The Valuation of Executive Stock Options when Executives Can Influence the Payoffs

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Abstract

It is widely believed that executives value stock options at considerably less than market or Black-Scholes-Merton values. This belief is contingent, however, on a subtle assumption that executives are, like shareholders, price takers with no ability to influence the underlying stock. But executives clearly have the ability to influence the stock, as that is the principal reason why they are granted the options. As such, executives are likely to be more willing to hold options than would ordinary investors, an important fact not captured in conventional models. We develop a model in which executives exert costly effort to alter the stock return distribution. We find that when executives act optimally, their options are worth much more than generally believed. We also find that consideration of effort can alter early exercise behavior, leading higher-quality executives to exercise later.

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1. Introduction

It is a widely accepted result that executives value stock options at lower than market or Black-Scholes-Merton values. This belief is due to the fact that executives are typically compelled to maintain large personal investments in their companies, their stock options cannot be traded, and the options usually contain vesting periods. These factors limit the executive's ability to diversify. Consequently newly-granted stock options are viewed as having less value to an executive than would tradable stock options have to an ordinary investor.

These arguments, however, ignore a significant factor that could greatly alter the executive's perception of the values of stock options. Models in the literature typically value these instruments from the perspective of the executive as simply a poorly diversified investor whose positions cannot be liquidated to achieve optimality. With the executive having no ability to materially change his portfolio, he values options and restricted stock at much lower than their conventional market values. In short, executives appreciate options and restricted stock less than do ordinary investors. But these models make one key and subtle assumption that is simply not true: they assume that executives are price-taking investors.

A price taker cannot influence the outcome of a risky decision. Financial models almost always assume that no investor can change the current or future price by trading or by any other actions. But executives can clearly influence the outcomes of their options. Indeed, the ability of executives to influence the stock price, the determinant of the option payoff, is the principal reason why the options are granted. If executives can influence the payoffs of their options, it is reasonable to believe that they will value their options more highly than if it is assumed they cannot. Indeed it is possible that executives might value their options more highly than would ordinary price-taking investors.

Naturally options are awarded to induce executives to take actions that will increase the value of the stock. It is certainly in the interest of the executive to push the price of the stock as high as possible. But executive actions that increase the price of the stock, such as working harder, have a cost in the form of disutility of effort. Executives cannot be presumed to be able or willing to push the stock price up without limit. Clearly they, as do all, trade off the rewards from extra effort against the costs.

It is interesting to note that in the literature, options have been regarded as somewhat less than ideal instruments for inducing executive effort. The reason is primarily that executives are

thought to value options at least than the cost to the firm. Consequently, options are considered to be fairly inefficient as incentive devices. But these conclusions have been drawn under the assumptions of conventional models of executive stock option value, that is, executives are assumed to be price takers. If we recognize that executives can influence the payoffs of their options, the incentives must surely be greater. Thus, academic research, which fails to provide strong support for the use of options as effective incentives, conflicts with practice, in which options are widely used. As we show in this paper, the reason may well be that models of executive stock options fail to capture a critical element of the attractiveness of these instruments to executives.

This paper examines the valuation of executive stock options under the assumption that executives can influence the outcomes of their options. We make the simple assumption that executives can shift the probability distribution of the stock. In accordance with the standard Modigliani-Miller approach, we assume that asset investment policy and thus risk is unchanged. In short, executives are able to render favorable outcomes more likely and unfavorable outcomes less likely. Upon being granted options, executives then decide on the effort they will expend to achieve this result. Their chosen effort is not observable, however, until a later time and consequently the stock price does not reflect this effort until it is observed. When that occurs, the stock registers an abnormal return. Executives must then maintain their effort to sustain a normal return thereafter. Following these assumptions we can then value their options. As we will see, their options are worth more than in the standard price-taking case. Indeed, in a few cases we will find that their options are worth more than would be valued by an outside investor. In other words, *executive stock options can even be worth more than the Black-Scholes-Merton value.* In addition, we examine their early exercise behavior in light of the incorporation of effort into the model.

In Section 2, we briefly review the previous research. In section 3, we develop a model for valuing stock options that incorporates the executive's quality and effort. In Section 4, we perform numerical comparative statics analysis of executive effort and option values. We summarize the implications and provide our conclusions in Section 5.

2. Previous Research

In the conventional principal-agent model, such as Grossman and Hart (1983), principals design a compensation package to maximize the expected payoff net of the cost of compensation subject to the constraint that agents choose their optimal efforts by maximizing their own expected utility. The effect of executive effort is reflected in the probability and/or the payoff of each outcome of the firm's projects. Starting from this fundamental intuition, many researchers

search for optimal sharing rules, or contracts, between agents and principals under different assumptions. For example, Holmstrom and Milgrom (1987) analyze the problem of intertemporal incentives in a continuous-time framework. They find that the principal problem can be solved under the static framework in which the executive can change the mean of the multivariate normal distribution and principals use a linear sharing rule. Following their analysis, we make a similar assumption about the effect of effort on the expected stock return.

Another stream in the executive compensation literature is to find the values of different components of the executive compensation package. This approach recognizes that executives are different from individual investors in several respects. For example, executives cannot sell short their firms' stock and legal requirements restrict their ability to hedge the risk of their stock and stock options. In addition, executives must follow other specific constraints, such as vesting periods or disclosure regulations. Therefore, conventional market-based valuation, with its few restrictions, is not appropriate for executive compensation. Lambert, Larcker, and Verrecchia (1991) recognize this problem and use a certainty equivalent approach to find option values while taking executive and firm characteristics into account. Hall and Murphy (2000, 2002), Henderson (2005), Hall and Knox (2004), and others also define the certainty equivalent values of stock options as executive values and analyze the relationship between executive values and other variables. Due to the differences between executives and individual investors, these authors conclude that undiversified risk-averse executives value their stock options at less than the market values of those options based on the Black-Scholes-Merton model.[1]

The conventional principal-agent model assumes that executives can exert effort to maximize firm value. The goal of a principal is to choose a compensation scheme to motivate an executive to exert the target, or desired, effort. If this is the case, then the effect of executive effort should affect the values of stock options to the executive. Cadenillas, Cvitanic, and Zapatero (2004) show that levered stock is an optimal compensation policy to induce optimal effort in many situations, such as for firms with high expected returns or large size or executives of high quality. They, however, do not analyze the effect of executive effort on executive option values. In addition, they do not consider the effect of other components of executive wealth, such as cash or the firm's stock, which can affect optimal effort. To bridge this gap, we take into

[1]There is an extensive body of literature that argues that executives value their options at lower than market value or firm cost. In addition to the papers cited above, see Kulatilaka and Marcus (1994) and Cai and Vijh (2005) for the comparison of option value between risk-averse and risk-neutral employees within a utility-based model and Detemple and Sundaresan (1999), Johnson and Tian (2000), Hall (2003), Meulbroek (2001), and Ingersoll (2006) for comparisons of option value to Black-Scholes-Merton value.

account cash and the firm's stock in the executive's portfolio and examine how effort affects executive option values.

In other research, Palmon et al (2004) develop an effort-based model in which cash flows are uniformly distributed and executive effort can shift the end points of the distribution. They use this model to examine the question of what is the optimal moneyness at which to issue options. They find that unless there are tax-related disadvantages, in-the-money options are better for shareholders.

Hodder and Jackwerth (2005) focus on similar issues. They assume that the executive has the ability to control the risk level of the firm, and develop a discrete-time model to value executive stock options. They find that the certainty equivalent values of these options are higher than the Black-Scholes-Merton values under some circumstances. Their model is closest in spirit to ours, but there are four major differences. First, they assume that the executive can dynamically control the stochastic process for the firm's value by using forward contracts to hedge the firm's risky technology. Under their assumption, however, the terminal return distribution can be trimodal, which is somewhat awkward and uncommon in financial models. In contrast, we assume that executives have the ability to influence firm performance by shifting the expected return without altering the risk.[2] Second, they incorporate the cost of executive effort by assuming a penalty function in the form of a lower boundary on firm value that triggers dismissal. We consider the cost of effort in the form of a quadratic disutility function commonly used in the literature. Third, in their analysis, the possibility of exercising early is independent of the time period, which is inconsistent with the general motivation of early exercise in which time is always a factor. The interest on exercise proceeds certainly affects the early exercise decision, and, therefore, should be taken into account in the behavior of early exercise. Finally, they assume that because hedging strategies are represented by the portion of the hedged assets, there is no difference in hedging strategies among executives. We argue that different executives have different abilities, which result in different outcomes from their effort. Therefore, we assume executives have different qualities that have diverse effects on firm values.

There are some researchers who focus on the effect of executive effort on the incentives of stock-based compensation. Schaefer (1998) develops a simplified agency model and derives the functional form for optimal effort and finds that optimal effort is positively related to firm size

[2]In this assumption, we want to reflect a means by which executive effort increases shareholder wealth. Executives can influence either expected return or stock volatility or both. To simply the analysis and distinguish our paper from others, we assume executives can influence only expected return. Holding risk constant to analyze the effect of a factor is a common approach in financial theory as in the original Modigliani-Miller analysis of the effects of financial leverage and dividend policy.

and marginal productivity of effort, but negatively related to risk aversion and the variance of firm value. Feltham and Wu (2001) analyze the incentive effects of stocks and options with consideration of executive effort. Under the assumption of a normally distributed terminal stock price, they find that the number of options granted to induce a certain level of effort increases with the exercise price when the effort does not influence the firm's operating risk. They conclude that the cost of compensation increases with the exercise price. If the effort influences both the mean and the variance, then conclusions about incentive effects of stock and options depend on the impact of effort on firm risk. When the impact is large, then the compensation cost decreases with the exercise price.

There are two major differences between the Feltham and Wu model and ours. In their model, the executive has only stock or options and, therefore, they do not consider the effect of the other component of the agent's wealth. This consideration can affect the number of shares of stock or options needed to induce executive effort in their analysis. In addition, their assumption of normality for the terminal stock price is not consistent with the conventional lognormal assumption of the stock price distribution, which is what we use.

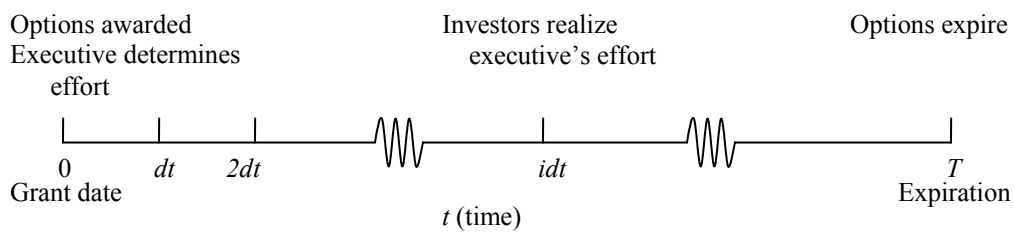
Lambert and Larcker (2004) use a principal-agent model to find the optimal contract and compare their results with those in Feltham and Wu (2001). They find that option-based contracts in general dominate restricted stock-based contracts and that most options in the optimal contracts are out-of-the-money options. In addition, they also note the invalidity of the first order condition in the agent's maximization problem. They argue that expected utility is not a concave function of executive effort when the convexity of the option's payoff dominates that of the agent's disutility of effort. We find that one major reason for this problem is because the number of options increases with the level of effort in their model. In contrast, we fix the number of options granted to an executive, and then find the optimal executive effort. Therefore, the first-order condition is valid in our model. To verify this result in Section 3, we examine the executive's expected utility within a reasonable range of executive effort and show that it is a well-behaved concave function of effort.

3. Theoretical Model

We start the analysis with a time line sequencing an option granted to an executive who then maximizes his negative exponential utility to determine his optimal effort. We assume that the executive effort decision is not immediately observable, but over time that effort is revealed and manifests in an abnormal return to the stock. Ultimately we will be able to value the option as the certainty equivalent cash award that he would accept in lieu of the optimal.

3.1. The Determination of Optimal Effort

As noted above, executive effort is not directly observable but should be reflected in the stock price when it leads to improved firm performance. When the effect of effort is realized, investors then adjust the expected return. Because the executive's quality and effort are private information, we assume that there is no stock price reaction on the grant date.[3] At a point prior to maturity, the executive's effort will be identified by investors and will be reflected in abnormal performance of the stock. The diagram below specifies the sequence of this process.



At time 0 the firm grants stock options with a maturity T. The executive then decides on his optimal effort over the lifetime of the options. At time 0, the current stock price is S_0 and the expected return is $E(r)$. At time idt, the effect of the executive's effort is reflected in the stock price and results in an abnormal return over the period of $t = 0$ to idt. From that point forward, investors then price the executive's effort into the stock, so no further abnormal returns arising from this grant would occur.[4]

Let S_t be the stock price that would exist in the absence of an option grant. We are interested in determining the stock price that would exist if the option is awarded and the executive chooses to put forth additional effort as a result of receiving the options. As noted, the market determines the results of the executive's effort at time idt and the stock price changes to

$$S^*_{idt} = S_{idt} q^{\delta}, \quad q \geq 1, \tag{1}$$

where S^*_{idt} is the stock price after taking the effort into account at time idt.[5] S_{idt} is the stock price on the date idt with minimum effort equal to one, q is the measure of executive effort over the

[3]The evidence on this issue is somewhat mixed and is complicated by several factors. Grant awards are rarely discovered by investors until a later date, grant dates can be manipulated around the announcement of good news (Yermack (1997)), and grand dates can be backdated (Lie (2005) and others). Nonetheless, we do not have to make any assumptions about how long it takes investors to recognize the executive's effort and adjust the stock price, so that effort could be realized fairly quickly.

[4]In a multiperiod framework, executives could receive additional grants that could motivate them to re-calibrate their efforts to the aggregate incentive package held by them and potentially lead to subsequent abnormal returns. Extension of this model to a multiperiod setting would be a fruitful arena for future research.

[5]This type of model has been widely used in the literature. Camara and Henderson (2005) use it to analyze the manipulation of stock price and accounting earning. Palmon et al (2004) assume use a similar model in which executives can exert effort to increase the upper and lower bound of the cash flow distribution.

period of time $t = 0$ to idt, and δ is the measure of executive quality, which is the elasticity of the stock price with respect to executive effort, $\delta \geq 0$.[6] Under the same effort, the higher the δ, the higher is the after-effort stock price. Thus, δ is a direct measure of the quality of the executive, and it is an observable parameter at the start.[7]

We need not specify when the effort is expended during the interval $t = 0$ to idt. It could occur early, late, or spaced evenly throughout. We assume that investors do not realize the results of the effort until the end of the interval, and these results then manifest in an abnormal return. Keep in mind that the executive continues to expend the effort after $t = idt$, but it generates no abnormal return beyond that point because investors are now aware of the executive's effort and build it into the stock price. In fact, if the executive fails to expend the effort after $t = idt$, there will be a negative abnormal return.

We assume the stock price without effort, S_t, follows the Geometric Brownian motion process

$$dS_t = \alpha S_t dt + \sigma S_t dw_t,$$

with α the drift, σ the standard deviation of the raw stock return, and w_t a standard Brownian motion. We assume the continuously compounded Capital Asset Pricing Model (CAPM) holds so that $\alpha = r_f + \beta(E(r_m) - r_f)$, where r_f is the risk-free rate, β is the measure of systematic risk, and $E(r_m)$ is the expected market return. In this paper, we also assume that early exercise decisions have no effect on the executive's choice of optimal effort.[8] Therefore, executives determine their optimal effort immediately after accepting the option. We can view Equation (1) in terms of the expected return by dividing by S_0 and taking the expectation of the log returns:

$$E \ln\left(\frac{S^*_{idt}}{S_0}\right) = E\left(\ln\left(\frac{S_{idt}}{S_0}\right)\right) + \delta \ln q. \tag{2}$$

Under the geometric Brownian motion assumption, the log return without effort follows a normal distribution with mean μ and standard deviation σ. We define μ^* as the after-effort expected log return and η as the incremental expected return resulting from executive effort:

Grout and Zalewska (2006) apply a similar assumption that the mean of the terminal firm value increases by ε when executives exert the additional effort. Also, they assume there is no impact on the variance of the distribution.

[6]When executives exert minimum effort, $q = 1$, the stock price is independent of executive quality. Later in this paper, when we refer to the elasticity of stock price, we mean the elasticity of stock price with respect to executive effort.

[7]This interpretation is the same as that in Cadenillas, Cvitanic, and Zapatero (2004). They mention that δ is an indicator of the quality of the executive.

[8]In Section 3.3., we analyze the effect of optimal effort on the early exercise decision. To limit the interaction between optimal effort and the early exercise decision, we make the assumption that there is no effect of early exercise on optimal effort.

$$\mu^* idt = (\mu + \eta) idt. \tag{3}$$

Therefore, we relate executive effort to incremental expected return as follows:

$$\eta idt = \delta \ln q \Rightarrow q = e^{\eta idt/\delta}. \tag{4}$$

From Equation (4), we know that the executive's effort is exponentially related to incremental expected return, the period of time, and the quality of the executive.[9] As noted, the market price converges to the price that reflects effort at $t = idt$. Any price prior to that time, such as S_{dt}, does not reflect effort. Nonetheless, the executive will have a private opinion of the price, S^*_{dt}, which by recursive evaluation, will equal, $S_{dt}q^{\delta}$.[10] Because the executive has a private awareness of his effort and its impact on the value of the stock, the executive will be more optimistic about the future prospects for the stock and the option than would other investors.

To understand what is happening, consider that while we hold the risk constant the executive's effort shifts the expected return upward. The executive's effort increases the probabilities of states in which higher returns are received and decreases the probabilities of states in which lower returns are received. But we must be careful about the terminology used. Because we observe the results of the executive's effort in the abnormal return, we must be cautious in our definition of the expected return. The executive shifts the distribution by η. If we incorporate η into the expected return, there would naturally be no abnormal return. Thus, prior to $t = idt$, we need to clearly distinguish the expected return without effort from the expected return with effort, the latter of which is not observed by investors.

From the conventional CAPM, we know that the expression $r_f + \beta(E(r_m) - r_f)$ is commonly referred to as an *expected* return but is more accurately described as a *required* return. In a perfect market, of course, the two are equal, but our model is characterized by information asymmetry, so we must distinguish these concepts. Because executives have private information about their quality and effort that can influence firm performance, the expected return from the executive's standpoint is designated as $E(r^*)$, which is the sum of $E(r)$ and η. We will call this measure the *expected return with effort*. Before the executive effort is revealed, $E(r)$ is different from $E(r^*)$ by the incremental expected return. At time idt, $E(r)$ converges to $E(r^*)$ and η is zero. Executives still exert the same level of optimal effort but there is no longer an incremental expected return.

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[9]If $\delta = 0$, then $\eta = 0$, and the stock price process becomes the original process with minimum effort of $q = 1$. Therefore, the case of $\delta = 0$ has the same effect on expected return as $q = 1$, even though the interpretations of these two cases are different.

[10]At time idt, $S_{idt}^* = S_{idt}q^{\delta}$. One period prior to time idt, $S_{(i-1)dt}* = E(S_{idt}*)e^{-(\alpha+\eta)dt}q^{\delta} = S_{(i-1)dt}q^{\delta}$. Continuing back to any time jdt gives $S_{jdt}^* = S_{jdt}q^{\delta}$, which is the executive's private assessment of the value of the stock and reflects the additional information he knows about his effort.

Upon receipt of the grant, the executive decides on the optimal effort. We assume the executive has three components of his portfolio, which are $\$c$ in cash, m shares of the firm's stock, and n stock options. The terminal wealth is

$$W_T = c\left(1+r_f\right)^T + mS_T^* + nMax\left(S_T^* - K, 0\right),$$

where T is the maturity of the options, r_f is the risk-free rate, S_T^* is the terminal after-effort stock price, and K is the exercise price of n options. As described, we assume that the executive can affect firm value by choosing his level of effort and therefore increasing the return of the stock, which increases the return on the executive's stock-based compensation.[11] If effort were costless, however, the executive would clearly increase the stock price without limit. Therefore, we must impose a cost to effort. We do this in the form of disutility function. Following the literature, we specify disutility as a quadratic function of effort.[12] To analyze the trade-off relationship mentioned above in the expected utility model, we represent executive effort and disutility in terms of incremental expected return. Therefore, the disutility function of effort is

$$C(q) = \frac{1}{2}q^2 = \frac{1}{2}e^{2\eta T/\delta}. \tag{5}$$

Recall that $q = e^{\eta idt/\delta}$ where idt is the period over which the effort converts into the abnormal return. As noted before, investors become aware of the results of executive effort at time idt and adjust the expected return to the expected return with effort. After that, there is no longer an abnormal return from executive effort but executives must maintain the same level of effort. Otherwise investors can identify the reduction of effort at the next observation point and there will be a negative abnormal return. To compute the disutility of effort over the entire option life, we assume that the total effort from time 0 to time T is the product of effort in each interval of idt, that is, $\eta T = \eta idt$. Thus, from Equation (4), total effort can be expressed as $q = e^{\eta T/\delta}$.

To examine the impact that effort has on option valuation, we must compare the results of this model and standard models that do not take effort into account. Thus, we need consider only

[11]In general, if the effort comes from the executive's ability, then it should have a long-term effect. In contrast, if the effort comes from inside information or stock price manipulation, then its effect should last only for a very short term. Because effort is unobservable, the market will know the executive's effort gradually through observing other proxies for the effort and updating the information in the stock price. If the effort comes from insider information or manipulation, then the stock price will reflect the information immediately after it becomes public. Therefore, the effect of this kind of effort exists in only the short term.
[12]Interested readers can refer to Baker and Hall (2004), and Cadenillas, Cvitanic, and Zapatero (2004). Some researchers use a modified version of a quadratic function in the agency model for tractability. See Prendergast (1999).

the disutility of the effort beyond $q = 1$. Therefore, we can write the disutility function (equation (5) as[13]

$$C(q) = \frac{1}{2}(q-1)^2 = \frac{1}{2}\left(e^{\eta T/\delta} - 1\right)^2 .$$

We assume the executive has negative exponential utility with coefficient of absolute risk aversion ρ:

$$U(W_t) = -\frac{1}{\rho} e^{-\rho W_t} .$$

The executive determines the optimal effort by maximizing the expected utility with respect to terminal wealth net of the disutility of effort, which is

$$\underset{q}{Max}\ E\left(U\left(W_T\right)\right) - C(q). \tag{6}$$

In the objective function, we assume additively separable utility for terminal wealth and the disutility of effort.

3.2. Executive Option Values

To operationalize the model, we need to be able to identify the effort that an executive would expend and, ultimately, the value of the option, taking that effort into account. In a utility-based model, the typical method of finding the value of an executive stock option is the certainty equivalent approach. The option value is viewed as the cash amount, *CE*, at the valuation date that has the same expected utility as the option. Therefore, without considering effort, the option value would be determined by solving for *CE* in the following equation,

$$\begin{aligned} &\int_0^\infty U\left(\left(c + CE\right)\left(1 + r_f\right)^T + mS_T\right) f\left(S_T\right) dS_T \\ &= \int_0^\infty U\left(c\left(1 + r_f\right)^T + mS_T + nMax\left(S_T - K, 0\right)\right) f\left(S_T\right) dS_T . \end{aligned} \tag{7}$$

The value of one option is, therefore, CE/n. This is the standard approach taken in models that value the option as a certainly equivalent without the effect of effort.

There are, however, some differences between our approach and that of others. Note that on the left-hand side, the executive receives a cash award, CE, while on the right-hand side, he receives options. The cash award is merely a technical mechanism for identifying the amount of cash that the executive would take that would have the same expected utility as the option and would, therefore, have the same value as the option. But if cash were actually granted in lieu of

[13]We assume that the cost of minimum effort is zero. Because the incremental expected return is zero in the minimum effort case, the original expected return is determined by co-movement with the market, which is out of the executive's control. Therefore, no extra cost is needed in the minimum effort case.

options, the cash would clearly imply lower incentives. The executive would have some incentives with a cash grant, however, because of stock ownership, but clearly stock and options would have more incentives than stock and cash. This problem, which is common to all models of this type, has been recognized by Hall and Murphy (2002) by acknowledging that the stock price distribution does not change when options are granted.

Our approach deals with this problem by incorporating the shift in the stock price that results from executive effort. Our left-hand side reflects cash and stock, with stock providing some incentive and, therefore, some shift of executive effort. Our right-hand side reflects options and stock, with both stock and options proving a shift of executive effort. The differences in incentives are accounted for in our model. Because the effort that maximizes expected utility determines the stock return distribution, which then determines the certainty equivalent value of the option, we first need to determine the optimal effort. Accounting for both the utility benefits of effort and the utility cost, our option value is *CE* in the following:

$$\begin{aligned}&\int_0^\infty U\left(\left(c+CE\right)\left(1+r_f\right)^T+mS_T^{1*}\right)f\left(S_T^{1*}\right)dS_T^{1*}-\frac{1}{2}\left(e^{\eta_1 T/\delta}-1\right)^2\\&=\int_0^\infty U\left(c\left(1+r_f\right)^T+mS_T^{2*}+nMax\left(S_T^{2*}-K,0\right)\right)f\left(S_T^{2*}\right)dS_T^{2*}-\frac{1}{2}\left(e^{\eta_2 T/\delta}-1\right)^2.\end{aligned} \tag{8}$$

In Equation (8), S_T^{1*} is the stock price after taking into account the optimal effort from stock ownership, and S_T^{2*} is the stock price after taking into account the optimal effort from both stock and option ownership. Then η_1 and η_2 are the corresponding incremental expected returns on the left-hand and right-hand sides respectively. We use this approach to find the CE for the continuous-time case of no early exercise. Details are provided later. When the options are exercisable early, we take a similar but slightly different approach.

3.3. Early Exercise

Other factors that can affect the executive option value are early exercise and the vesting schedule. Early exercise and optimal effort combine to raise an interesting question about which comes first. Does the ability to exercise early affect effort or does effort affect the early exercise decision? It is impossible to definitively state that one is cause and one is effect. We assume that effort is chosen independent of the early exercise feature, which means that optimal effort would be no different for American options than for European options. But we then assume that effort can affect the decision of when to exercise early. If early exercise occurs, we invest the proceeds in the risk-free asset until the maturity date of the options. The executive will exercise early when the expected utility of early exercise is higher than that from holding the options.

As is commonly required when analyzing the early exercise of standard options, a numerical method is used to capture the early exercise decision. We use a binomial tree. The expected utility at each node in a binomial tree after time t is

$$E\left(U\left(W_{t-1}\right)\right) = Max\left\{pE\left(U\left(W_t^u\right)\right) + (1-p)E\left(U\left(W_t^d\right)\right), E\left(U\left(W_{t-1}^E\right)\right)\right\},$$

Where $U(W_t^u)$ and $U(W_t^d)$ are the utilities at time t with up and down moves respectively, and $U(W_{t-1}^E)$ is the utility from early exercise. Following this rule, we can find the expected utility considering early exercise at time 0. Then, the value of the options is the cash amount received at time 0 and invested in the risk-free asset that provides the same expected utility.

Our objective is to determine how option values differ when we account for the ability of the executive to influence the payoff of the option. We will do that by assuming a reasonable range of values of the inputs required by the model.

3.4. Parameter Assumptions and Estimates

We examine a range of reasonable values for the model inputs. There are twelve parameters in the model, which can be grouped into Black-Scholes-Merton variables, CAPM variables, and executive properties.

3.4.1. Black-Scholes-Merton Variables

The Black-Scholes-Merton variables are the current stock price, S_0, the exercise price, K, the risk-free rate, r_f, the volatility of the stock return, σ, and the time to maturity, T.[14] Using Standard and Poor's ExecuComp, we find that between 2000 and 2005 more than 99% of stock options were granted at-the-money. The mean exercise price in 2005 was $32.47 and the median was around $29.20. We use $30 as the exercise price. We use at-the-money options as the benchmark but also consider out-of-the-money and in-the-money options by examining stock prices of $20 and $40, respectively. These options can reflect the effects of issuing premium and discount options. For the risk-free rate, the three-month T-bill rate is 4.95% and 10-year treasury maturity rate is 5.09% in July 2006. We use 5% as the risk-free rate in our simulations.

The average volatility reported in ExecuComp between 2000 and 2005 to compute the Black-Scholes-Merton value is 47%. We use 50% as the benchmark volatility and 30% and 70% to represent lower and higher volatility companies, respectively. The most common time to maturity for original issue executive stock options is ten years. To examine how option values change over their lives, we examine ten-, seven-, and five-year options. Although companies do not typically issue seven- and five-year options, examining these maturities can give us an idea of the effect of issuing options shorter than the standard period. In addition, the common but

[14]To simplify the analysis and focus on the issue of optimal effort, we assume no dividends.

simplistic view that options are exercised with certainty after a fixed number of years can be somewhat applicable to this assumption, though we will address early exercise directly.

3.4.2. CAPM Variables

There are three variables in the traditional CAPM, which are the risk-free rate, the expected market return, and the systematic risk, beta. The risk-free rate is the same as that mentioned in the previous section. We use the value-weighted return on all NYSE, AMEX, and NASDAQ stocks as a proxy for the expected market return. The average market return from 1992 to 2005 is 11.88%.[15] Therefore, we use 12% as the benchmark for the expected market return. To observe how changes in market conditions affect executive effort and the values of executive stock options, we also run the simulations under 10% and 14% expected market returns. We use a beta of 1.0 as the benchmark and betas of 0.5 and 1.5 to show the results for firms with different levels of systematic risk. To conserve space and focus on the most important variables, we do not report or discuss the results with high and low values of beta and the expected market return. The variation is not found to be sufficiently large to justify any attention.

3.4.3. Executive Characteristics

In this model, as well as in many similar models in the literature, there are three components of the executive's personal wealth: cash, the firm's stock, and stock options. As noted, we assume the executive has negative exponential utility, which has the characteristic of constant absolute risk aversion. Moreover, the elasticity of the stock price is also a crucial component in our model in relation to others. Therefore, we establish a benchmark value for the elasticity of the stock price, the amount of non-option wealth, the number of shares of stock and options, and the coefficient of absolute risk aversion.

Kim, Lin, and Park (2008) estimate that the change in stock return divided by the change in sales deflated by market value of equity is 0.8599. Multiplying this figure by their mean market value of equity of 1,419.41 and dividing by their mean sales figure of 1,997.53 gives 0.611 as an estimate of the elasticity of stock price with respect to sales. For convenience, we round off to 0.625. Bitler at el. (2005) estimate the elasticity of sales with respect to working hours as 0.40. We then multiply these two figures to get 0.25 as a benchmark estimate of the elasticity of stock price with respect to working hours, which is our proxy for the elasticity of stock price with respect to effort. We also use 0.1 and 0.5 to represent low and high quality executives. Obviously our model assumes that executives do not learn or improve over time, but

[15]The data comes from the data library on Kenneth French's website. The data range from 1992 to 2005 and are consistent with the data in the ExecuComp database. The average market return from 1927 to 2005 is 12.20%.

this assumption seems appropriate for an initial effort to understand how executive quality and effort affect option values.

Due to vesting requirements and possibly a negative signaling effect, most executives hold more than the optimal level of their firms' stock. Therefore, the stock component of non-option wealth should be higher than the optimal level in the benchmark. Based on the optimal holding of risky assets from Merton (1969), the optimal holding in the benchmark is 24%.[16] We assume the executive invests 40% of his wealth in the firm's stock as a benchmark to show that the executive bears higher than optimal firm-specific risk. In addition, we extend the stock-wealth ratio to 30% and 50% for low and high stock holdings. From ExecuComp, we find that the average stock wealth in year-end 2005 is $39.2 million, but the median is $1.6 million. Using the median stock wealth and 40% stock-wealth ratio assumption, we use $4 million as a benchmark for total non-option wealth. The less and more wealthy executives are assumed to have $2 and $6 million in their non-option wealth, respectively. The number of shares of stock is equal to the stock wealth divided by the current stock price.

From ExecuComp, we find that the median number of options granted in executive compensation is 21,000 and the mean is 78,970. So the distribution of options granted is highly skewed. When we use only the CEO in the database, the median and mean are 60,000 and 191,000 respectively. We use the median of these grants and set the number of granted options equal to 40,000 and use 20,000 and 60,000 options to observe the effect of low and high option grants.

The last executive parameter is the coefficient of risk aversion. From Pratt (1964), the relation between absolute risk aversion, *ARA*, and relative risk aversion, *RRA*, is

$$ARA_t = \frac{RRA_t}{W_t}.$$

The commonly used value of *RRA* is from 2.0 to 4.0. We use *RRA* = 2.0 as the benchmark and *RRA* = 1.0 and 3.0 to represent lower and higher relative risk aversions. Because negative exponential utility has the characteristic of constant absolute, rather than relative, risk aversion, the coefficient of *ARA* is 0.0000005 in the benchmark and 0.00000025 and 0.00000075 are for *RRA* = 1.0 and *RRA* = 3.0, respectively.

3.4.4. Other Issues and Variables

[16]The optimal holding of risky assets is the expected return divided by the product of relative risk aversion and the variance of the stock returns. In our benchmark case, the expected return is 12% and variance is 25%. Assuming the coefficient of relative risk aversion is 2, the optimal holding of the firm's stock is 24%.

We use a monthly time step, which means $h = 12$, in the binomial model.[17] The optimal effort and executive option values are lower than those in the continuous-time model. The difference does not change our qualitative results. As noted previously, Lambert and Larcker (2004) identify a technical issue concerning the validity of the first-order condition in solving the agent's problem. Because we use the first-order condition to find the optimal effort, we examine whether this problem occurs in our model. To verify that the executive's expected utility is well-behaved, we use the parameters in the benchmark and examine the relationship between expected utility and effort. We find the expected utility is a well-behaved concave function with respect to executive effort. Therefore, within the parameters chosen, the first-order condition is valid in our model. When we examine early exercise, we specify vesting periods of two, three, and four years, and focus only on 10-year options.

4. Simulation Results

To identify the effects of our model, we conduct simulations using the various ranges of input parameters. These simulations are based on the condition that upon receipt of options, the executive maximizes his expected utility by choosing the optimal level of effort. Given the chosen effort along with the other input variables, the value of the option can be derived as the certainty equivalent. A description of the full simulation procedure is contained in Appendix I.

4.1. Optimal Executive Effort

The optimal effort for the various parameter values is shown in Table 1. The columns labeled "Lower value," "Benchmark, and "Higher value" refer to the assumption of the lower of the three values of the input we vary, as shown in the first column. Within each column of the three main columns, we show results for three maturities: ten, seven, and five years. Within each cell, three moneyness levels are shown vertically with the top value representing the lowest stock price (out-of-the-money) and the bottom value representing the highest stock price (in-the-money). Six variables are indicated in the first column of Table 1. Combined with variation in maturity and moneyness, we can observe the effects of eight variables on effort.

In all cases, optimal effort is greater than one, which means that it is always optimal for the executive to exert effort beyond that required to maintain the current stock price. Thus, at that minimum level of effort, the marginal benefit of effort exceeds the marginal cost. As expected, the optimal level of effort varies widely for different inputs, being in some cases three times as high as in others.

We find that optimal effort decreases with moneyness, which means that out-of-the-money options induce more effort than in-the-money options. With out-of-the-money options,

[17]The qualitative results do not change when we use a weekly time step, where $h = 52$.

there is a lower probability of the options expiring with value, so executives recognize that additional effort is required. But in all cases the differences in effort by moneyness are quite small. Therefore, moneyness does not appear to be a strong factor in motivating executives, but we will return to this issue later.

We also see that optimal effort decreases with maturity, and the effect is quite strong. That is, longer term options result in lower effort. When the maturity is shorter, there is less time for the option to expire in-the-money. The executive is, therefore, motivated to work harder. In contrast, if the maturity is longer, such as ten years, there is more time for the options to expire in-the-money. The executive's odds of success are much more favorable for a given level of effort with a longer-term option. Moreover, the executive benefits from the mere passage of time and the positive drift of the stock price. In contrast, with a shorter time to expiration, the executive perceives that additional effort on his part may be necessary to boost the option's payoff. In addition, disutility of effort is greater the longer the period over which the effort is expended. In short, the executive is far more willing to work harder over a short period of time than over a longer one, and, therefore, companies might wish to consider shortening the original issue terms of their options.[18]

Effort changes in a slightly positive manner with the stock-wealth ratio and in a slightly negative manner with non-option wealth. This result makes sense: if the executive's options represent only a small portion of his overall wealth, he will not be motivated to work hard. Effort also varies sharply and inversely with risk aversion. This result is also logical as more risk averse executives are less inclined to make the effort. The number of options exerts a moderately negative effect on effort. Hence, the more options awarded, the less effort exerted., though the effect is not strong.

Executive effort has a concave relationship with the elasticity of stock price, with the greatest effect occurring at medium elasticity. This implies that medium quality executives work harder than low or high quality executives. This result has different explanations for different quality executives. From Equation (4), executive effort is negatively related to δ, which means executives with high quality exert less effort. That is what we observe from benchmark quality to high quality. For executives with low quality, one unit of their effort from low quality executives has the same disutility as others but less influence on stock price. It is optimal for them to exert less effort.

[18]Recall that we do not allow executives to reset their effort as the option approaches maturity. Effort is determined at the grant date. But they suggest that if we allowed executives to reset their effort through time, effort would probably increase if moneyness and all other factors were held constant.

Optimal effort in general has a moderately positive relation with volatility. Volatility is well-known to have somewhat counterintuitive effects on executive stock option values. On the one hand, higher volatility makes options more valuable, but on the other, these options are valued by expected utility, not by arbitrage. Therefore, higher volatility also has a negative effect on option value, as well as stock value.[19] Therefore, the effect of volatility on effort reflects the combined effects of the stock and the options. From Table 1, holding the firm's stock, which is represented by the stock-wealth ratio, has a slightly positive effect but the number of options has a slightly negative effect on effort.

In Table 1 we observed that optimal effort is moderately negatively related to moneyness. That is, out-of-the-money options lead to greater effort. We explore this result further here by computing the optimal effort with respect to different exercise prices and show this relationship in Figure 1. There are two major findings. First, we observe a positive asymptotic relationship between optimal effort and exercise price in the 30% and 50% volatility cases. This result implies that out-of-the-money options induce more effort than in-the-money options for low to medium volatilities, which is consistent with our findings in Section 4.1. When the exercise price is higher, it decreases the option's payoff. Therefore, greater effort is required to make the options expire in-the-money. Figure 1 also shows that options with positive exercise prices induce more effort than those with an exercise price of zero. This result implies that restricted stock, which is equivalent to an option with zero exercise price, induces less effort than ordinary stock options, which of course have positive exercise prices.

In the high volatility case, however, we find a counterexample to our previous conclusion. Beyond a point, effort decreases with exercise price. The benefit of high volatility is such that a higher exercise price does not induce greater effort. A similar finding was reported by Feltham and Wu (2001). From the 70% volatility case in Figure 1, the highest effort exists at the exercise price of $14 rather than the at-the-money exercise price of $30. In addition, optimal effort increases with exercise prices from $0 to $14. When the exercise price exceeds $14, however, effort decreases with the exercise price because the marginal effect of the effort decreases. In this range, the positive effect of wealth reduction on effort is dominated by the negative effect of increasing exercise price on effort. Therefore, the executive exerts less effort if the exercise price is higher than $14. We illustrate the trade-off relationship between both effects on the executive's effort in Appendix II.

[19]Of course, with traded and arbitrageable options, volatility exerts a strictly positive effect on option value. The negative impact of volatility on expected utility is embedded entirely in the stock price. Thus, arbitrage-based valuation of traded options will reflect only a positive effect of volatility.

To summarize our findings, the variables that have the greatest impact on effort are maturity, non-option wealth, and risk-aversion with greater effort found with shorter maturity options, executives with lower non-option wealth, and less risk-averse executives. Slightly greater effort is found for medium quality executives, executives of more volatile firms, executives with higher stock-wealth ratios, and executives with fewer options. Out-of-the-money options are slightly more effective at motivating executives to work harder.

4.2. Option Values

To determine how executive effort affects the values of the options, we compute option values with optimal effort, $\delta > 1$, and with minimum effort, $\delta = 1$. We summarize the values with optimal effort (Panel A) and without (Panel B) and the Black-Scholes-Merton values (Panel C) under different parameters in Table 2.

First, we find an interesting and counterintuitive result. Executive option values are inversely related to time to expiration, which is opposite that of standard option valuation intuition.[20] Holders of standard options, of course, benefit from longer time to expiration. If they need to liquidate prior to expiration, they can sell their options. But executives in need of liquidity cannot sell their options. Hence, the liquidity penalty is a heavy one that dominates the time value benefit. As expiration approaches, however, the option value increases because the liquidity penalty is smaller. We will see later that this effect will vanish when the options can be exercised early.

Second, comparing Panels A and B, we find as expected that option values increase after taking optimal effort into account. In some cases, the differences are quite large. For example, our benchmark 10-year option is worth \$1.31 without accounting for effort and \$2.97 after accounting for effort. The seven-year option is worth \$1.76 without effort and \$4.85 with, and the five-year option is worth \$2.17 without effort and \$7.32 with. This result casts a new light on the standard view on executive option valuation whereby executives do not value these options at their market values because of illiquidity, vesting, and the poorly diversified nature of their personal wealth. But those models ignore the fact that executives have influence on the outcomes of the options. They know that with additional effort to the point where marginal effort equals marginal cost, these options can be made more valuable. Thus, we should not attempt to determine executive option values by treating executives as though they were ordinary investors merely constrained by poor diversification and illiquid holdings.

[20]This result is not unique to our model. When we use 50% volatility in the model of Hall and Murphy (2002) and keep other parameters the same as their assumptions, the option value also decreases when we change the maturity from 10 years to 15 years.

Third, the effect of effort is such that we even observe cases in which *executive option values are greater than Black-Scholes-Merton option values*. For example, we find in Panel A that executive option values are higher than Black-Scholes-Merton option values when the volatility or risk aversion is low and the maturity is seven years or less. This result means that executives like some options even better than do ordinary investors, in spite of the options being illiquid. When the ability to affect the payoff is considered, it should not be surprising that executives like options better than other models suggest.

This result has an interesting implication for the issue of accounting for executive stock options. The Financial Accounting Standards Board (FASB) suggests that the fair value of a stock option be estimated by taking into account the expected life of the option (FAS123R, A26-A30). The Black-Scholes-Merton model is widely used as a methodology. Suppose, for example, that the expected life of a new 10-year option is five years, the volatility is 30%, and all other inputs are their benchmark values. From Table 3, Panel C, we see that the Black-Scholes-Merton value would, therefore, be $10.69. But the executive would perceive the option as a 10-year option worth $12.52. If the executive accounts for the fact that he expects to exercise this option in five years, he sees it as having a value of $23.94, more than twice the recorded expense. This effect is not seen without accounting for effort, wherein the executive would value the option at only $4.65 as a 10-year option and $5.25 as a five-year option. Thus, the recorded option expense would be about considerably less than the value perceived by the executive.[21]

Table 3 shows the ratio of option value with effort to value without effort. We see that option values with optimal effort are consistently higher than those with minimum effort, in most cases more than twice as high. This result holds for both in-the-money and out-of-the-money options for all most variations of parameters, the exception primarily with low elasticity. Interestingly, the ratios decrease with maturity, which is consistent with the pattern we observed for optimal effort in Table 1. We know that the executive exerts more effort when the maturity is shorter. This additional factor magnifies the difference between option values with and without optimal effort. Of course it is not surprising that the difference in values is greater when the executive's quality is high. Even though high quality executives exert less effort, they still value their options higher than other executives. The ratios are also higher the lower the volatility, the lower the non-option wealth, the higher the stock-wealth ratio, the lower the number of options, and the lower the risk aversion, but elasticity and volatility are the only variables that have a strong effect. The effect of moneyness is inconsistent but in most cases, it has an inverse effect.

[21]In Table 2, we do not permit early exercise but to do so would make the point even stronger. As we will show in Table 5, the value of this option if it is exercisable early is at least $15.

As noted above, option value to the executive can exceed the FASB-prescribed value obtained using a maturity-reduced Black-Scholes-Merton model. But even without reducing the maturity, the executive could value the option at more than the Black-Scholes-Merton value. As is customarily done in the literature, we assume that the Black-Scholes-Merton values represent the firm cost. We then compute the ratio of option value with effort to firm cost and show the results in Table 4. These ratios are low in the cases of ten-year maturities, but they increase with shorter maturity options. This result is consistent with the previous analysis of the "negative" time value effect, or liquidity discount of the executive values. The Black-Scholes-Merton values are the values of European options. We know there is no liquidity discount for the non-tradable constraint in the Black-Scholes-Merton value. Therefore, the time value is positive and increases with maturity in the Black-Scholes-Merton formula. In contrast, the liquidity discount can dominate the positive time value in executive options. Hence, these cases are more likely to occur in options with longer maturities. The effect of effort could dominate the liquidity discount so that the option values of executive stock options would be higher than Black-Scholes-Merton values. Hence, the commonly-stated argument that that the Black-Scholes-Merton model gives too high a value for executive stock options is not generally correct. In some cases the Black-Scholes-Merton model can even *understate* the values of these instruments. This means that the executive would place a higher value on this illiquid instrument than would an outside investor place on a perfectly liquid version of the same option. This seemingly irrational result arises strictly because outside investors are price takers and executives are not. Executives have the ability to affect the payoffs of their options and that power has value that is not accounted for in standard approaches to executive option valuation.

4.3. Early Exercise

Now we allow the options to be exercisable prior to expiration. We focus on 10-year options that vest in zero, two, and four years. Using the binomial model, we summarize the values of the options after taking early exercise into account and the ratios of option values with effort to those without effort in Table 5.

First note that as expected, we find that the longer the vesting period, the lower the value of the option. In comparing the European option values in Table 2 with the American option values in Table 5, we see that, as expected, the ability to exercise early increases the values of the options regardless of whether effort is considered. Some of the differences are extremely large. For example, for the high volatility case, the 10-year European-style option is worth $0.48 with no effort and $0.99 with effort. Even with a four-year vesting period, that option is worth $9.88 with no effort tand $11.86 with effort. Consistent with intuition, the early exercise premium is

larger for longer maturities. In Table 5 we again see that all numbers in Panel A, with effort, are greater than those in Panel B, without effort. Panel C shows the ratio of values with effort to without. Even though the addition of early exercise reduces the difference between option value with effort and option value without effort , we still find many option values with consideration of effort more much higher than those without effort.

From the literature, two important factors that should substantially affect the decision to exercise early are the executive's stock holdings and his risk aversion. When executives are more risk averse or have high stock holdings, they may value the benefits of exercising now to reduce exposure to the stock at more than the value of future volatility. Thus, they may choose to exercise at somewhat lower stock prices. In the previous results with effort, however, we saw that elasticity, which proxies for executive quality, can play an important role in the determination of option values and this effect can dominate these two factors. To examine this question, we estimate liquidity premiums by comparing option values with effort but without early exercise to those with effort and with early exercise. In other words, the early exercise premium is a proxy for the liquidity premium. These results are shown for different elasticities, maturities, and vesting periods in Table 6.

We see in Panel C that as expected, the liquidity premium increases with maturity. It is also inversely related to the vesting period. We know that the liquidity premium can be viewed as either the cost of being unable to exercise a European option early or the benefit of being able to exercise an American option early. With a longer vesting period, this cost or benefit is lower because exercise is deferred. The liquidity premium is also directly related to elasticity meaning that there is a greater advantage to exercising early for higher quality executives. In other words, higher quality executives have the ability to make the stock perform well, so the ability to exercise early and capture that effect over having to wait until expiration is beneficial.

To verify the relation between the elasticity of stock price and the decision to exercise early, we compute the threshold price, which is the critical stock price for the decision to exercise early. We summarize these threshold prices with respect to the elasticity of stock price that implies different executive quality in Figure 2 for 10-year options. In Panel A we see that the threshold prices are positively related to the elasticity of stock price and, hence, executive quality. For example, in year 3, the executive with the lowest quality, $\delta = 0.1$, will exercise the options when the stock price is above \$58.58, but the executive with the highest quality, $\delta = 0.5$, will wait until the stock price is over \$73.73. This result is consistent with expectations. Suppose executives know their ability and the effort they exert. Holding the options longer can increase their expected wealth, which also increases their expected utility. Based on the higher expected

utility of continuing to hold the option, the threshold prices should be higher for more capable executives. The behavior, however, is not significantly different in the last year. From this result, we expect the executive who exerts more effort will exercise at a higher stock price. Therefore, the effect of the effort would interact with that of risk aversion or stock holdings on the behavior of early exercise, which is examined in the following analysis.

From the results in Section 4.1, we know the executive would exert more effort with higher stock holdings but less effort with higher risk aversion. Hence, the decision to exercise early with respect to different risk aversion should be similar with the finding in the literature without consideration of executive effort. The more risk-averse executive would exercise stock options at lower stock prices, after taking effort into account. In Panel B, we show the threshold stock prices for two different executive qualities (elasticities) of 0.25 and 0.50 and risk aversions of 0.0000005 and 0.00000075. First we see that the more risk-averse executive exercises at a lower stock price regardless of his quality. We also see that the lower quality executive with the same risk aversion exercises at a lower stock price regardless of risk aversion. The highest threshold stock price is for the less risk averse and higher quality executive, and the lowest is for the more risk averse and lower quality executive. Thus, higher quality executives will hold out for a higher stock price, and this effect is even greater the less risk averse they are. But lower quality executives may hold out for a higher stock price if they are less risk averse.

An existing result in the literature states that less diversified executives would exercise at a lower stock price compared with ordinary diversified investors, who are of course price takers. Because an executive can reduce his personal firm-specific risk by early exercise, he would do it as soon as the options are vested. But this result does not consider the executive's ability. From the previous analysis, we know that there is a counteracting effect between stock wealth and executive effort resulting from the stock-wealth ratio on the behavior of early exercise. The change in the threshold stock price with respect to the stock-wealth ratio is shown in Panel C. We consider two stock-wealth ratios, 40% and 50%, and two elasticities, 0.25 and 0.5. Think of these four cases as low exposure (40%) and low quality (0.25), low exposure (40%) and high quality (0.5), high exposure (50%) and low quality (0.25), and high exposure (50%) and high quality (0.5). For any maturity level and exposure, high quality executives will exercise at higher prices than low quality executives. For any quality, low exposure executives will exercise at higher prices than will high exposure executives. Quality seems to dominate exposure, however, as high quality and high exposure executives exercise at higher prices than low quality and low exposure executives.

5. Conclusions

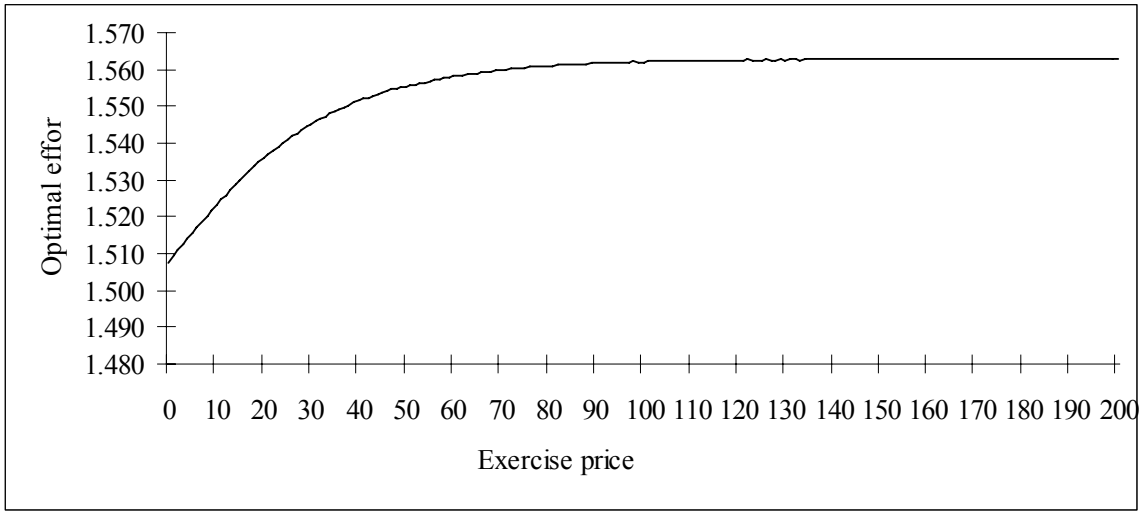
It has become widely accepted that the values of executive stock options are less, and often far less, than the values of traded options. But unlike ordinary investors, executives are not price takers. While it is true that executives cannot sell their options and liquidity is at least partially limited by vesting requirements, it is also true that executives have the ability to influence the payoffs of their options. Indeed the principal reason why executives are granted options is to motivate them to take actions that will increase the stock price. Thus, we should not value their options as though executives are merely passive participants in corporate performance.

In this paper we model the process by which executives of varying quality choose their optimal effort and how this factor translates into the values of stock options. We find that executive ability and the effort that executives undertake can greatly affect the valuation of their stock and options. Executive effort increases the values of the stock and options beyond the value that would be assigned under traditional executive stock option valuation models that assume that executives have no effect on the payoffs of their options. In fact, in a few cases in which short-term options are awarded, the values of stock options can even exceed their Black-Scholes-Merton values. In addition we find that consideration of executive effort can influence early exercise behavior, as higher quality executives exercise at a higher stock price. This result implies that executives may postpone their decision to exercise their options until their effort is reflected in the stock price.

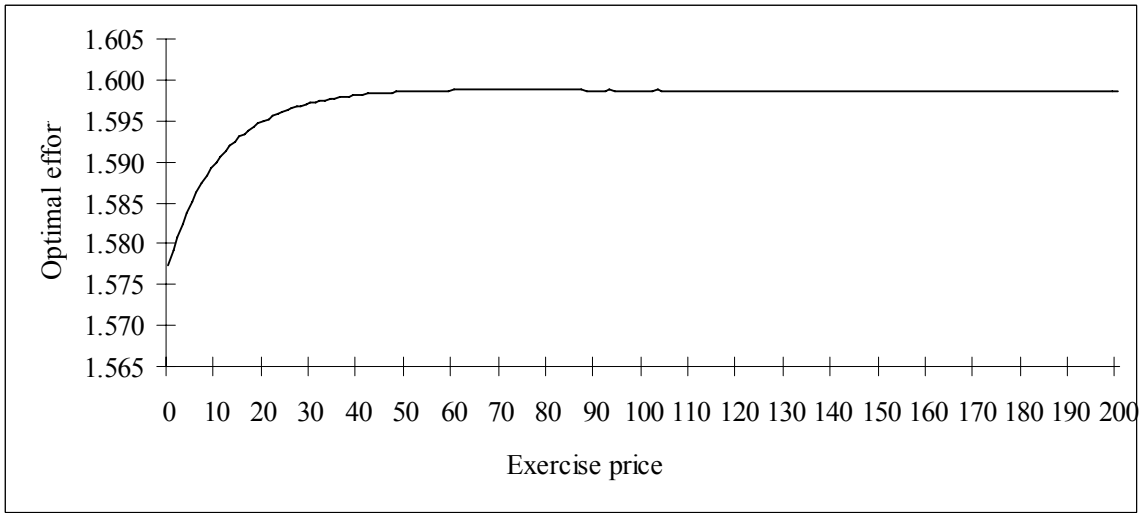
By incorporating the influence that executives have on the payoffs of options and the cost of executive effort, we do not treat executives as ordinary investors. It should not be surprising, therefore, that executives are far more willing to hold options than they are found to be when we use traditional models to value these options. Hence, executives value options more highly than they would if they were treated, as in most models in the literature, like ordinary investors. Recognizing their ability to influence the payoffs of their options, we must acknowledge that executives may well like their options even more than do outsider investors.

Figure 1. The Relationship between Optimal Effort and Exercise Price

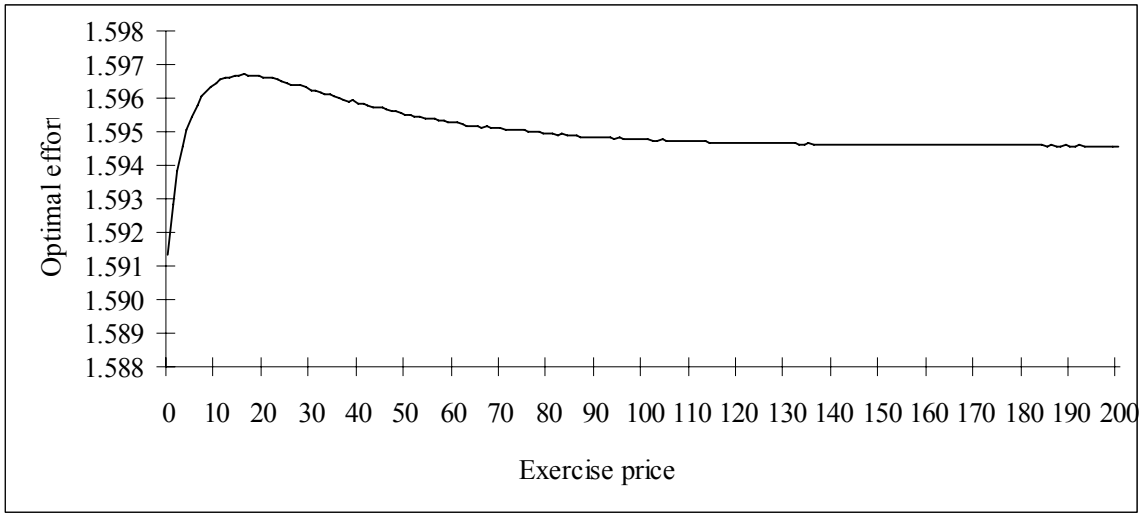
Panel A. Volatility = 30%



Panel B. Volatility = 50%



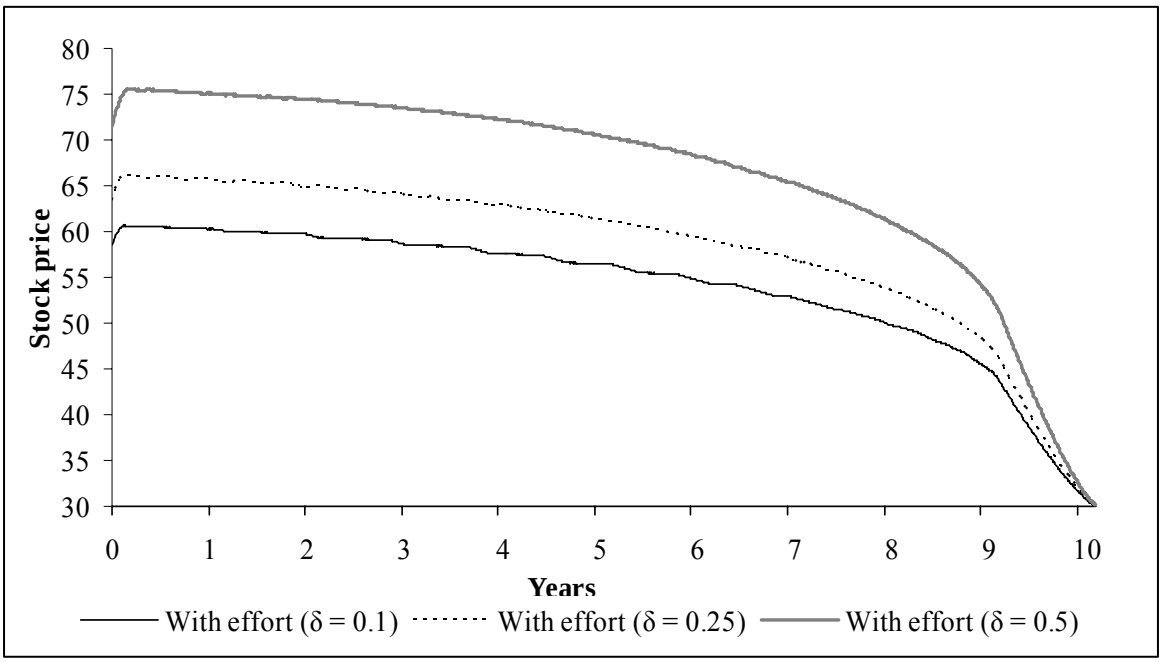
Panel C. Volatility = 70%



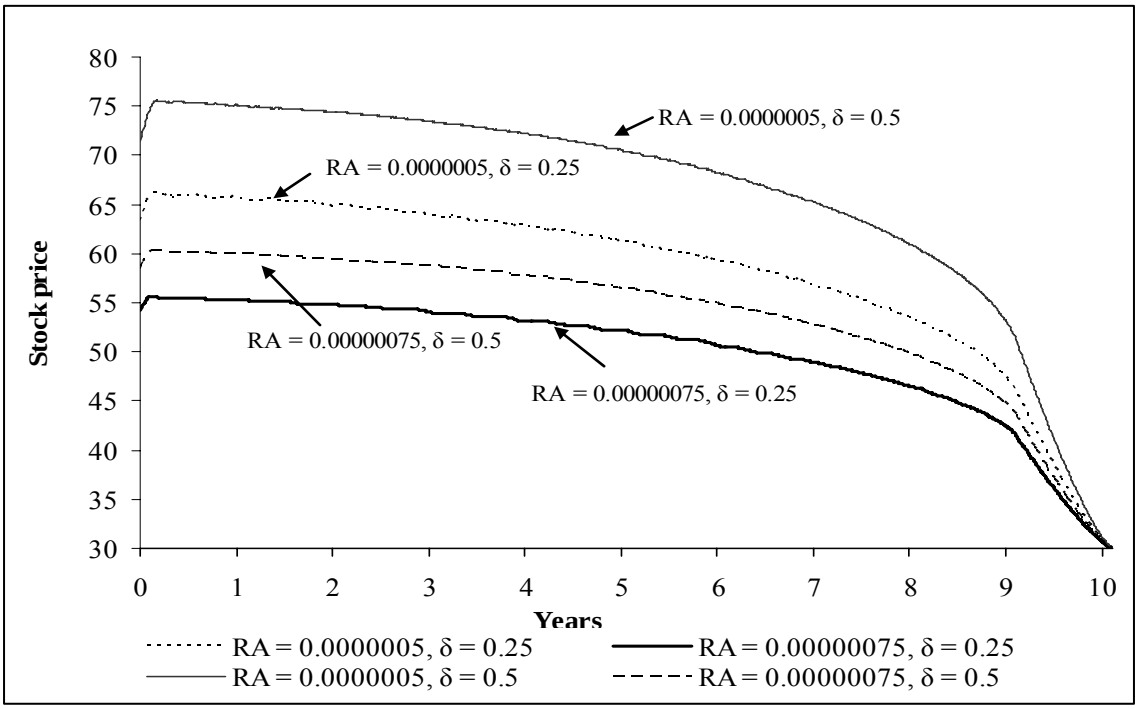
Expected utility is computed under the benchmark assumptions of maturity = 10 years, volatility = 50%, 30%, and 70%, expected market return = 12%, beta = 1, elasticity of stock price = 0.25, non-option wealth = $4 million, stock-wealth ratio = 40%, and coefficient of absolute risk aversion = 0.0000005. The exercise price changes $1 at a time from $0 to $60. The total Black-Scholes-Merton value of these options is $400,000 under different exercise prices. Therefore, the number of options is generated by using $400,000 divided by the Black-Scholes-Merton value under different exercise prices. The current stock price is $30.

Figure 2. Early Exercise Threshold Prices for Different Executive Quality, Risk Aversion, and Stock-Wealth Ratios

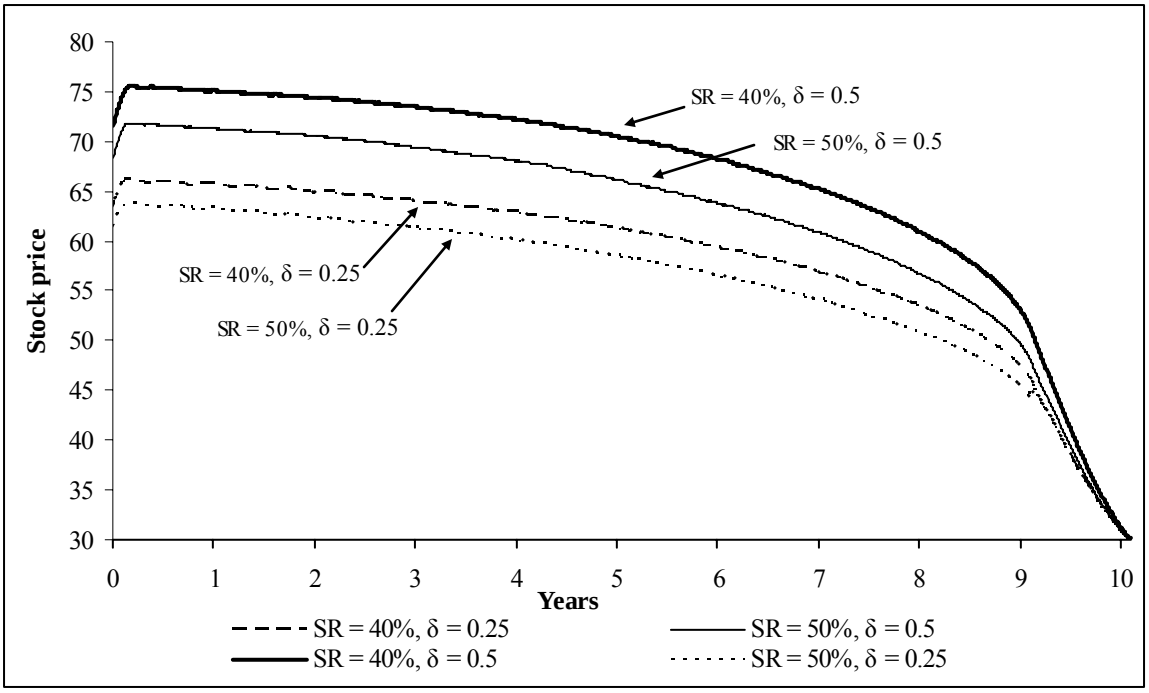
Panel A. Elasticity (Executive Quality)



Panel B. Risk Aversion



Panel C. Stock-Wealth Ratio



The threshold price is the critical price for the decision to exercise early. When the stock price is higher than the threshold price, the executive will exercise his options. Otherwise, the executive will hold these options and re-evaluate whether to exercise at the next time step. We change the elasticity of stock price but keep all other parameters as their benchmark values. Because the threshold price is a stepwise function, we smooth the curve by using linear interpolation. In addition, we also increase the number of time steps each year from 12 to 100.

Table 1. Optimal Executive Effort

The numbers in each cell are (vertically) the optimal effort when the current stock prices are $20, $30, and $40 respectively. When changing one parameter at a time, we keep all other parameters as their benchmark values. The bold values are the optimal effort of the at-the-money options. Benchmark values are shown only once because they do not change when elasticity, non-option wealth, stock-wealth ratio, number of options, and absolute risk aversion change. Benchmark values for these variables are the middle of the three vertical values for each corresponding variable in the left column.

	Lower value			Benchmark			Higher value		
	T = 10	T = 7	T = 5	T = 10	T = 7	T = 5	T = 10	T = 7	T = 5
Volatility (30%, 50%, 70%)	1.557	1.931	2.570	1.598	1.997	2.681	1.595	2.002	2.702
	1.547	**1.911**	**2.524**	**1.598**	**1.994**	**2.671**	**1.596**	**2.003**	**2.703**
	1.536	1.888	2.476	1.596	1.989	2.655	1.596	2.003	2.701
Elasticity (0.1, 0.25, 0.5)	1.541	1.905	2.527				1.602	1.973	2.577
	1.542	**1.906**	**2.530**				**1.598**	**1.962**	**2.547**
	1.542	1.906	2.529				1.593	1.949	2.517
Non-option wealth (2,000,000, 4,000,000, 6,000,000)	1.691	2.152	2.966				1.515	1.862	2.440
	1.688	**2.145**	**2.946**				**1.514**	**1.861**	**2.434**
	1.685	2.135	2.920				1.513	1.858	2.426
Stock-wealth ratio (30%, 40%, 50%)	1.563	1.975	2.629				1.636	2.020	2.739
	1.561	**1.972**	**2.615**				**1.635**	**2.017**	**2.732**
	1.559	1.965	2.596				1.633	2.012	2.719
Number of options (20,000, 40,000, 60,000)	1.599	1.998	2.682				1.598	1.997	2.680
	1.598	**1.996**	**2.676**				**1.597**	**1.992**	**2.665**
	1.597	1.993	2.667				1.595	1.985	2.645
Absolute risk aversion (0.00000025, 0.00000050, 0.00000075)	1.748	2.256	3.170				1.486	1.813	2.351
	1.747	**2.252**	**3.156**				**1.486**	**1.811**	**2.345**
	1.745	2.246	3.139				1.484	1.807	2.333

Table 2. Executive Option Values with and without Optimal Effort

The numbers in each cell are the option values when the current stock price is $30, which is an at-the-money option. When changing one parameter at a time, we keep all other parameters as their benchmark values. We use the optimal effort in the previous table to compute the option values in Panel A. When we change a parameter other than volatility, the Black-Scholes-Merton values are the same as those in the benchmark case in Panel C. Therefore, we report these values only once.

Panel A. Executive option value with optimal effort									
	Lower value			Benchmark			Higher value		
	T = 10	T = 7	T = 5	T = 10	T = 7	T = 5	T = 10	T = 7	T = 5
Volatility	$12.52	$17.89	$23.94	$2.97	$4.85	$7.32	$0.99	$1.71	$2.77
Elasticity	$1.88	$2.74	$3.71				$5.71	$10.07	$16.35
Non-option wealth	$7.01	$10.63	$15.04				$1.49	$2.49	$3.90
Stock ratio	$4.21	$6.61	$9.70				$2.20	$3.66	$5.66
Number of options	$3.52	$5.71	$8.56				$2.57	$4.22	$6.40
Absolute risk aversion	$9.18	$13.85	$19.47				$1.28	$2.16	$3.38
Panel B. Executive option value without optimal effort									
	Lower value			Benchmark			Higher value		
	T = 10	T = 7	T = 5	T = 10	T = 7	T = 5	T = 10	T = 7	T = 5
Volatility	$4.65	$5.11	$5.25	$1.31	$1.76	$2.17	$0.48	$0.74	$1.02
Elasticity	$1.31	$1.76	$2.17				$1.31	$1.76	$2.17
Non-option wealth	$2.91	$3.66	$4.24				$0.67	$0.93	$1.20
Stock ratio	$1.91	$2.49	$2.99				$0.93	$1.27	$1.60
Number of options	$1.54	$2.05	$2.51				$1.14	$1.54	$1.91
Absolute risk aversion	$3.61	$4.48	$5.11				$0.59	$0.83	$1.08
Panel C. Black-Scholes-Merton value									
	Lower value			Benchmark			Higher value		
	T = 10	T = 7	T = 5	T = 10	T = 7	T = 5	T = 10	T = 7	T = 5
Volatility	$15.64	$12.94	$10.69	$20.14	$17.32	$14.83	$23.75	$21.09	$18.55

Table 3. Ratios of Executive Option Values with Optimal Effort to Those without Optimal Effort

The numbers in each cell are the ratios of executive option values when the current stock prices are $20, $30, and $40 respectively. When changing one parameter at a time, we keep all other parameters as their benchmark values. The bold values are ratios of the at-the-money options. The ratios are computed as executive option value with effort divided by executive option value without effort. When we change a parameter other than volatility, the Black-Scholes-Merton values are the same as those in the benchmark case. Therefore, we report these ratios only once.

	Lower value			Benchmark			Higher value		
	T = 10	T = 7	T = 5	T = 10	T = 7	T = 5	T = 10	T = 7	T = 5
Volatility (30%, 50%, 70%)	3.23	4.61	6.74	2.48	3.09	3.94	2.20	2.52	3.08
	2.69	**3.50**	**4.56**	**2.26**	**2.76**	**3.38**	**2.05**	**2.31**	**2.71**
	2.42	3.00	3.72	2.11	2.50	2.95	1.90	2.15	2.44
Elasticity, (0.1, 0.25, 0.5)	1.39	1.56	1.76				5.12	7.14	10.11
	1.43	**1.56**	**1.71**				**4.34**	**5.74**	**7.54**
	1.38	1.48	1.60				3.86	4.91	6.28
Non-option wealth (2,000,000, 4,000,000, 6,000,000)	2.58	3.25	4.19				2.29	2.97	3.83
	2.41	**2.91**	**3.55**				**2.23**	**2.68**	**3.24**
	2.26	2.66	3.15				2.06	2.41	2.82
Stock ratio (30%, 40%, 50%)	2.39	2.98	3.80				2.56	3.19	4.08
	2.20	**2.65**	**3.25**				**2.37**	**2.89**	**3.53**
	2.07	2.43	2.88				2.19	2.59	3.05
Number of options (20,000, 40,000, 60,000)	2.50	3.13	4.01				2.46	3.06	3.89
	2.28	**2.79**	**3.41**				**2.25**	**2.75**	**3.35**
	2.13	2.52	2.98				2.10	2.48	2.94
Absolute risk aversion (0.00000025, 0.0000005, 0.00000075)	2.79	3.54	4.60				2.08	2.76	3.58
	2.54	**3.10**	**3.81**				**2.17**	**2.61**	**3.14**
	2.38	2.82	3.36				1.99	2.34	2.73

Table 4. Ratios of Executive Option Values with Effort to Black-Scholes-Merton Option Values

The numbers in each cell are the ratios of executive option values to Black-Scholes-Merton option values when the current stock prices are $20, $30, and $40 respectively. When changing one parameter at a time, we keep all other parameters as their benchmark values. The bold values are ratios that are greater than one. When we change a parameter other than volatility, the Black-Scholes-Merton values are the same as those in the benchmark case in Panel C. Therefore, we report these values only once.

	Lower value			Benchmark			Higher value		
	T = 10	T = 7	T = 5	T = 10	T = 7	T = 5	T = 10	T = 7	T = 5
Volatility (30%, 50%, 70%)	0.73	**1.43**	**2.67**	0.04	0.07	0.40	0.03	0.05	0.10
	0.80	**1.38**	**2.24**	0.07	0.10	0.49	0.04	0.08	0.15
	0.81	**1.32**	**2.00**	0.08	0.13	0.53	0.05	0.10	0.18
Elasticity, (0.1, 0.25, 0.5)	0.06	0.11	0.18				0.22	0.48	**1.02**
	0.09	0.16	0.25				0.28	0.58	**1.10**
	0.11	0.19	0.29				0.32	0.62	**1.12**
Non-option wealth (2,000,000, 4,000,000, 6,000,000)	0.31	0.58	**1.04**				0.04	0.09	0.17
	0.35	0.61	**1.01**				0.07	0.14	0.26
	0.35	0.61	0.96				0.10	0.18	0.32
Stock ratio (30%, 40%, 50%)	0.17	0.32	0.59				0.07	0.14	0.27
	0.21	0.38	0.65				0.11	0.21	0.38
	0.23	0.40	0.66				0.14	0.25	0.43
Number of options (20,000, 40,000, 60,000)	0.12	0.24	0.45				0.09	0.19	0.35
	0.18	0.33	0.58				0.13	0.24	0.43
	0.21	0.38	0.63				0.15	0.27	0.46
Absolute risk aversion (0.00000025, 0.0000005, 0.00000075)	0.40	0.75	**1.32**				0.03	0.07	0.15
	0.46	0.80	**1.31**				0.06	0.12	0.23
	0.48	0.81	**1.27**				0.08	0.16	0.27

Table 5. Executive Option Values with Consideration of Early Exercise

The numbers in each cell are the option values when the current stock price is $30, which is an at-the-money option, and the maturity is 10 years. When changing one parameter at a time, we keep all other parameters as their benchmark values. We use the optimal effort in the binomial model without consideration of early exercise to compute the option values in Panel A. We assume the options are exercisable after they are granted and use a monthly time step in the binomial model. When we change a parameter other than volatility, the Black-Scholes-Merton values are the same as those in the benchmark case. Therefore, we report these values only once.

Panel A. Executive option values with optimal effort and early exercise (1)

	Lower value			Benchmark			Higher value		
	0 Years	2 Years	4 Years	0 Years	2 Years	4 Years	0 Years	2 Years	4 Years
Volatility	$16.00	$15.97	$15.67	$11.59	$10.91	$9.26	$11.86	$10.04	$7.46
Elasticity	$9.75	$8.94	$7.23				$15.30	$14.79	$13.26
Non-option wealth	$10.78	$10.06	$8.39				$12.49	$11.85	$10.20
Stock ratio	$11.13	$10.42	$8.76				$12.13	$11.47	$9.81
Number of options	$15.13	$14.69	$13.30				$9.89	$9.04	$7.25
Absolute risk aversion	$12.71	$12.11	$10.54				$10.84	$10.09	$8.37

Panel B. Executive option values without optimal effort and with early exercise (2)

	Lower value			Benchmark			Higher value		
	0 Years	2 Years	4 Years	0 Years	2 Years	4 Years	0 Years	2 Years	4 Years
Volatility	$8.62	$8.49	$7.89	$8.78	$7.85	$6.13	$9.88	$7.89	$5.41
Elasticity	$8.78	$7.85	$6.13				$8.78	$7.85	$6.13
Non-option wealth	$8.12	$7.10	$5.38				$9.45	$8.60	$6.90
Stock ratio	$8.40	$7.42	$5.70				$9.15	$8.28	$6.57
Number of options	$10.25	$9.51	$7.94				$7.98	$6.90	$5.10
Absolute risk aversion	$9.35	$8.52	$6.88				$8.34	$7.33	$5.56

Panel C. Ratio of executive value with effort to that without effort (1)/(2)

	Lower value			Benchmark			Higher value		
	0 Years	2 Years	4 Years	0 Years	2 Years	4 Years	0 Years	2 Years	4 Years
Volatility	1.86	1.88	1.99	1.32	1.39	1.51	1.27	1.33	1.38
Elasticity	1.11	1.14	1.18				1.88	2.01	2.16
Non-option wealth	1.33	1.42	1.56				1.31	1.36	1.42
Stock ratio	1.32	1.40	1.54				1.38	1.44	1.51
Number of options	1.48	1.54	1.67				1.38	1.44	1.51
Absolute risk aversion	1.36	1.42	1.53				1.29	1.34	1.40

Table 6. Liquidity Premium with Respect to Elasticity of Executive Effort

The numbers in each cell in Panels A and B are the option values when the current stock price is $30, which is an at-the-money option. The options in Panel A are European-type options with different time to maturity. Those in Panel B are American-type options, and we assume different vesting periods of zero, two, and four years. The numbers in Panel C are liquidity premiums, which are the values with early exercise (Panel B) minus those without early exercise (Panel A). Except the elasticity, all other parameters are maintained at their benchmark values.

Panel A. Option value with effort but no early exercise (1)									
	T = 10	T = 7	T = 5	T = 10	T = 7	T = 5	T = 10	T = 7	T = 5
	δ = 0.1			δ = 0.25			δ = 0.5		
Option value	$1.88	$2.74	$3.71	$2.97	$4.85	$7.32	$5.71	$10.07	$16.35

Panel B. Option value with effort and early exercise consideration (2)									
	T = 10	T = 7	T = 5	T = 10	T = 7	T = 5	T = 10	T = 7	T = 5
	δ = 0.1			δ = 0.25			δ = 0.5		
0 year vesting	$9.75	$9.34	$8.99	$11.59	$11.99	$12.71	$15.30	$17.74	$21.57
2 years vesting	$8.94	$8.41	$7.91	$10.91	$11.28	$11.96	$14.79	$17.27	$21.13
4 years vesting	$7.23	$6.38	$5.37	$9.26	$9.29	$9.38	$13.26	$15.39	$18.68

Panel C. Liquidity Premium (2) – (1)									
	T = 10	T = 7	T = 5	T = 10	T = 7	T = 5	T = 10	T = 7	T = 5
	δ = 0.1			δ = 0.25			δ = 0.5		
0 year vesting	$7.88	$6.61	$5.28	$8.62	$7.14	$5.39	$9.59	$7.66	$5.22
2 years vesting	$7.06	$5.68	$4.20	$7.94	$6.43	$4.64	$9.09	$7.20	$4.78
4 years vesting	$5.35	$3.64	$1.66	$6.29	$4.44	$2.06	$7.56	$5.32	$2.33

Appendix I. The Estimation of Optimal Effort and Executive Option Value

There are two steps in the simulations. First, we find optimal executive effort by solving Equation (6). Second, applying this solution in the certainty equivalent approach in Equation (8), we find the option value. We now describe the procedure used to compute optimal effort and option value.

I.A. Optimal Effort

To determine the optimal effort, which is the solution for η in Equation (6), we must first specify the terminal stock price. Following Hall and Murphy (2002), we assume the distribution of stock prices in T years is lognormal with volatility σ and expected return $[r_f + \beta(r_m - r_f) + \eta - \sigma^2/2]T$. Applying the solution for the after-effect stock price in Section 3.1.2, we have

$$S_T^* = S_0 e^{\left(\alpha - \frac{1}{2}\sigma^2 + \eta\right)T + \sigma z_T}.$$

We then construct the distribution of possible terminal stock prices with effort by generating random values of the Weiner process, z_T, in the range of -6 to +6 standard deviations and then convert to the stock price following the above equation. Inserting the terminal stock price with effort into the terminal wealth, W_T, in Section 3.1., we have the distribution of possible terminal wealth that reflects cash of c, m shares of stock, and n stock options. Finally, given the negative exponential utility function with coefficient of absolute risk aversion, ρ, and all necessary parameters, r_f, r_m, β, σ, T, S_0, K, c, m, n, δ, and ρ, we can solve for η in Equation (6) by using the first-order condition, which is

$$\sum_0^\infty U'(W_T) f\left(S_T^*\right) - \left(e^{\eta T/\delta} - 1\right) e^{\eta T/\delta} \frac{T}{\delta} = 0,$$

where $f(S_T^*)$ is the density of S_T^*. Because there is no closed-form solution for η in the above condition, we simulate with different values for η until the first-order condition is less than 0.0000001. The first η that meets this criterion is the optimal executive effort reported in Table 1 under given parameters.

I.B. Option Value

To find the option value using the certainty equivalent approach in Equation (8),

$$\int_0^\infty U\left(\left(c + CE\right)\left(1 + r_f\right)^T + mS_T^{1*}\right) f\left(S_T^{1*}\right) dS_T^{1*} - \frac{1}{2}\left(e^{\eta_1 T/\delta} - 1\right)^2$$
$$= \int_0^\infty U\left(c\left(1 + r_f\right)^T + mS_T^{2*} + nMax\left(S_T^{2*} - K, 0\right)\right) f\left(S_T^{2*}\right) dS_T^{2*} - \frac{1}{2}\left(e^{\eta_2 T/\delta} - 1\right)^2,$$

we need the optimal effort, which is a function of η_2, in the first step. Based on the optimal effort, we find the possible terminal after-effort stock prices, S_T^{2*}, and the expected utility net of the

disutility of effort, which is the right-hand side of Equation (8). The certainty equivalent amount, *CE*, is the total value of *n* stock options. We need to solve for *CE/n* that is the value of one option. Applying the same method to find the optimal effort with *m* shares of stock but zero stock options, we find the optimal effort without options, which is a function of η_1. Based on the optimal effort without options, we find the possible terminal after-effort stock prices, S_T^{1*}. Finally, we compute the expected utility on the left-hand side of Equation (8) by changing *CE* until

$$\frac{\int_0^\infty U\left(\left(c+CE\right)\left(1+r_f\right)^T + mS_T^{1*}\right) f\left(S_T^{1*}\right) dS_T^{1*} - \frac{1}{2}\left(e^{\eta_1 T/\delta} - 1\right)^2}{\int_0^\infty U\left(c\left(1+r_f\right)^T + mS_T^{2*} + nMax\left(S_T^{2*} - K, 0\right)\right) f\left(S_T^{2*}\right) dS_T^{2*} - \frac{1}{2}\left(e^{\eta_2 T/\delta} - 1\right)^2} - 1 < 0.00001 \ .$$

The first *CE* that satisfies this criterion is the total value of *n* options. The value of one option is *CE/n*.

Appendix II. Illustration of the Trade-Off between Effort and Exercise Price

Figure 2 relates optimal effort to exercise price. In this appendix we explain why the relationship is as it appears. To relate exercise price to effort, we must relate exercise price to expected utility. In the expected utility maximization model, we need to know only the terminal wealth to compute the expected utility. Therefore, the executive's expected utility is a linear combination of the utilities from three cases of the option's payoff, in-the-money (ITM), at-the-money (ATM) and out-of-the-money (OTM). Based on the assumptions about the executive's utility and terminal wealth in Section 3, the terminal wealth and utility are follows:

$$W_T = \begin{cases} c\left(1+r_f\right)^T + \left(m+n\right)S_T^* - nK & ITM \\ c\left(1+r_f\right)^T + mS_T^* & ATM \text{ and } OTM \end{cases},$$

$$U_T = -\frac{1}{\rho}e^{-\rho W_T} - \frac{1}{2}\left(e^{\eta T/\delta} - 1\right)^2 .$$

The partial derivative of the utility with respect to the incremental expected return is

$$\frac{\partial U_T}{\partial \eta} = \begin{cases} e^{-\rho W_T}\left(m+n\right)S_T^* T - \left(e^{\eta T/\delta} - 1\right)e^{\eta T/\delta}\frac{T}{\delta} & ITM \\ e^{-\rho W_T} mS_T^* T - \left(e^{\eta T/\delta} - 1\right)e^{\eta T/\delta}\frac{T}{\delta} & ATM \text{ and } OTM \end{cases}.$$

Let us align the possible terminal stock prices into *h* possible outcomes ranked in ascending order with probability p_i and the case of at-the-money occurs in the outcome $a < h$ with exercise price $K = K_1$. The expected utility is

$$E\left(U_T^{K_1}\right) = \sum_{i=1}^{a-1} p_i U_{OTM}^{K_1} + \sum_{i=a+1}^{h} p_i U_{ITM}^{K_1} + p_a U_{ATM}^{K_1} .$$

The partial derivative of $E\left(U_T\right)$ with respect to incremental expected return is

$$\frac{\partial E\left(U_T^{K_1}\right)}{\partial \eta} = \sum_{i=1}^{a-1} p_i \frac{\partial U_{OTM}^{K_1}}{\partial \eta} + \sum_{i=a+1}^{h} p_i \frac{\partial U_{ITM}^{K_1}}{\partial \eta} + p_a \frac{\partial U_{ATM}^{K_1}}{\partial \eta} .$$

This is the marginal effect of the executive effort. The same equation holds for $K = K_2 > K_1$, which is

$$\frac{\partial E\left(U_T^{K_2}\right)}{\partial \eta} = \sum_{i=1}^{b-1} p_i \frac{\partial U_{OTM}^{K_2}}{\partial \eta} + \sum_{i=b+1}^{h} p_i \frac{\partial U_{ITM}^{K_2}}{\partial \eta} + p_b \frac{\partial U_{ATM}^{K_2}}{\partial \eta} .$$

From the above equation, we know the at-the-money case occurs in the outcome $b < h$, $b > a$. Then, the effect of increasing the exercise price on the marginal expected utility with respect to incremental expected return is

$$\frac{\partial E\left(U_T^{K_2}\right)}{\partial \eta} - \frac{\partial E\left(U_T^{K_1}\right)}{\partial \eta} = \sum_{i=1}^{a-1} p_i \left(\frac{\partial U_{OTM}^{K_2}}{\partial \eta} - \frac{\partial U_{OTM}^{K_1}}{\partial \eta} \right) + p_a \left(\frac{\partial U_{OTM}^{K_2}}{\partial \eta} - \frac{\partial U_{ATM}^{K_1}}{\partial \eta} \right) + \sum_{i=a+1}^{b-1} p_i \left(\frac{\partial U_{OTM}^{K_2}}{\partial \eta} - \frac{\partial U_{ITM}^{K_1}}{\partial \eta} \right)$$
$$+ p_b \left(\frac{\partial U_{ATM}^{K_2}}{\partial \eta} - \frac{\partial U_{ITM}^{K_1}}{\partial \eta} \right) + \sum_{i=b+1}^{h} p_i \left(\frac{\partial U_{ITM}^{K_2}}{\partial \eta} - \frac{\partial U_{ITM}^{K_1}}{\partial \eta} \right)$$

The first two terms on the right hand side are equal to zero. The third and fourth terms are negative under our parameter sets and they result from increasing the exercise price. Therefore, we define that both terms have a negative effect from increasing the exercise price. The last term on the right-hand side is positive, and it results from the reduction of terminal wealth following the increase in exercise price. We define this term as a positive effect from the wealth reduction. When the exercise price is very low, such as $K_1 = 1$ and $K_2 = 2$, a and b are very small. Therefore, the positive effect in the last term dominates the negative effect from increasing the exercise price. When the exercise price, however, is much higher than the current stock price, then the probability of expiring in-the-money is decreasing. This means that the total value of the last term becomes smaller. Hence, the negative effect in the third and fourth terms dominates the positive effect in the last term. When the marginal effect of executive effort decreases, the executive would reduce his effort to achieve the maximization of expected utility.

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