35***</td>
<td>0.31%</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.41%</td>
<td>3.74***</td>
<td>0.21%</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.09%</td>
<td>0.67</td>
<td>-0.23%</td>
</tr>
<tr>
<td></td>
<td>5 (increases)</td>
<td>-0.61%</td>
<td>-5.83***</td>
<td>-0.55%</td>
</tr>
<tr>
<td><strong>Three-factor</strong></td>
<td>1 (decrease)</td>
<td>0.05%</td>
<td>0.32</td>
<td>-0.11%</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.71%</td>
<td>5.91***</td>
<td>0.45%</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.13%</td>
<td>1.01</td>
<td>0.34%</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>-0.25%</td>
<td>-1.58</td>
<td>-0.22%</td>
</tr>
<tr>
<td></td>
<td>5 (increases)</td>
<td>-1.05%</td>
<td>-6.84***</td>
<td>-0.69%</td>
</tr>
</tbody>
</table>

**Hedge portfolio**

| | | **Four-factor** | | **Three-factor** | |
|---|---|---|---|---|
| | 1 minus 5 | 1.04% | 10.92*** | 0.57% | 3.16*** | |
| | 1 minus 5 | 1.10% | 11.86*** | 0.58% | 3.16*** | |

***, ** Statistically different from zero at the one and five percent significance level, respectively.
Table 3: Firm-specific stock price responses to contemporaneous innovations in dispersion of analyst forecasts

For each firm, $i$, we estimate the following regression:

$$R_{i,t} - R_{f,t} = \alpha_i + \pi_i \delta(disp)_{i,t} + \beta_i (R_{m,t} - R_{f,t}) + s_i \text{SMB}_t + h_i \text{HML}_t + u_i \text{UMD}_t$$

where $R_{i,t}$ is the rate of return for firm $i$ at time $t$, i.e., the rate of return between month $t-1$ and $t$, $R_{f,t}$ is the rate of return on a one-month Treasury bill at time $t$, $R_{m,t}$ is the value-weighted return of the stock market at time $t$, SMB$_t$ is the difference in returns between portfolios of large stocks and portfolios of small stocks, HML$_t$ is the difference in returns between portfolios of stocks with high book-to-market ratios and those with low book-to-market, and UMD$_t$ is the difference in the returns of stocks with high (positive) momentum and those with low (negative) momentum. $\delta(disp)_{i,t}$ is the change in dispersion of analyst forecast for firm $i$, between time $t-1$ and time $t$. For robustness, alternative specifications are also estimated using a model that does not include the UMD factor.

Shown below is the cross-sectional average of firm specific coefficients $\pi_i$, with t-statistics computed in the cross-section.

<table>
<thead>
<tr>
<th>Model estimated</th>
<th>Cross-sectional average of $\pi_i$, the coefficient of $\delta(disp)_{i,t}$</th>
<th>Point estimate</th>
<th>t-stat</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full model including UMD</td>
<td>-0.03484</td>
<td>-4.16***</td>
<td></td>
</tr>
<tr>
<td>Reduced model without UMD</td>
<td>-0.03958</td>
<td>-4.66***</td>
<td></td>
</tr>
</tbody>
</table>

***, Statistically different from zero at the one percent significance level.
Table 4: Cumulative Abnormal Returns around stock repurchase announcements

Mean and median Cumulative Abnormal Returns (CARs) are reported for firms announcing stock repurchases. CARs are measured for a three-day window centered on the repurchase announcement day. Results are reported for the entire sample of 1604 firms. CARs are estimated by summing the relevant daily returns of the event firm during the announcement window and subtracting corresponding return of the CRSP equally weighted market index, as explained in Brown and Warner (1985). T-statistics and p-values are shown in brackets, for mean and median CARs, respectively.

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Cumulative Abnormal Returns</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.02719</td>
</tr>
<tr>
<td></td>
<td>([t=11.26]^{***})</td>
</tr>
<tr>
<td>Median</td>
<td>0.01688</td>
</tr>
<tr>
<td></td>
<td>([p&lt;0.001]^{***})</td>
</tr>
</tbody>
</table>

\[***, **, *\] Significantly different from zero at the 1%, 5%, and 10% level, respectively (two tail test).
Table 5: Changes in Beta and its estimation error around stock repurchase announcements

Pre- and post repurchase-announcement betas and estimation errors are calculated using the following OLS market model:

\[ R_{i,t} = \alpha_i + \beta_i R_{m,t} + e_{i,t} \]

Two independent OLS regressions are estimated for each of the 1604 sample firms: (1) the 36-month period before and (2) the 36-month period after the repurchase announcement month (the month containing the announcement is excluded). At least twelve months of returns are required in both the pre- and post-announcement periods. The mean and median pre- and post-announcement betas (\( \beta \)) and estimation errors of beta (\( \sigma^2_\beta \)) are reported below. The estimation error (\( \sigma^2_\beta \)) is the standard error of \( \beta \) in the above regression. Also reported are the mean changes in beta and estimation errors across the pre- and post-announcement periods, along with t-statistics (calculated cross-sectionally), and the corresponding median changes along with p-values.

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Beta (( \beta ))</th>
<th>Estimation error of beta (( \sigma^2_\beta ))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Before</td>
<td>After</td>
</tr>
<tr>
<td>Mean</td>
<td>1.06698</td>
<td>0.83533</td>
</tr>
<tr>
<td></td>
<td>([t=-8.56]***)</td>
<td>([t=-12.62]***)</td>
</tr>
<tr>
<td>Median</td>
<td>1.00359</td>
<td>0.76932</td>
</tr>
<tr>
<td></td>
<td>([p&lt;0.001]***)</td>
<td>([p&lt;0.001]***)</td>
</tr>
</tbody>
</table>

***, **, *  Significantly different from zero at the 1%, 5%, and 10% level, respectively (two tail test).