Weather Radar Theory and Practice

Signal Processing Assignment #2
Reflectivity Factor and Statistical Properties of Weather Radar Data

Due: October 31, 2007 by the end of class

1 Learning Objectives

The following are the learning objectives for this assignment:

- For students to learn how *reflectivity factor* relates to received radar power.
- For students to learn how to conduct a rudimentary radar calibration.
- For students to learn the challenges of using reflectivity factor for estimation of rainfall rate and its associated uncertainties.
- For students to investigate the statistical characteristics of weather radar signals, including probability density functions and correlation structure.

2 Introduction

The *Weather Radar Equation* describes the relationship between transmitted power of the radar and the expected returned power, under certain assumptions. Given this equation, it is possible to design radar characteristics to obtain particular levels of sensitivity. The weather radar equation is given by

$$E[P_r] = \frac{P_t g^2 \lambda^2 \eta c \tau \pi \theta_1^2}{(4\pi)^3 r^2 l^2 16 \ln 2} \tag{1}$$

where P_t, g, η, τ, θ_1, and l are the transmit power, antenna gain, reflectivity, pulse length, beamwidth, and loss factor, respectively. One of the more important parameters is the reflectivity, which is given by

$$\eta = \frac{\pi^5}{\lambda^4} |K_w|^2 \int_0^\infty D^6 N(D)\, dD \tag{2}$$

where $N(D)$ is the dropsize distribution and K_w is the complex dielectric factor of water. The integral factor is defined as the *Reflectivity Factor*

$$Z = \int_0^\infty D^6 N(D)\, dD \tag{3}$$

and is usually given in units of dBZ relative to 1 mm^6 m^{-3}. Z can vary from negative values to as high as 60 dBZ for large hail.

From the weather radar equation, it is possible to convert P_r to reflectivity factor in dBZ by the following simple manipulation

$$10\log[Z] = 10\log[P_r] + 20\log[l] + 20\log[r] - 10\log[C] \tag{4}$$

where C is called the system calibration factor. As part of this assignment, you will further investigate the PAR data from June 2005, and will attempt to estimate Z. In addition, you will use your estimates of reflectivity factor to estimate rainfall rate using standard $Z - R$ relationships.

In addition to your investigation of reflectivity factor, you will study the statistical properties of the weather radar signals. Due to the random nature of weather radar signals, it is assumed that the in-phase (I) and quadrature (Q) signals are uncorrelated and Gaussian distributed. Further, it is easily justified that the phase of the signal is uniformly distribution over $\pm\pi$. Exploiting these two results, the distributions of the amplitude and power can be shown to have Rayleigh and exponential distributions, respectively. Using data from the PAR, you will validate these assumptions by estimating the probability density functions for these parameters. Finally, the temporal correlation characteristics will be studied by calculating the the auto-correlation function (ACF) for actual data under differing meteorological conditions.

3 Hands-On Activities

1. If you have not already done so, acquire the PAR data (three files in `mat` format) from the following website using the login (`wxradarstudent`) and password (`metrece5673`).

   `http://arrc.ou.edu/~rpalmer/wxradartheory/`

2. In the previous module, you may have used loops to estimate the power for each gate and radial. Especially in Matlab, this is not a computationally efficient method of programming. Given that Matlab uses vectors as its basis, it is possible to implement many loop operations by using matrix multiplications, for example. Edit your program from the previous module and use the following code for the estimation of power.

   ```
   pow = real(mean(X(:,:,1:num_pulses).*conj(X(:,:,1:num_pulses)),3));
   pow_dB = 10*log10(pow);
   ```

   Are the results the same? Do you notice any speed increase when running the two versions of the program? You can check this more accurately by using the command `cputime`.

3. You will now take the *power* estimates from the previous task and convert them to *reflectivity factor* in units of dBZ using equation (4). As discussed in class, you will need to compensate the power estimates for both range and system calibration. For the S-band PAR, it is justified to ignore losses due to attenuation. The system calibration offset can be obtained by comparing your power values (corrected for range) to the reflectivity factor from a well-calibrated WSR-88D radar. The most convenient site would be KTLX in Oklahoma City. I have provided KTLX images on `http://arrc.ou.edu/~rpalmer/wxradartheory/` for the same dates and times as the PAR data. What system calibration offset did you calculate in dB? Was it consistent for all three data sets? If not, why not? Show plots to justify your discussion.

4. Even with your *calibration* procedure using KTLX, it is likely that the reflectivity factor values will have differences when compared to KTLX over the entire domain. Why would this be the case? What polarizations are used with the PAR and WSR-88D? Describe regions of storms where you would expect the largest difference.

5. Using the reflectivity factor map for one of the three PAR data sets (your choice), provide the estimated rainfall rate map in mm hr^{-1}. Use at least two different $Z - R$ relationships and compare the results.

6. Using the `xcorr` command in Matlab, calculate and plot the ACF for several examples of I/Q signals from the data set. Recall from the example in class that you may need to *detrend* the data before estimating the ACF. In the previous module, you plotted the I/Q signals for cases of weather, clutter, noise, etc. You can use that code and calculate the ACF for these same I/Q examples. Compare and contrast the decorrelation times for these different examples. Which type of signal should decorrelate faster?

7. Using the `plot` command, with point option

   ```
   plot(real(vtmp),imag(vtmp),'.')
   ```

   where `vtmp` is the I/Q sequence, plot example *scatter* diagrams of $I + jQ$. Given that there are only 50 pulses in each signal, it may be necessary to concatenate the signals from several adjacent pixels to get a representative plot. Are the signals correlated? How can you tell from the scatter diagram? It is important to have the same scale on the I and Q axes for comparison. You can use the commands

   ```
   axis([-2000 2000 -2000 2000]);
   axis square;
   ```

   where 2000 is simply the upper/lower limits on the axes.

8. You will now use the `hist` command to estimate the probability density functions (pdf) for various products of the radar signal. As mentioned in class, the I and Q signals are assumed to have a Gaussian pdf. Verify this assumption and provide representative examples.

9. Calculate the amplitude $|V|$, phase γ, and power $P = |V|^2$ for the same examples as above. Estimate and plot the pdfs for each parameter. As discussed in class, amplitude $|V|$, phase γ, and power $P = |V|^2$ should have Rayleigh, uniform, and exponential pdfs, respectively. Is this the case? Compare various parts of the observation region.

Graduate Students Only

1. In contrast to rainfall *rate*, hydrologists are often interested in the total volume of water (m^3) which reaches the ground, over a certain region. Over the entire 90° region observed by PAR, estimate the *total accumulated water* that reached the ground in units of m^3 for a 10 min period. Describe any assumptions that must be made for this estimate to be valid.

2. Compare and contrast the PAR rainfall rate estimates to surface station (rain gauge) measurements near the PAR observation region. Provide a plot of this comparison. Give detailed explanations for possible sources of error in this comparison and comment on improved methods of comparison. Would it be possible to use rain gauge data to calibrate the reflectivity factor of the radar? Suggest a procedure.